

# Space Harmonics Modelling and Analysis of a 3 Phase Asymmetrical Unevenly Chorded Winding

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## Abstract

By modifying the design specifications, a higher energy-efficiency can be attained and undesirable operational conditions within the induction motor can be mitigated. When analyzing motor design and torque generation mechanism, it becomes evident that, despite a sinusoidal supply voltage and frequency, variations occur at each angular position within the motor due to the influence of its geometric structure. This phenomenon, stemming from the distribution of windings, gives rise to a structural feature known as space harmonics. Previous studies showed that it is possible to reduce or eliminate, the disturbing effects of space harmonics by structural changes. This study focuses on the possibility of application of an asymmetrical spaced stator slots with chorded double-layer 3-phase winding which is open to accommodating different numbers of turns from different phases in the same slot. The calculations were performed and results are shared for the impact of slot and winding distribution on space harmonics.

## 1. Introduction

Induction motors have enjoyed widespread adoption for an extended period due to their facile manufacturability, cost-effective nature, and straightforward controllability. Various applications, encompassing domains such as motion systems, electric vehicles, space missions, cranes, elevators, and more, necessitate the deployment of induction motors. In the realm of electric motor typology, paramount considerations include high efficiency, controllability, and robustness across diverse operating conditions. It is on account of these aforementioned attributes that induction motors find favor in these applications.

By reconfiguring the design specifications of induction motors, one can attain motion solutions that are more energy-efficient and rectify undesired operational anomalies within the motor. Upon scrutinizing motor design and torque generation, it becomes apparent that, notwithstanding a sinusoidal supply voltage and frequency, fluctuations manifest at distinct angular positions within the motor, a phenomenon stemming from the geometric structure's influence on electrical behavior. This situation, arising from co design and winding distribution, gives rise to a structural facet termed "space harmonics," along with the associated ramifications [1-4].

The recognition of space harmonics' influence became evident several decades following the inception of induction motors. Given that the presence of these harmonics is contingent upon the specific structural attributes of the machine, their elimination while preserving the fundamental operation proves to be an

insurmountable challenge. Previous investigations into these perturbing effects [5, 6] have predominantly centered on altering the conventional arrangement of windings [5-7] or manipulating the distribution of phase conductors along the stator core, while retaining the same slot configuration and permitting a designated variation in the number of conductors within the slots [8, 9]. Additionally, a study delved into skew optimization was achieved by using three-dimensional analytical structure and space harmonics were eliminated in concentrated windings for induction machines [10-12]. Moreover, a study represented into the use of a fractional slot winding for a permanent magnet AC machine stator, employing two different slot shapes, without disrupting the phase distribution [6].

In order to demonstrate the effects of slot arrangements in a distributed structure, slot positions were provided for an asymmetric spaced slot configuration that not having equal angles between each slot for a 36-slot 3-phase reference induction motor [13] and double layer structure is provided with arbitrary chording from reference motor for two different designs. Additionally, in the second winding layer, the windings were modeled for a distributed and chorded winding slot structure to calculate space harmonics using MATLAB. This allowed to illustrate the impact on the space harmonics related to a distributed, chorded and multi-level winding having different numbers of turns in each layer belonging to different phases.

## 2. Space Harmonics Expression

When analyzing the machine structure and torque generation, it becomes evident that variations occur at different positions within the machine, despite the sinusoidal supply voltage and frequency, owing to its geometric structure.

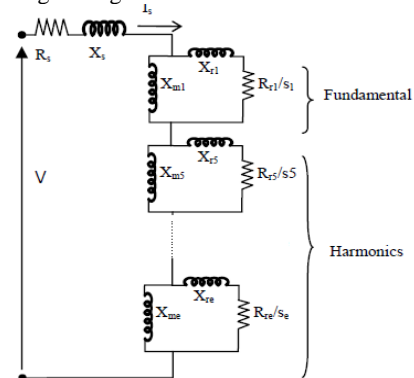


Fig. 1. Equivalent circuit for space harmonics.

An equivalent circuit, as illustrated in Fig. 1, has been developed to incorporate the space harmonic components and is employed for the analysis of their harmonic influences. Unlike the arrangement for time harmonics, the harmonic branches within the circuit are connected in series, given that space harmonics share a uniform frequency [2]. This equivalent circuit approach paves the way for the calculation of separated adverse effects of space harmonics.

When explaining the behavior of an AC electric motor, the ampere-turn distribution of the machine comes to the forefront. The mathematical representation of the ampere-turn description involves the summation of harmonics. To calculate a specific harmonic component, it is necessary to establish a conductor distribution function and subsequently derive harmonics based on the position ( $\theta$ ) and time ( $t$ ). The ampere-turn calculation can be expressed by the multiplication of these harmonics by the current magnitude. Due to variations in position and current, these harmonics may assume different values at distinct locations within the machine. The total ampere (i)-turn ( $N$ ) for R, S, T phases can be computed using Eq. 1.

$$F_{Total}(\theta, t) = N_R(\theta)i_R(wt) + N_S(\theta)i_S(wt) + N_T(\theta)i_T(wt) \quad (1)$$

To compute these space harmonics from ampere-turn calculations, (2), (3) and (4) specify the use of the ampere-turn function  $F$ , employing the Fourier series approach. The air-gap mmf function with the specified ampere-turn values can be decomposed into time-independent positional values of space harmonics using the Fourier series. As a result, these coefficients signify the impact of various harmonic orders, contingent upon ampere-turn distribution considering the location of each slot ( $a_i$ ) and the harmonic order 'e' where 'k' represents the order of different winding layers.

$$f_{total}(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos(nwt) + b_n \sin(nwt)) \quad (2)$$

$$a_e = \frac{1}{e\pi} \sum_{i=1}^{2q-1} (F_{1i}(\sin(ea_{1(i+1)}) - \sin(ea_{1i}))) + (F_{2i}(\sin(ea_{2(i+1)}) - \sin(ea_{2i}))) + \dots + (F_{ki}(\sin(ea_{k(i+1)}) - \sin(ea_{ki}))) \quad (3)$$

$$b_e = \frac{1}{e\pi} \sum_{i=1}^{2q-1} (F_{1i}(\cos(ea_{1(i+1)}) - \cos(ea_{1i}))) + (F_{2i}(\cos(ea_{2(i+1)}) - \cos(ea_{2i}))) + \dots + (F_{ki}(\cos(ea_{k(i+1)}) - \cos(ea_{ki}))) \quad (4)$$

### 3. Space Harmonics Calculations

A previously studied and optimized 3-phase induction motor stator core having unevenly distributed asymmetrical slot positions was chosen as reference in this study, but the conductors were placed to the slots evenly as a single layer winding different than the previous study. [14-16]. The reference motor has 36 slots and 2 poles. Each phase has 6 coils and all phase spreads intersperses to each other. The mmf distribution was formulated and solved by using a routine developed in MATLAB. First the mmf distribution created by this winding was analyzed and then the winding was converted to a double-layer winding. In the

second part of the study, firstly 5 and then 10 conductors from each slot were shifted to the nearest and closest slot to shorten the coils. By this way, all coils were made chorded and the newly distributed windings were then analyzed separately in terms of mmf harmonic spectrum for the possibility of the application of chording to an asymmetrically distributed winding. No numerical optimization was targeted and but only the results for chording effects were. In short, only the effect of chording the coils with the same number of conductors, but with different coil pitches, are taken into account.

In the analysis, for the three mentioned windings, mmf harmonics spectra for the first 50 existing harmonics, excluding the triple (non-effective) and even harmonics (absent) were considered. To calculate the harmonic coefficients of the windings, slot distribution of the winding combination was obtained. Winding positions of the 3-phase asymmetric slot motor that is not having equal angular distances between each stator slot is presented in Fig. 2. The asymmetrical winding distribution in single phase bands and the symmetrical structure between phases can also be seen. The positions of the first three slots are [ $8^\circ$   $25^\circ$   $42^\circ$ ] and the numbers of turns per slot are [30 30 30], then the rest goes symmetrically.

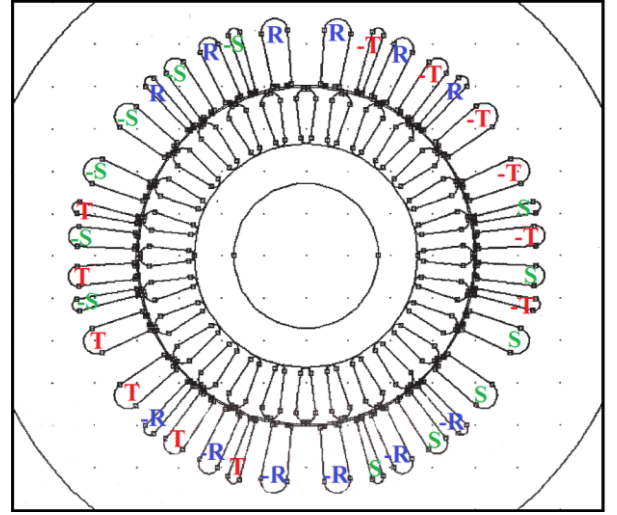


Fig. 2. Winding positions for 3-phase motor with asymmetric spaced slots and distributed winding.

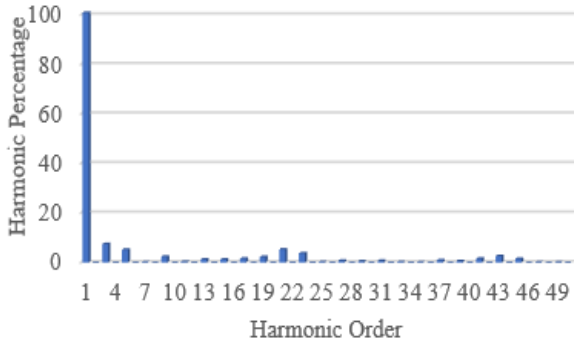
In order to illustrate the effects on space harmonics, the harmonic coefficients of the reference motor were initially computed within the mathematical framework provided. As an illustrative example, it was calculated with the assumption that a current of 1A is passing through each turn and there would be a peak ampere-turn value of 90. The distribution of the Ampere-turn as a step function and its calculated harmonic coefficients are presented in Fig. 3 and Fig. 4, respectively.

To initialize a new technique for further elimination of space harmonics than that of the previous studies [13-16], chording the coils by shortening the coil-pitches by the same number of turns for all coils. This needs the winding to be a double-layer winding. The difference of this design is the shortening distance for all coils are all different since the distribution in a quarter is asymmetrical for each phase. The phase-slot-layer combination is given in Fig. 5. 5 turns in each slot among evenly distributed 90 turns in slots per quarter wave were shifted to the previous slot, of which can be belonging to another phase as given in Fig. 5. The originality of the study is chording the coils by dividing the

saved slot space into two while all chorded coils can have different numbers of turns in different layers that can be different from other coils where all coil-pitches are also different from each other. The impact of the space harmonics created by this novel winding was then calculated.



**Fig. 3.** Ampere-turn step distribution of reference motor (180 turns per phase per pole-pair, 90 A-turns peak value).

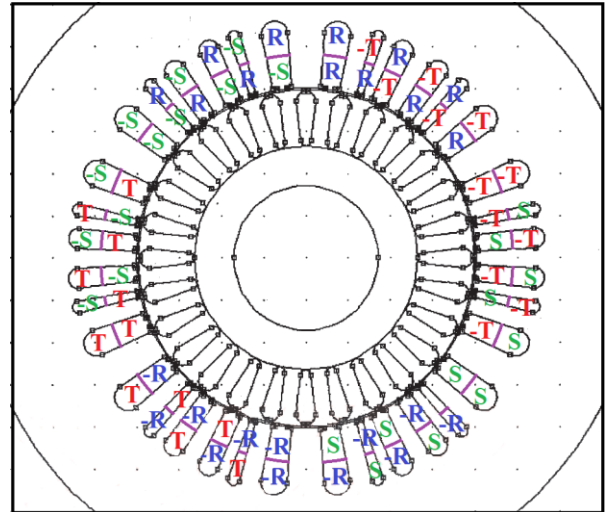


**Fig. 4.** Normalized harmonic MMF spectrum of reference motor.

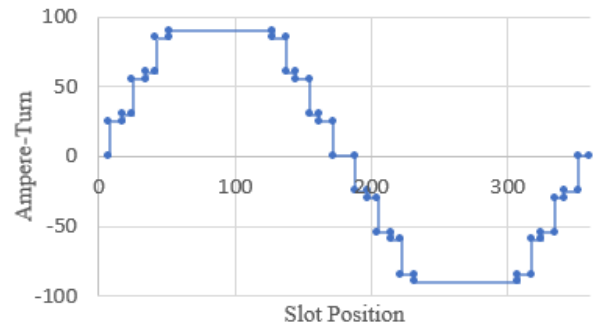
As can be seen in Fig. 6, this increased the numbers of steps in the mmf wave resulting the mmf to become more sinusoidal. The harmonic content of the 1<sup>st</sup> chorded winding (without any optimization) obtained by the created MATLAB routine is given in Fig. 7.

Same action was taken for chording all coils by 10 turns to visualize the change in mmf distribution and same technique and created MATLAB routine were used for the slot and winding distribution given in Fig. 5. The mmf distribution and harmonic content for the double-winding with 10 turns moved to the previous slot are given in Fig. 8, Fig. 9, respectively.

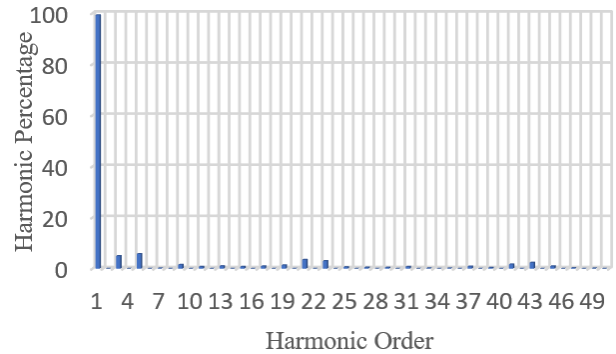
The reduced harmonic impacts for various orders of space harmonics are illustrated in Table 1 and the numerical change of harmonics in the spectra are also presented in comparison in Table 2, along with a comparison of harmonic coefficients across all designs. It is better to remember that all phases accommodated 180 turns and the peak value of mmf functions are all 90 A-turns.



**Fig. 5.** Winding positions for 3 phase motor with asymmetric spaced slots for double layer chorded 1<sup>st</sup> and 2<sup>nd</sup> design.



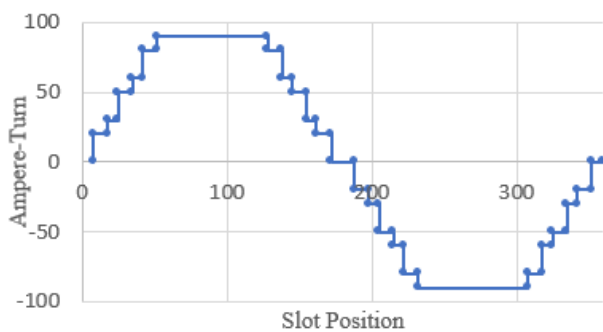
**Fig. 6.** Ampere-turn step distribution of asymmetric winding double layer chorded 1<sup>st</sup> design (5-turns were shifted).



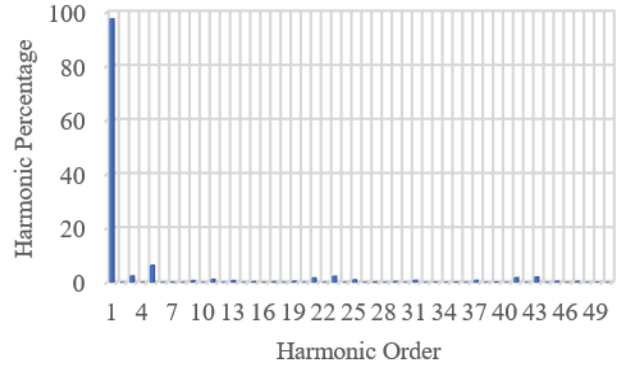
**Fig. 7.** Normalized harmonic MMF spectrum of asymmetric winding double layer chorded 1<sup>st</sup> design.

**Table 1.** Harmonic percentages of different designs

Harmonic Order	Reference Motor	1 <sup>st</sup> Design	2 <sup>nd</sup> Design
1.00	100.83	99.21	97.60
3.00	7.44	4.96	2.48
5.00	5.15	5.78	6.41
7.00	0.17	0.15	0.13
9.00	2.35	1.56	0.78
11.00	0.30	0.77	1.23
13.00	1.23	1.00	0.77
15.00	1.19	0.79	0.40
17.00	1.57	0.95	0.34
19.00	2.21	1.34	0.47
21.00	5.27	3.51	1.76
23.00	3.69	3.01	2.33
25.00	0.25	0.63	1.02
27.00	0.69	0.46	0.23
29.00	0.48	0.43	0.39
31.00	0.67	0.75	0.84
33.00	0.26	0.17	0.09
35.00	0.15	0.14	0.14
37.00	0.90	0.89	0.87
39.00	0.53	0.35	0.18
41.00	1.52	1.70	1.89
43.00	2.62	2.37	2.12
45.00	1.45	0.97	0.48
47.00	0.10	0.25	0.40
49.00	0.14	0.11	0.09



**Fig. 8.** Ampere-turn step distribution of asymmetric winding double layer chorded 2<sup>nd</sup> design (10-turns were shifted).



**Fig. 9.** Normalized harmonic MMF spectrum of asymmetric winding double layer chorded 2<sup>nd</sup> design.

**Table 2.** Harmonic change comparison of designs

Harmonic Order	Reference vs 1 <sup>st</sup> Design (%)	Reference vs 2 <sup>nd</sup> Design (%)
1.00	-1.60	-3.21
3.00	-33.33	-66.67
5.00	12.28	24.56
7.00	-9.60	-19.19
9.00	-33.33	-66.67
11.00	156.65	313.29
13.00	-18.44	-36.88
15.00	-33.33	-66.67
17.00	-39.29	-78.57
19.00	-39.29	-78.57
21.00	-33.33	-66.67
23.00	-18.44	-36.88
25.00	156.65	313.29
27.00	-33.33	-66.67
29.00	-9.60	-19.19
31.00	12.28	24.56
33.00	-33.33	-66.67
35.00	-1.60	-3.21
37.00	-1.60	-3.21
39.00	-33.33	-66.67
41.00	12.28	24.56
43.00	-9.60	-19.19
45.00	-33.33	-66.67
47.00	156.65	313.29
49.00	-18.44	-36.88

#### 4. Conclusions

A design comparison is introduced to investigate space harmonics and their effects. It has been previously demonstrated that mitigating the undesirable effects of space harmonics is achievable through the implementation of a specific design solution. This paper presents a comparison between three windings of which the first one is the reference motor having a previously used mentioned stator structure. First 180 turns per phase was distributed to the slots evenly to the unevenly distributed slots and then 5 and 10 turns from each slot were moved to the previous slot. No optimization was applied for calculating the number of turns to be shifted.

The mentioned 3 motors were then analyzed in terms of mmf distribution and harmonic content. The computed disturbance values in the reference asymmetric motor and the two double-layer asymmetrical motors were presented. The aim is to emphasize the practicality of these design considerations. The originality of this chorded winding is the presentation of chorded coils having different coil-pitches. The presented new motor designs successfully diminish the space harmonic effects over a wide range, despite certain harmonic impacts experiencing an increase.

Depending on the structure of harmonic calculations it is shown that even harmonics does not occur. Results show that higher performance improvement can be achieved in wide range with changing winding structure by using the proposed method than that of achieved by using conventional methods. However, some harmonics are rising while many of the investigated harmonic order decrease with presented method.

This method is applicable to windings with varying layers, poles, and phases. In subsequent research, the selected design or calculation method will be employed to optimize the winding in accordance with a reference study. By achieving the design structure, the distinct contributions of these designs can be harnessed when applied to different types of motors.

To provide better understanding and solutions on induction motor performance with eliminating space harmonic impacts, further investigations on optimizing slot and winding structure of an asymmetric winding machine need to be performed on proposed method. Further success can be obtained by optimizing the numbers of turns to be shifted for all individual coils which can provide more harmonic elimination.

## 5. References

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