A General MMF and Winding Factor Calculation Model for Electrical Machines

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Abstract

Calculation of MMF and winding factor of electric machines is a tedious task. Additionally, there is not a common unified method to calculate the winding factors of different types of electric machines. This is due to the lack of a universal formula for the calculation of the winding factors of both Fractional Slot Concentrated Windings (FSCWs) and Distributed Windings. It is even harder to build a generalized model in case of non-uniformities such as unequally pitched slots and different numbers of turns. However, it is possible to make such a model by using generalized winding models and complex Fourier Expansion. In this paper, a universal MMF, Fourier Coefficients, and Winding Factor calculation model is developed. The developed model is implemented to the practical stage and a calculator tool is developed in the Python environment. With this tool, it is possible to calculate and visualize the MMF, Fourier Coefficients, and Winding Factors for any winding function.

1. Introduction

Winding design is one of the most crucial parts of the electric machine design process. Considering the vast number of winding types, it becomes evident to importance of unified and simplified methodologies in winding design. Even if it is impossible to combine all winding design processes into a single algorithm, it is still beneficial for engineers to create inclusive models concerning their design goals.

Calculation of the magnetomotive force (MMF) and winding factor of a winding is a common need for almost all electric machine windings. If the requirements are a bit more challenging, it becomes inevitable to work with harmonics. However, it is not always possible to calculate these easily. And sometimes in special cases, it is not possible to obtain them analytically, without case-specialized formulas.

In this study, a universal model is developed to obtain the MMF (winding function), implement Fourier expansion into this, and calculate its winding factors. Calculation of the winding factors for an arbitrary winding function is only possible with the Fourier expansion anyway. The abovementioned quantities are calculated by modelling the machine space and the winding with the required detail level. There are similar modelling techniques in the literature to obtain these quantities for different types of electric machine windings. The earliest studies paving the way to model the machine space and using Fourier analysis to examine MMF go back to studies of Burbidge in the late fifties. [1]. Despite giving some flexibility, modeling the machine space is not enough to analyze arbitrary winding functions. To achieve this task, it is beneficial to use complex numbers and matrix notation. One of the groundbreaking studies in this field belongs to Stepina [2].

Despite using it for many beneficial industrial purposes like pole amplitude modulation, modelling the machine space is not sensitive enough without exact slot positions. If the slot widths are not uniform, the problem becomes even more complex. After the 2000's with the purpose of obtaining optimized MMF and less harmonic content in electric machines, improved winding factors topic started to popularize again. The study has become one of the milestones of the topic by achieving performance increase in induction machines by using accordingly adjusted slot widths [3].

However, neither of these studies provided a universal model for calculating the magnetomotive force, Fourier coefficients, and winding factor for an arbitrary winding. On the other hand, it is fair to say the purposes of the predecessors were to improve the theory itself and obtain fruitful results for superior designs rather than developing tools. In this study with the condition of properly defined; Fourier coefficients and winding factor of any function can be calculated.

2. General Information

The winding function is the representation of the airgap MMF distribution of an electrical machine [4]. It is convenient to work with winding functions because the MMF itself is the natural function of the winding structure. This method provides a chance for a mathematical approach. It is also important to mention the unit machine concept before continuing. A unit machine is the minimum periodic unit to form the entire machine. A machine of "Q" slots and "p" pole pairs constituting t unit machines, where "t" is used as the machine periodicity and equals the greatest common divisor of "Q" and "p". It should be known that the outputs of the unit machine will be also valid for its multiplexes. In this study, calculations are made for unit machines and winding functions normalized with respect to their number of turns. $F(\theta)$ is used to refer magnetomotive force and in case of 1 Ampere-turn for each coil arm, it is equal to the winding function, $m(\theta)$.

It is common to calculate the winding factor of a winding by calculating several factors such as distribution, pitch and skewing factors. However, this method restrains us from obtaining a universal model which is usable for an arbitrary winding. For instance, calculation of the distribution factor of the FSCWs is too complex to simplify it by this type of formula. Nevertheless, Alberti et al. have given such a formula by using star of slots [5]. On the other hand, for an undismantled winding factor formula, multiple conditions must be taken into consideration such as: number of layers, harmonic distribution, and number of phase belts per phase [6,7].

3. Developed Model

The required inputs and definitions of the developed model are given below:

- Number of phases (m)
- Number of pole pairs (p)
- Number of stator slots (Ns)
- Slot position angles in the winding function (γ)
- Magnitudes of each section in the winding function (NI)

All other internal variables of the routine are calculated from these parameters. While creating the winding function, the sections between two adjacent slots were referred individually. Since there would be inherently two alternance, it is enough to receive magnitudes and slot position angles for the program for the half of each pole. One of the essential advantages of the developed tool is its flexibility and practicability. The created routine can be used for

- distributed and concentrated windings,
- integer and fractional-slot windings,
- single and multi-layer windings.

There are also similar studies in the literature to provide these types of models and tools for practical usage. Di Tomasso et al. developed software and accomplished to produce accurate results for two-phase and three-phase machines [8]. Also, the study of Yokoi et al. provides a solid background for deriving the winding factor in a generalized form for fractional slot concentrated winding machines [9].

Nonetheless, the originality of this study is based on its validity for completely arbitrary winding functions thanks to the structure of the developed model. It is known that most of the functions can be expressed approximately by Fourier series in a condition of convergence. Similarly, any winding function can be expressed in terms of the Fourier series. As an analogy, just like drawing pictures with the Fourier series for art works, it is also possible to calculate aimed quantities (harmonic coefficients, winding factors) for any drawings (winding functions) in the code.

4. Design & Simulation

The routine consists of three parts. The first part of the algorithm contains a classical MMF drawing model. However, for it is possible to be valid for arbitrary winding functions it should work for any number of slots and angle. This is achieved by defining required internal variables. To draw the MMF for arbitrary winding function the program should know the exact number of sections in the winding function which is equal to the number of slots for a rotating machine. It is beneficial to remember in case of a linear machine the number of sections would be number of slots minus one. Because each slot creates a transition angle regarding its angle in the machine space and despite in a circular pattern the number of sections would be equal to the number of dividers, in a finite linear pattern it would not. Since the program also received the magnitudes corresponding to each section, it is now relatively easy to calculate magnitudes for each section. This is achieved by cascade iterations using the number of slots and magnitudes. Formation of the winding function between the two slot positions [m(θ)] can be expressed with the equation below:

$$m(\theta) = m_{q'+1} + \sum_{i=1}^{q'} (m_i - m_{i+1}) u(\alpha_i) \tag{1}$$

Note that the angles α_i are the slot position angles, $u(\alpha)$ is the elementary building block, q' is the number of transition angles, and m_i is the magnitude of the winding function of the relevant section. In the algorithm, to obtain the simplest version, (2) was used instead of (1). However, for a complex analysis, the first equation is still needed. Since a winding function is based on a sine or cosine function as the fundamental component, the same section is repeated twice for windowing (one at the beginning and one at the end of the period). In this case, the number of elements in magnitudes (m) will be the number of transition angles (α) plus one.

$$m(\theta) = \sum_{i=1}^{q'+1} m_i \tag{2}$$

In the second part, Fourier analysis was applied to the defined winding function. Yet, it is crucial to determine the symmetry types to obtain the correct Fourier coefficients for an arbitrary function. For this reason, the program also contains some extra internal variables and sub-algorithms. The crux of this part is to determine whether the function has an odd/even symmetry. If it has one of them, the routine determines the type and continues to the analysis. On the other hand, other types of symmetries can also be exploited to increase the efficiency of the software. Fourier expansion of the winding function is implemented according to well-known Fourier formulas:

$$m(\theta) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\theta) + bn \sin(n\theta) \tag{3}$$

$$a_0 = \frac{1}{T} \int_{-\frac{T}{2}}^{+\frac{T}{2}} m(\theta) d\theta \tag{4}$$

$$a_n = \frac{2}{T} \int_{-\frac{T}{2}}^{+\frac{T}{2}} m(\theta) \cos(n\theta) d\theta \tag{5}$$

$$b_n = \frac{2}{T} \int_{-\frac{T}{2}}^{+\frac{T}{2}} m(\theta) \sin(n\theta) d\theta \tag{6}$$

To implement this analysis to the code, numpy and scipy modules were used.

In the third and last part of the code, winding factors for each harmonic were calculated. To grant this, magnitudes of the harmonics should be determined first.

$$M_n = \frac{2}{n\pi} \sum_{i=1}^{q'} (m_i - m_{i+1}) sin(n\alpha_i) \tag{7}$$

M_n is the magnitude of n'th-order harmonic of the winding function. Notice that this equation is structurally similar to (1). The difference between two neighboring levels was determined by the MMF and polarity of the corresponding slot. It is also required to find the p_0-th order component, the working harmonic. Because it might be beneficial to express winding factors in terms of normalized values. For most of the conventional electric machines, it is the 1'st order term (and/or fundamental component). The working harmonic term is used here to refer to torque producing component of the machine.

$$M_{p_0} = \frac{2}{p_0\pi} \sum_{i=1}^{q'} (m_i - m_{i+1}) sin(p_0\alpha_i) \tag{8}$$

In this way, a winding function can be expressed by its harmonics as in (9).

$$m(\theta) = \sum_{n=1}^{\infty} M_n \, cos(\theta) \tag{9}$$

In this analysis, cosine function was used to take the starting point of the magnetic axis as a reference point. It was also considered during the creation of the software architecture. Besides, calculation of the total number of turns (N_{tot}) in all coils was required to calculate the winding factor as in (10).

$$N_{tot} = \sum_{i=1}^{q'} |m_i - m_{i+1}| \qquad (10)$$

Consequently, winding factors for each harmonic (k_{w_n}) can be given by (11):

$$k_{w_n} = \frac{n M_n \pi}{2\, N_{tot}} \qquad (11)$$

Notice that it is possible to calculate it without M_{p_0}. If we write p_0 instead of "n" we could calculate the winding factor of 1'st order which is the working harmonic of the machine.

$$k_w = \frac{p_0 M_{p_0} \pi}{2\, N_{tot}} \qquad (12)$$

By using this relation, it is possible to express all equation system in terms of per-unit values by normalizing them to p_0 and k_w. However, here in this study, this was not preferred.

5. Results & Interpretation

The software can be used to calculate different winding functions. Because of its allegation of universality, it needs to be improved with continuous feedback. Although the equations were proven to be universal theoretically, it is always necessary to repair and debug the problems caused by different practical issues.

5.1. Simulation Results for FSCW

To emphasize the universal structure of the software, two examples computed by the developed software were chosen to present here, of which the first one is an FSCW machine with the given values in Table-1.

Table 1. Winding parameters for examined FSCW

Number of phases (m)	3
Number of slots (Q_s)	9
Number of slot per phase per pole (q)	3/8
Number of poles (2p)	8
Layer	2

The outline of this machine is given in Fig.1.

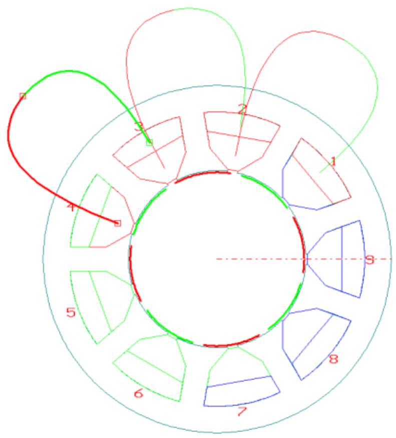

Fig. 1. Three phase fractional slot concentrated winding machine outline for Qs=9, 2p=8, q=3/8.

The winding function of one phase computed by the routine is given in Fig. 2, the calculated harmonics of the winding are visualized in Fig. 3. and the harmonic spectrum of the mentioned winding can be seen in Fig. 4. Winding factors are given in Figure-5.

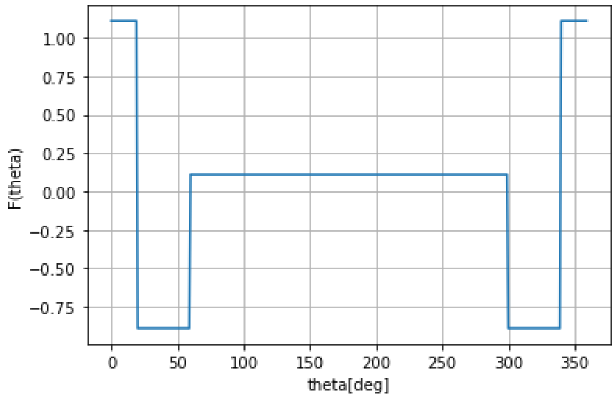

Fig. 2. Single phase MMF waveform of the FSCW machine given in Table 1

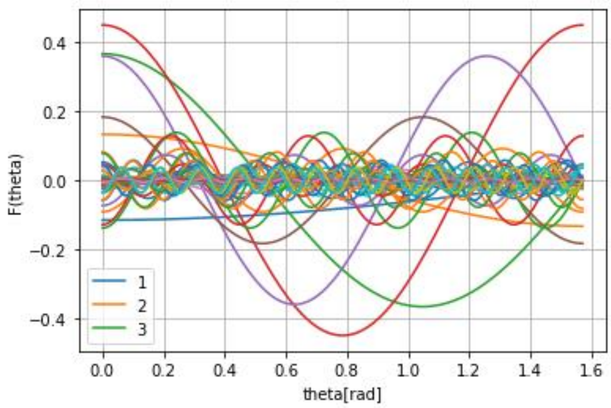

Fig. 3. Winding harmonics of the FSCW machine

It must be kept in mind that even order harmonics exist since the winding function does not have half-wave symmetry. Due to the third order harmonics are not eligible to produce a rotating field, they are not shown in the spectrum. It must be noticed that the colors of the harmonics are different in Fig. 3 and Fig. 4 because of the drawing method.

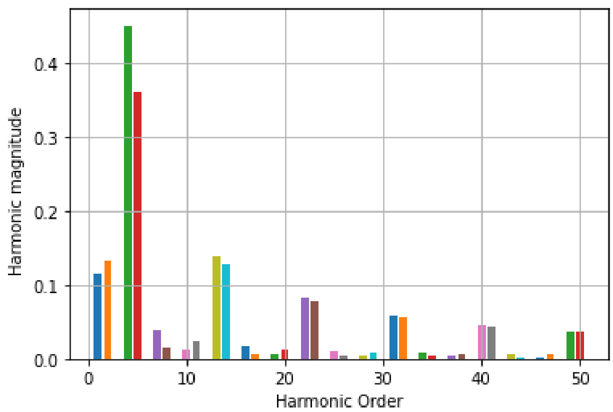

Fig. 4. The MMF harmonic spectrum of the FSCW

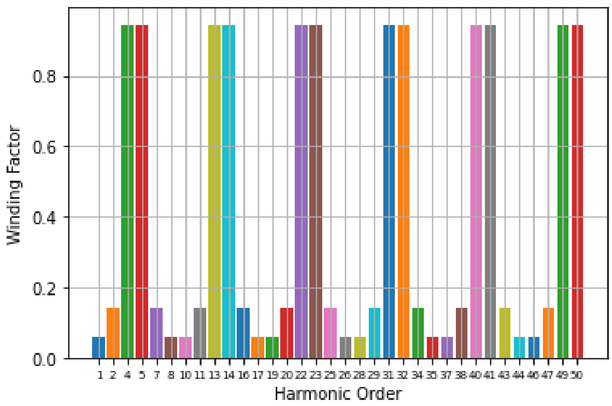

Fig. 5. The winding factors of the FSCW (Qs=9, 2p=8, q=3/8)

It can be seen that the winding factors for different order harmonics follow a certain pattern. There are different groups called "family" consisting of different harmonic orders. This phenomenon can be exploited for novel improvements in windings.

5.2. Simulation Results for Distributed Winding

The second example is a distributed winding machine with the given parameters in Table-2. However, this data was taken from a previous study [3] and the machine has different slot widths and numbers of turns per slot to improve the winding performance. This structure was deliberately chosen to prove the comprehensive architecture of the developed software.

Table 2. Winding parameters for examined non-uniform slotted distributed winding machine

Number of phases (m)	3
Number of slots (Q_s)	36
Number of slot per phase per pole (q)	6
Number of poles (2p)	2
Layer	1
Slot position in quarter wave	[48° 65° 82°]
MMF Values in quarter-wave	[100 81 47]

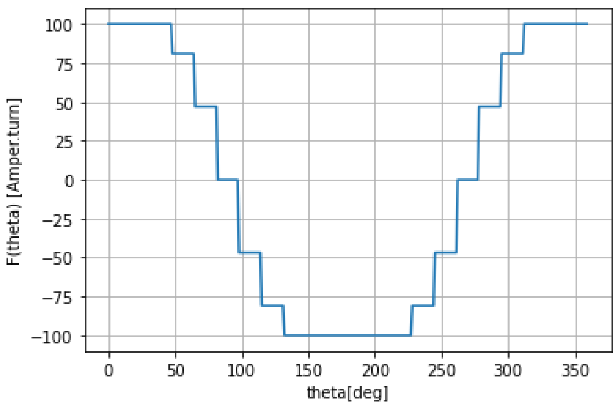

Fig. 6. Single-phase MMF waveform of the non-uniform slotted 3- phase distributed winding machine for the given configuration (q=6)

The winding function for single-phase was obtained by the software and can be seen in Fig. 6. The design of this winding was discussed with all its details in [3,10]. It must be noticed that the graphics for this winding are not normalized and were drawn with respect to nominal MMF values. It was used in this way to help the reader to compare the results with that of the original study [3, 10].

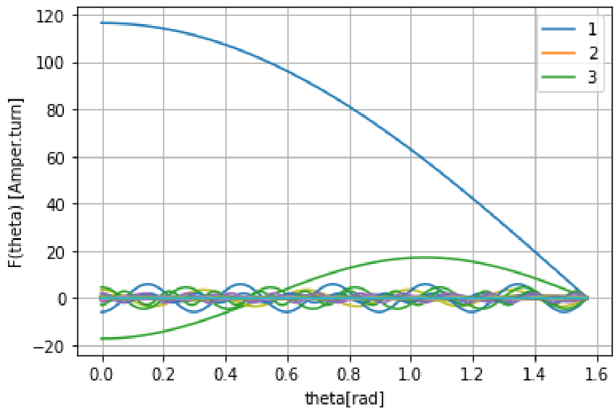

Fig. 7. Harmonics of the non-uniform slotted 3- phase distributed winding machine

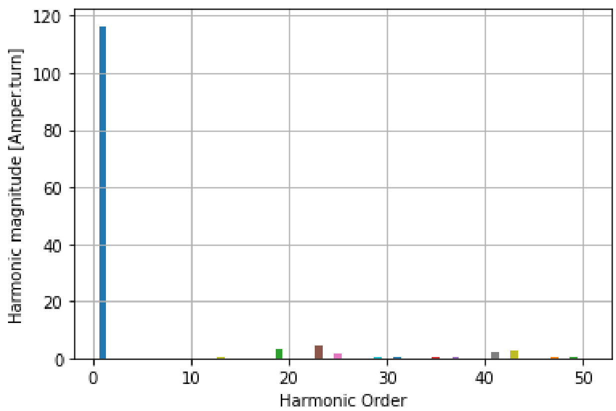

Fig. 8. MMF harmonic spectrum of the non-uniform slotted 3- phase distributed winding machine

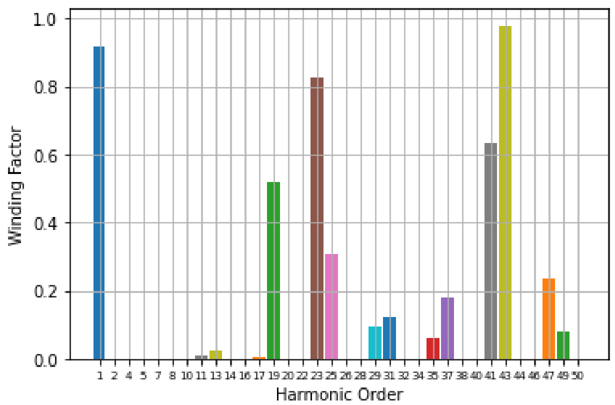

Fig. 9. Winding factors of the non-uniform slotted 3- phase distributed winding machine

The income of the optimized structure of stator slots and winding can be seen from the harmonic waves as in Fig. 7. The harmonic spectrum of this winding is given in Fig. 8. The winding factors calculated for different orders are shown in Fig. 9.

The simulation results of the examined distributed winding design are in great harmony with the outputs of the original study. It is clear that the prepared routine for this study can catch the results for a non-traditional winding where PWM and PAM techniques were used together.

6. Conclusions

In this study, a universal routine that can calculate MMF, Fourier coefficients, and winding factors was successfully developed. The mathematical background for this routine was explained in detail. By courtesy of the created software, it is possible to examine any winding function with details. It was seen that the routine outputs showed consistency with many different well-known winding topologies both for FSCW and distributed windings including unconventional ones.

To present the performance of the routine, 2 different 3-phase windings consisting of a fractional slot concentrated winding and an asymmetrical single layer winding accommodating unevenly distributed slots and number of turns were analyzed thoroughly by this routine. Winding functions, harmonic waveforms, harmonic spectra and harmonic winding factors were all given in comparison. The routine worked very well even on an unconventional non-uniform pitched winding and reproduced its previous results while adding additional features.

The developed algorithm offers a holistic perspective on the winding factor subject. Besides expediting the winding design process. It is aimed by the authors, to study unique winding designs by using the flexibility of the developed algorithm for further research.

6. References

[1] R.F Burbidge, "A Rapid Method of Analysing the MMF Wave of a Single or Polyphase Winding", *The Institution of Electric Engineers*, no. *280S,* Jan, 1958.

[2] J. Stepina, "Matrix Analysis of Space Harmonics of Asymmetrical Stator Windings", *IEEE Proceedings*, vol. 1*34, Pt.B,* no. *4,* Jul, 1987.

[3] D.A. Kocabaș, "Novel Winding and Core Design for Maximum Reduction of Harmonic Magnetomotive Force in AC Motors", *IEEE Transactions on Magnetics*, vol. *45,* no. *2,* Feb, 2009.

[4] N. Tang, I.P. Brown, "Framework and Solution Techniques for Suppressing Electric Machine Winding Space Harmonics by Varying Slot Distribution and Coil Turns", *IEEE Transactions on Magnetics*, vol. *54,* no. *5,* May, 2018.

[5] L. Alberti, N. Bianchi, "Theory and Design of Fractional-Slot Multilayer Windings", *IEEE Transactions on Industry Applications*, vol. *49,* no. *2,* Apr, 2013.

[6] N. Tang, I.P. Brown, "Family Phenomenon in Electric Machine Winding MMF Space Harmonics: Attribution and Applications", *IEEE Transactions on Magnetics*, vol. *55,* no. *5,* May, 2019.

[7] A. O. Di Tomasso, F. Genduso, R. Miceli " A New Software Tool for Design, Optimization, and Complete Analysis of Rotating Electrical Machines Windings ", *IEEE Transactions on Magnetics*, vol. *51,* no. *4,* Apr, 2015.

[8] A. O. Di Tomasso, F. Genduso, R. Miceli "A Software for the Evaluation of Winding Factor Harmonic Distribution in High Efficiency Electrical Motors and Generators", in *2013 Eight International Conference and Exhibition on Ecological Vehicles and Renewable Energies (EVER)*, Monte Carlo, Monaco, 2013, pp. 1-6.

[9] Y. Yokoi, T. Higuchi, Y. Miyamoto "General Formulation of Winding Factor for Fractional-Slot Concentrated Winding Design", *IET Electric Power Applications*, vol. *10,* Iss. *4,* pp. 231-239, 2016.

[10] D. A. Kocabaş, "Asenkron Makinalarda Uzay Harmoniklerinin Etkilerini Azaltmaya Katkılar", Ph.D. thesis, Graduate School, Istanbul Technical University, Istanbul, Turkey, 2005.