A New Numerical Approach for Simulation of Power Electronic Converters

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Abstract

Discrete simulation of Power Electronic Converters (PECs) can be important for several purposes such as condition and health monitoring, controller design, and analysis of these devices. In this study, an improved Trapezoidal Rule with Second Order Backward Differentiation Formula (TR-BDF2) and Predictor-Corrector (PC) approach is proposed for the simulation of PECs. The average of the function evaluation values is used along with the standard TR–BDF2 and PC method for the improvement and applied to a single-phase full-wave rectifier. The simulation results have shown that the tracking capability of the TR–BDF2 is improved and the numerical oscillations seen with the PC no longer appear in the proposed approach. Furthermore, the simulation results reveal that the improved approach provides promising results in terms of computational efficiency with respect to the fourth order Runge–Kutta (RK4) method, making it an alternative tool for simulating power electronic converters.

1. Introduction

Due to the high complexity of modern power electronic systems, simulation is crucial and indispensable. It offers invaluable insights into circuit performance, which would be costly, time-consuming or sometimes impossible to obtain through a laboratory prototype [1]. Therefore, simulation tools have been extensively used for the design and improvement of electrical systems over many decades [2].

For the simulation of PECs or other types of converters, an equation or equation set that represents the behavior of the circuit is needed. Later, these equations can be solved by using analytical or numerical techniques. Although deriving the exact analytical solution to a given problem is valuable, it would be overwhelming for non-linear and complex systems, such as power electronic circuits [3]. Consequently, numerical methods are used in most of the simulation tools. Several numerical methods used for the simulation of electrical systems can be listed as follows: Forward and Backward Euler, Trapezoid, Taylor's expansion and Runge-Kutta, Adams-Basforth-Moulton, Predictor-Corrector, Backward Differentiation Formulas (BDFs), Gear [1, 4]. For instance, the Electromagnetic Transients Program (EMTP) is used for the simulation and analysis of power systems, where EMTP uses nodal analysis with the trapezoidal rule integration for solving high-speed transient effects in power systems [5, 6]. Simulation Program with Integrated Circuit Emphasis (SPICE) is used for simulation and analysis of analog circuits, where SPICE uses gear and/or trapezoidal integration methods [7, 8]. On the other hand, Matlab/Simulink uses the combination of Tustin and Backward Euler (TBE) method to perform a discrete simulation of electrical systems, while mitigating the numerical oscillations seen with the Tustin method, it provides better accuracy than the Backward Euler method [9].

The trapezoidal integration method is commonly used in dynamic simulations, especially in power system transient simulations, due to its simplicity and moderate accuracy, which is of second order [10]. However, it has several drawbacks. One major issue is that it is less accurate compared to other methods available. Additionally, when simulating PECs with fixed-time steps, problems like numerical oscillations and errors caused by abrupt discontinuities during switching events are often encountered [11, 12]. Several existing techniques for handling these discontinuities are analyzed in [10], in terms of accuracy. In their study, they have also proposed new approaches derived from existing technologies to improve accuracy [10]. While the most accurate results are obtained by TR_BE_I_V (Backward Euler method combined with trapezoidal integration, interpolation and variable time-step), it has been concluded that the TR_BE_SS_I (fixed time-step Backward Euler method combined with trapezoidal integration, interpolation and simultaneous switching) is the best trade-off between complexity and accuracy [10]. In another power electronics study presented in [13], five numerical techniques, namely Euler, Runge-Kutta, and Trapezoidal approximation with 1st, 2nd, and 3rd order, are investigated for the performance comparison in a predictive controller for an AC-DC power converter. Results revealed that the different discretization techniques result in different total harmonic distortion at the line side, and the 1st order trapezoidal approximation is a good trade-off between achieved performance and ease of implementation. Furthermore, it has been seen that the TR-BDF2 method (also Matlab ODE23tb solver [14]) is employed for challenging simulation of PECs, such as cascaded full-bridge multilevel converter used in [15]. It is noteworthy to mention that the use of TR-BDF2 can also be seen in many disciplines, including electronics, biology, mechanical engineering, and option pricing [16]. As seen from the reviewed literature, identifying a good numerical technique capable of accurately simulating power-electronics circuits is a difficult task, with the goal of finding the best balance between accuracy, simplicity, and computing time.

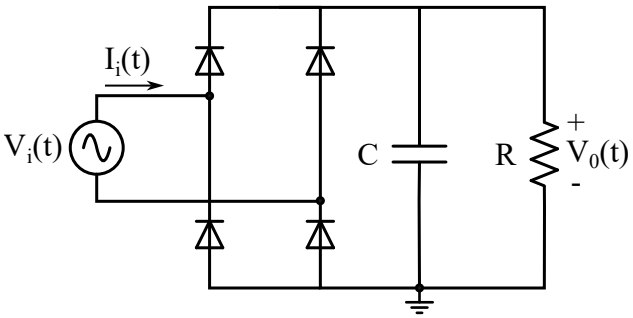
In this study, improved TR–BDF2 and PC approaches are proposed for the simulation of PECs. The average of the function evaluation values is used as intermediate steps along with the standard TR-BDF2 and Predictor-Corrector methods for this improvement. Hereinafter, the proposed approaches are called I-TR-BDF2 and I-PC. To validate the effectiveness of the proposed approach, the simulation of a simple AC/DC power electronic converter depicted in Fig.1 is considered. The proposed approach offers several distinct features. Firstly, to the best of the authors' knowledge, the use of the average of the function evaluation values represents a new numerical approach derived from the existing methods, which may be used for power electronics circuit simulation. Secondly, the accuracy of the proposed approaches is obtained comparable to the RK4 method. Furthermore, it is

demonstrated that the proposed approach improves the tracking capability of the standard TR-BDF2 and eliminates the numerical oscillations seen with the classical PC method for the simulation of the selected circuit. Finally, our approach exhibits a comparatively lower computational complexity than the RK4 method.

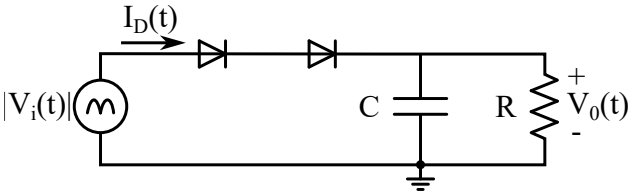
The remainder of this paper is organized as follows. In Section 2, the methodology of the proposed approach is introduced. In Section 3, simulation results are given. The paper is concluded with Section 4.

2. Methodology

This section describes the mathematical expressions for the full-wave rectifier depicted in Fig.1a and explains how to solve this problem by the proposed approach. Furthermore, other numerical methods are briefly discussed.



(a) Full wave rectifier using a diode bridge with capacitor filter.



(b) Its model

Fig. 1. A simple AC/DC power electronic converter, adapted from [17].

2.1. Analysis of the full-wave rectifier

When KCL is applied to full-wave rectifier model in Fig.1b, the diode current $I_D\,(t)$ can be obtained as follows:

$$I_D\,(t) = C\,\frac{dV_o\,(t)}{dt} + \frac{V_o\,(t)}{R} \tag{1}$$

where, V_o, R, and C, t are the output voltage, the resistance of the load, the capacitance of the capacitor filter, and time, respectively. The diode current can be also expressed by:

$$I_D\,(t) = I_s\left(exp\left(\frac{V_d(t)}{nV_T}\right) - 1\right) \tag{2}$$

where, I_s, V_T, and n are the inverse saturation current, thermal voltage, and non-ideality factor of the diode, respectively [17]. It is worth mentioning that all diodes are assumed as ideal and identical. Moreover, the input voltage is assumed as purely sinusoidal and given by:

$$V_i\,(t) = V_{max}\,sin\,(\omega t) \tag{3}$$

where, V_{max} and ω are the peak value and angular frequency of the input voltage, respectively. ω is given by $2\pi f$, where f is the line frequency. Finally, the mathematical model, which represents the behavior of the full-wave rectifier circuit, can be obtained by substituting (3) and (2) into (1) as follows:

$$\frac{dV_o\,(t)}{dt} = -\frac{V_o\,(t)}{RC} + \frac{I_s}{C}\left(exp\left(\frac{|V_{max}\,sin\,(\omega t)\,| - V_o\,(t)}{nV_T}\right) - 1\right). \tag{4}$$

As noticed, the Equation (4) is not linear and can be solved by numerical methods. In this study, three different well-known numerical methods are briefly explained and our proposed approach is introduced to solve Equation (4).

2.2. TR-BDF2 Method

This method includes the combination of the trapezoidal rule and the second-order Backward Euler differentiation formula for the solution of initial value problems encountered in engineering. The formula for the approach can be given as [18]:

$$y_{n+\gamma} = y_n + \gamma\frac{\Delta t}{2}\left(f_n + f_{n+\gamma}\right) \tag{5a}$$

$$y_{n+1} = \frac{1}{\gamma\,(2-\gamma)}y_{n+\gamma} - \frac{(1-\gamma)^2}{\gamma\,(2-\gamma)}y_n + \frac{1-\gamma}{2-\gamma}\Delta t\,f_{n+1} \tag{5b}$$

where, γ is a parameter in the range of $\gamma \in (0, 1)$ and it determines whether the method will be A or L stable. $f_{n+\gamma}$ represents the function evaluation $f\left(t_n + \gamma\Delta t, y_{n+\gamma}\right)$. Δt is the step size.

2.3. Predictor-Corrector Method

This method includes the application of an explicit Adams–Bashforth and an implicit Adams–Moulton method, respectively. In this study, a second-order explicit Adams–Bashforth method is first applied to predict the solution, and then a second-order implicit Adams–Moulton method is considered to correct the predicted solution [19]. The predictor step can be given as:

$$\hat{y}_{n+1} = y_n + \frac{\Delta t}{2}\,(3f_n - f_{n-1}) \tag{6}$$

where f_n represents the function evaluation $f\,(t_n, y_n)$. $\hat{y}$ represents the predicted value of the solution. The final step is to correct this prediction based on a second order Adams–Moulton method and it can be expressed as:

$$y_{n+1} = y_n + \frac{\Delta t}{2}\left(f_n + \hat{f}_{n+1}\right) \tag{7}$$

where $\hat{f}_{n+1}$ represents the function evaluation $f\,(t_n + \Delta t, \hat{y}_{n+1})$.

2.4. Fourth Order Runge–Kutta Method

Classic fourth-order Runge–Kutta method can be given as [1]:

$$k_1 = f(t_n, y_n) \tag{8a}$$

$$k_2 = f\left(t_n + \frac{\Delta t}{2}, y_n + \frac{\Delta t}{2} \cdot k_1\right) \tag{8b}$$

$$k_3 = f\left(t_n + \frac{\Delta t}{2}, y_n + \frac{\Delta t}{2} \cdot k_2\right) \tag{8c}$$

$$k_4 = f\left(t_n + \Delta t, y_n + \Delta t \cdot k_3\right) \tag{8d}$$

$$y_{n+1} = y_n + \frac{\Delta t}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right) \tag{8e}$$

2.5. Proposed Approach

The proposed I-TR-BDF2 approach is a combination of TR-BDF2 and the average of the function evaluation values for the current and next steps. The corresponding equations for the I-TR-BDF2 method in this case become

$$f_{n+\gamma} = \frac{1}{2}\left(f_n + f_{n+\gamma}\right) \tag{9a}$$

$$y_{n+\gamma} = y_n + \gamma\frac{\Delta t}{2}\left(f_n + f_{n+\gamma}\right) \tag{9b}$$

$$f_{n+1} = \frac{1}{2}\left(f_n + f_{n+1}\right) \tag{9c}$$

$$y_{n+1} = \frac{1}{\gamma\left(2-\gamma\right)}y_{n+\gamma} - \frac{\left(1-\gamma\right)^2}{\gamma\left(2-\gamma\right)}y_n + \frac{1-\gamma}{2-\gamma}\Delta t\, f_{n+1}. \tag{9d}$$

Likewise, the equation for the I-PC can be expressed as:

$$f_n = \frac{1}{2}\left(f_n + f_{n-1}\right) \tag{10a}$$

$$\hat{y}_{n+1} = y_n + \frac{\Delta t}{2}\left(3f_n - f_{n-1}\right) \tag{10b}$$

$$\hat{f}_{n+1} = \frac{1}{2}\left(\hat{f}_{n+1} + f_n\right) \tag{10c}$$

$$y_{n+1} = y_n + \frac{\Delta t}{2}\left(f_n + \hat{f}_{n+1}\right) \tag{10d}$$

The modification to the original TR-BDF2 and PC algorithms are given in Equations (9a), (9c), (10a), (10c), and it just includes the average of the function evaluations at each step. This modification improves the tracking capability of the TR-BDF2 and the numerical oscillations seen with the PC method no longer appear in the proposed approach.

3. Simulation Results and Discussion

This section gives the results of the aforementioned numerical solutions for a full-wave rectifier circuit, where the specifications of the circuit are given in Table 1.

Based on the given specifications, the circuit is firstly simulated by Matlab Simulink discrete-solver "Tustin/Backward Euler (TBE)", and the results obtained from this simulation are considered as the reference waveforms. Secondly, for comparison of the interested waveforms such as input current $I_i(t)$ and output voltage $V_o(t)$ in Fig. 1a, the circuit is simulated by TR-BDF2, PC, and proposed I-TR-BDF2 and I-PC approaches, respectively. For all simulations, step size and simulation time are chosen as 1 μs and 0.1 s, respectively.

Table 1. Specifications of the full-wave rectifier.

V_{max}	180 V
f	50 Hz
R	300 Ω
C	28 μF
V_T	25.8 mV
I_s	$1x10^{-14}$ A
n	1

Fig. 2, Fig. 3, and Fig. 4 show the simulation of the input current by TR-BDF2, PC, and RK4, respectively, compared to the Simulink discrete-solver. As can be seen from Fig. 2, the TR-BDF2 method could not perfectly follow the reference waveform obtained from Simulink, resulting in low accuracy. As for the PC method, undesired numerical oscillations and overshoots in amplitude are observed when each diode pair starts to conduct as in Fig. 3. Therefore, both TR-BDF2 and PC methods have not met the requirements of accurate simulation for the given problem. However, as seen from Fig. 4, the results obtained by RK4 are accurate as Simulink discrete-solver.

The simulation of the output voltage by TR-BDF2, PC, and RK4 are given in the same plot as shown in Fig. 5, compared to the Simulink discrete-solver. As noticed from Fig. 5, each method gives a similar result to the Simulink waveform. For better clarity, a zoomed version of the obtained waveforms between 0.014 s and 0.016 s is given as a subplot, which shows how closely these methods track the Simulink discrete-solver.

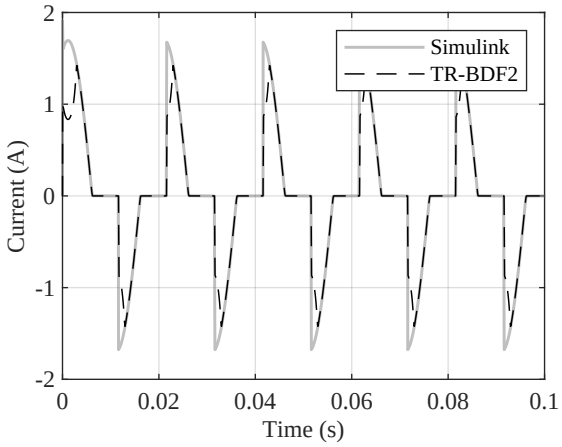


Fig. 2. Input current simulations by Simulink and TR-BDF2.

The simulation of the input current by the proposed approaches I-TR-BDF2 and I-PC is given in Fig. 6 and Fig. 7, respectively, compared to the Simulink discrete-solver. As can be seen from Fig. 6, the tracking capability of the TR-BDF2 method in Fig. 2 has been improved by the proposed I-TR-BDF2 approach. Similarly, as shown in Fig. 7, numerical oscillations seen with the PC

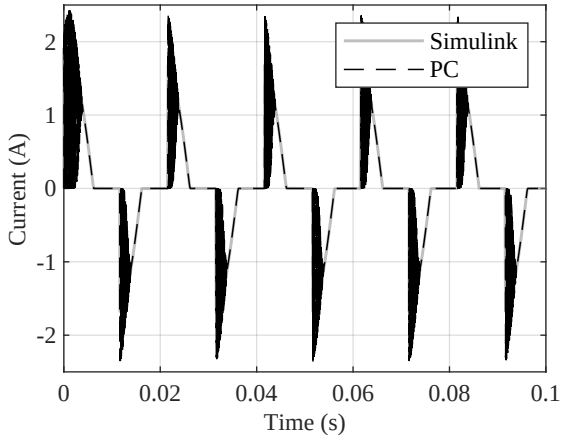


Fig. 3. Input current simulations by Simulink and PC.

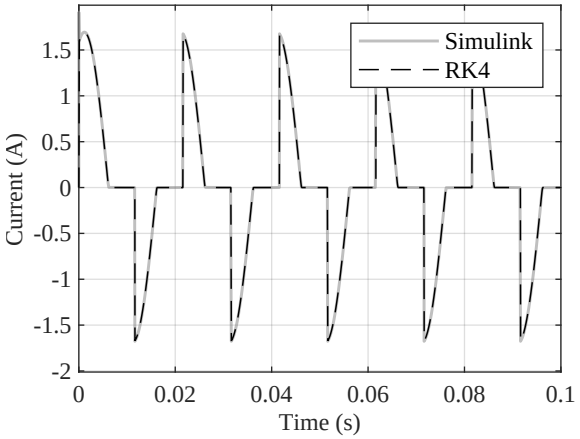


Fig. 4. Input current simulations by Simulink and RK4.

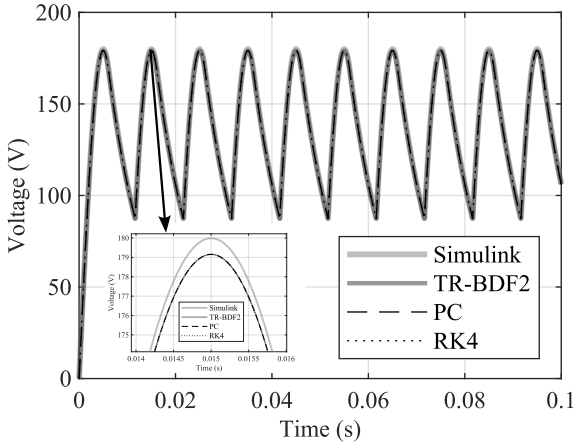


Fig. 5. Output voltage simulations by Simulink, TR-BDF2, PC and RK4.

method in Fig. 3 are no longer present in the proposed I-PC approach. Hence, it has been concluded that averaging the function evaluation values has boosted the performance of classical methods. Furthermore, the simulation of the output voltage by the proposed approaches is obtained nearly the same as TR-BDF2 and PC methods, thus, they are not included in this study.

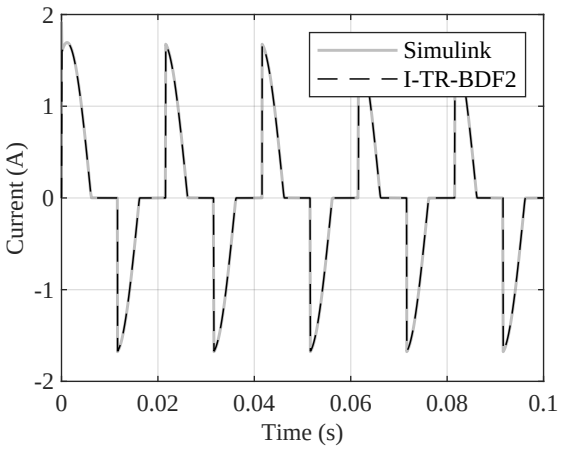


Fig. 6. Input current simulations by Simulink and I-TR-BDF2.

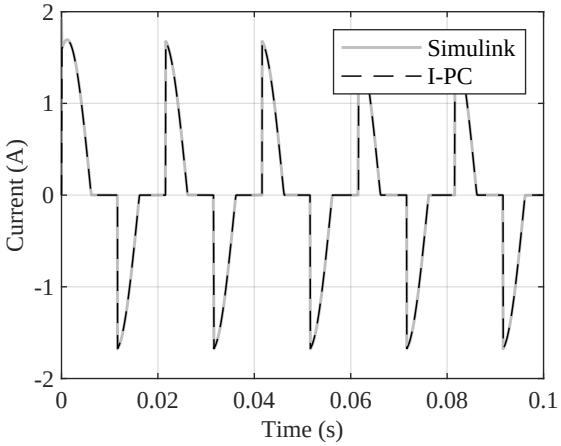


Fig. 7. Input current simulations by Simulink and I-PC.

In order to obtain the computational efficiency of the TR-BDF2, PC, RK4, and proposed approaches, each method is simulated 1000 times for reliable results. Computing time is recorded at each step for each method. Afterwards, the averages of the computing times are taken and presented in Table 2 for comparison reasons. Furthermore, to evaluate the overall accuracy of each method during the simulation interval, we have considered the relative rms error (RRE) presented in [20] as follows:

$$e_{rms} = \frac{1}{X_{rms}} \sqrt{\frac{1}{n_s} \sum_{k=1}^{n_s} (x[t_k] - \hat{x}[t_k])^2} \tag{11}$$

where, X is the interested waveform that is chosen as input current for our case, X_{rms} is the rms value of the input current, $x[t_k]$ is the reference value acquired by Simulink discrete-solver at a given time point $t_k \in [0, 0.1]$, n_s is the number of simulation points, and $\hat{x}[t_k]$ is the obtained value by the considered numerical method at a given time point t_k. Error results based on RRE are also given in Table 2. All simulations are carried out in Matlab 2023 on a Victus-HP Laptop with a processor of i5-12500H @2.50 GHz and 16 Gigabytes RAM, and the stopwatch timer of this software

is used.

As seen from Table 2, the RK4 method exhibits the highest computing time, while other methods demonstrate lower computing times than the RK4. Despite TR-BDF2 and PC having lower computing times than the RK4, they suffer from the drawbacks of low accuracy (high RRE) and poor tracking capability. In addition, a reduction of 19.24% and 16.60% in computing time compared to RK4 has been obtained by I-PC and I-TR-BDF2, respectively. On top of that, it has been observed that the I-TR-BDF2 and I-PC have almost the same computing time as TR-BDF2 and PC, while also achieving accurate results as Simulink discrete-solver and RK4.

Table 2. Comparison based on computing time and accuracy.

Method	Computing time (s)	RRE
*RK*4	0.0265	0.0647
*TR – BDF*2	0.0219	0.2957
PC	0.0214	0.4873
*I – TR – BDF*2	0.0221	0.0643
I – PC	0.0214	0.0656

4. Conclusions

The simulation results show that the proposed approach can be an alternative tool for simulating power electronic converters and is expected to have practical applications in power electronic systems, e.g. in real-time or off-line condition monitoring of converters, digital twin applications, harmonic analysis at the grid side. In future work, the proposed approaches will be applied for the simulation of other power electronic converters, such as Thyristor Controlled Reactor (TCR) and PWM-controlled DC-AC converters. Researchers from other disciplines can also utilize the proposed numerical approach for the solution of their problems that include a differential equation (or equation set) similar to this study. Furthermore, the MATLAB code of the proposed approach is uploaded to Github as an open source [21].

5. References

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