Performance Analysis of a Multi-antenna Based Cooperative Relaying Scheme over Fading Channels in Spectrum Sharing Environment

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***Abstract*—This paper presents an outage analysis of a cooperative relaying scheme over flat Rayleigh and Nakagami-m fading channels. In the proposed scheme, secondary transmitters cooperatively relay the primary traffic. Each secondary transmitter, equipped with multiple antennas, is divided into one of two clusters: a cooperative cluster (CC) and a non-cooperative cluster (NCC). Cluster head (CH) of the CC will be selected as a best decode-and-forward (DF) relay and will forward the primary information. Results show that the proposed scheme outperforms both non-cooperative, conventional single-antenna systems and random relay selection schemes in terms of outage probability. In each case, theoretical results are verified with Monte-Carlo simulation results.**

***Index Terms*—cognitive radio, cooperative relaying, multi-antenna, Rayleigh and Nakagami-m fading.**

I. Introduction

Cognitive radio (CR) was first coined by J. Mitola III [1]; it has been widely employed by the wireless community to solve the spectrum inefficiency problem. Cooperative diversity is another important technology for wireless networks that has recently been incorporated into cognitive radio networks (CRNs) to combat fading and to improve performance. Moreover, deployment of multiple antennas at wireless nodes may increase capacity without causing bandwidth expansion and significantly enhance the transmission reliability [2].

CR spectrum sharing methods that treat secondary users (SUs) as cooperative relays are proposed in [3]–[6]. In [3], [4], spectrum sharing protocols in which the primary system is a dual-hop DF selective relaying network are presented. Nodes in the secondary network can either cooperatively relay the primary traffic or act as secondary access transmitters. However, all of the nodes in [3], [4] are single-antenna systems. Multi-antenna based protocols are presented in [5], [6]. In [5], a cooperative overlay scheme in which a secondary transmitter (ST) base station equipped with multiple antennas cooperatively relays the primary traffic is proposed. In [6], the authors treat both the ST and the secondary receiver (SR) as relays that cooperate when sending the primary transmission. However, all of these studies [3]–[6] considered only the Rayleigh fading channel. In some wireless systems such as micro-cellular systems, fading is not as severe as Rayleigh fading; therefore, such systems may not be well described by the Rayleigh or Rician models. The Nakagami-m model proposed in [7] can be used to describe wireless systems with different levels of fading severity. When m=1, Nakagami-m fading corresponds to Rayleigh fading. In [8], [9], the authors analyzed the outage probability (OP) of the secondary network with cooperative relays exploiting single antennas at all the nodes over Nakagami-m fading channels.

In this work, we have analyzed the outage performance of a multi-antenna based cooperative relaying scheme in a CR network in which only the CH of the CC relays the primary traffic over either a Rayleigh or a Nakagami-m fading channel. Moreover, using the appropriate number of idle or inactive STs as relays for the primary system instead of using dedicated relays can reduce network establishment costs.

So, in this work, our contributions can be summarized as two-fold:

- Exploitation of multi-antenna in cooperative ST to relay the primary traffic cooperatively over either a Rayleigh or a Nakagami-m fading channels in CR spectrum sharing environment.
- Proposed cooperative relaying scheme shows performance improvement in terms of OP compared to non-cooperative, random relay and conventional single antenna relay selection schemes.

The rest of this paper is organized as follows. In section II, we introduce the cooperative relay based system model. Best relay or CH selection scheme is also presented in this section. In section III, performance analysis of the proposed scheme is illustrated. Theoretical results verified with simulation results are presented in section IV, and finally we conclude this paper in section V.

II. System Model

We consider a cooperative CR spectrum sharing environment consisting of a primary system with one primary transmitter-receiver (PT-PR) pair and a secondary system with M secondary transmitter-receiver (ST-SR) pairs. The proposed

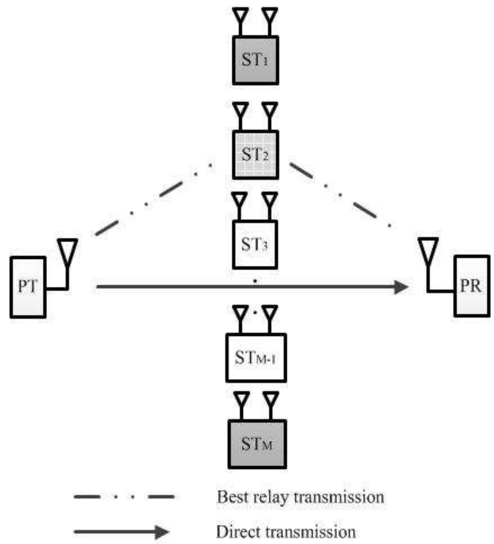

Fig. 1: System model under consideration. The dashed-dotted line indicates that the best relay (CH) transmits only when cooperation is required. For simplicity, SRs are not shown.

system model is shown in Fig. 1. Secondary transmitters are grouped into two clusters. Secondary transmitters ST$_j$,$j \in$ *{1, 2,..., L}*, which are idle or inactive, form the cooperative cluster (CC) indicated in gray in Fig. 1. STs in CC may cooperate in the primary system to forward primary information when the data rate between the PT and PR falls below R_P. We denote the number of nodes in CC by $L \in M$. R_P is the predetermined transmission rate or target rate of the PT. One of the nodes in the CC that achieves R_P can be selected as a best DF relay. We consider best relay (BR) to be the cluster-head (CH) in the CC. On the contrary, active STs, which are ready for their own transmission or are not able to cooperate with the primary system, form the non-cooperative cluster (NCC). Nodes in the NCC are indicated in white in Fig. 1. All of the nodes except the STs are equipped with single antennas. Specifically, we consider two antennas in each of the STs. In each node of the CC, one antenna can be used for cooperating with the primary transmission and another antenna can be used for its own secondary transmission. On the contrary, both antennas can be used for secondary transmission in each node of the NCC. However, in this work, our main aim is to analyze the performance of the primary system with cooperative relaying over Rayleigh and Nakagami fading channels; therefore, the secondary transmission policy is beyond the scope of this paper. Note that, in the rest of the presentation, cluster will refer only to CCs.

During cooperative transmission, primary transmission is performed over two transmission phases via the CH (or best) relay. Let $h_{i,j}$ be a channel co-efficient and SNR be the average signal-to-noise ratio between any two nodes i and j. Assume that all of the signals follow the independent fading paths. Additionally, assume that the channel coefficients remain static during both transmission phases.

The primary user's transmission policy can be divided into (i) non-cooperative or direct transmission (DT) and (ii) cooperative transmission (CT), both of which are described in detail in the following sections.

A. Direct Transmission (DT)

When the data rate from PT to PR satisfies R_P, i.e., $R_{DT} \geq R_P$ then the PT directly transmits to the PR without seeking any cooperation from the SUs. Therefore, the achievable rate of PT$\longrightarrow$PR over the direct link can be calculated as

$$R^{DT}_{PT-PR} = \log_2(1 + |h_{PT-PR}|^2 SNR) \tag{1}$$

B. Cooperative Transmission (CT)

If DT fails to satisfy R_P, i.e., $R_{DT} < R_P$, then the primary user (PU) will seek cooperation from the SUs by sending link layer control messages such as request to send (RTS) and clear to send (CTS) [10]. Assume that all of the secondary nodes are able to estimate the instantaneous channel state information (CSI), i.e., channel coefficient $h_{i,j}$, from these messages. However, for the best ST$_j$ (CH) selection, we use the achievable rate of each cooperative link instead of the instantaneous CSI of the links. Therefore, each ST$_j$ computes the achievable rates for PT$\longrightarrow$ST$_j$ and ST$_j$ $\longrightarrow$PR as follows

$$R^{ST_j}_{PT-ST_j} = \max_{n \in \{1,\, 2\}} \left(\frac{1}{2}\log_2(1 + |h^n_{PT-ST_j}|^2 SNR)\right) \tag{2}$$

$$R^{ST_j}_{ST_j-PR} = \frac{1}{2}\log_2(1 + |h_{ST_j-PR}|^2 SNR) \tag{3}$$

where n is the number of antennas at ST. The scaling factor $\frac{1}{2}$ in (2) and (3) arises because the overall transmission is divided into two phases.

1) Cluster Head (CH) Selection: In the proposed cooperative relaying scheme, S denotes the set of nodes forming the CC and can be represented as

$$S = \{j | j \in L, (\min\{R^{ST_j}_{PT-ST_j}, R^{ST_j}_{ST_j-PR}\} > R_P)\} \tag{4}$$

Each node in CC, $ST_j, j \in S$, starts a count-down timer with an initial value

$$T_j = \frac{B}{\min\{R^{ST_j}_{PT-ST_j}, R^{ST_j}_{ST_j-PR}\} > R_P} \tag{5}$$

where B is a constant that is dependent on the unit of time. In communication systems, the communication channel quality may change with time. Coherence time is the duration over which the channel impulse response is considered to be constant. Therefore, once T_j is measured, it remains valid until the next coherence time and T_j is calculated for each coherence time. After the expiration of a coherence time, RTS and CTS messages will be sent to initiate calculation of both the new CSI and the new value of T_j.

Equation (5) indicates that the node whose timer expires first will be selected as a CH (the best relay in the CC). As soon as the timer of the CH reaches zero, it will transmit a flag signal to inform other relays in S to back off and ask the PT-PR pair to identify its presence. Thus, our proposed protocol selects the CH, R_{CH}, if it satisfies the following condition

$$R^{ST_j}_{CH} = arg\max_{j \in L}(\min\{R^{ST_j}_{PT-ST_j}, R^{ST_j}_{ST_j-PR}\} > R_P) \tag{6}$$

After CH selection, the PT transmits the message to the CH in the first time slot. If the CH is able to decode the message successfully, it will then forward the message to the PR in the second time slot. Otherwise, the CH remains silent and the system declares an outage.

III. Performance Analysis

In this section, we derive the OP of the proposed multi-antenna based DF cooperative relaying scheme over Rayleigh and Nakagami-m fading channels. The OP of the DT can be formulated as follows

$$P_{OUT}^{DT} = P_r\{R_{PT-PR}^{DT} < R_P\} \tag{7}$$

Additionally, it is clear that an outage of the primary system with cooperative relaying (that is, no DT) occurs if and only if none of the relays satisfy R_P, i.e., $|S|$=0. Thus, the primary OP with cooperative relaying can be formulated as

$$\begin{aligned} P_{OUT}^{C} &= P_r\{|S|=0\} \\ &= P_r\{\max_{j\in L} \min(R_{PT-ST_j}^{ST_j}, R_{ST_j-PR}^{ST_j}) < R_P)\} \\ &= \prod_{j=1}^{L} P_r\{\min(R_{PT-ST_j}^{ST_j}, R_{ST_j-PR}^{ST_j}) < R_P)\} \\ &= \prod_{j=1}^{L}[1-(1-Y)(1-Z)] \end{aligned} \tag{8}$$

where $Y = \prod_{k=1}^{n} P_r(R_{PT-ST_j^k}^{ST_j^k} < R_P)$ and $Z = P_r(R_{ST_j-PR}^{ST_j} < R_P)$. Hence, the OP of the proposed network considering both DT and cooperative transmission can be written as

$$P_{OUT}^{P} = P_{OUT}^{DT} \times P_{OUT}^{C} \tag{9}$$

A. Rayleigh Fading

In Rayleigh flat fading, $\alpha_{i,j}$ is the link gain between any two nodes i and j. We assume that the channel coefficient $h_{i,j}$ is a complex Gaussian distribution with zero mean and variance $\lambda_{i,j}$, i.e., $h_{i,j} \sim CN(0,\lambda_{i,j})$. Therefore, the link gain $\alpha_{i,j}$ is denoted by $\alpha_{i,j} = |h_{i,j}|^2$, where $\alpha_{i,j}$ is an exponentially distributed random variable with mean value or scale parameter $\lambda_{i,j}$ ($\lambda_{i,j}$ >0) [11]. Therefore, the OP of the corresponding link in the Rayleigh fading channel can be formulated as

$$\begin{aligned} P_r\{R_{PT-PR}^{DT} < R_P\} &= P_r\{|h_{PT-PR}|^2 < \rho_{PT-PR}\} \\ &= 1 - exp(-\frac{1}{\lambda_{PT-PR}}\rho_{PT-PR}) \end{aligned} \tag{10}$$

$$\begin{aligned} P_r\{R_{PT-ST_j^k}^{ST_j^k} < R_P\} &= P_r\{|h_{PT-ST_j^k}^{k}|^2 < \rho_{PT-ST_j^k}\} \\ &= 1 - exp(-\frac{1}{\lambda_{PT-ST_j^k}}\rho_{PT-ST_j^k}) \end{aligned} \tag{11}$$

$$\begin{aligned} P_r\{R_{ST_j-PR}^{ST_j} < R_P\} &= P_r\{|h_{ST_j-PR}|^2 < \rho_{ST_j-PR}\} \\ &= 1 - exp(-\frac{1}{\lambda_{ST_j-PR}}\rho_{ST_j-PR}) \end{aligned} \tag{12}$$

where $\rho_{PT-PR} = \frac{2^{R_P-1}}{SNR}$, $\rho_{PT-ST_j^k} = \frac{2^{2R_P-1}}{SNR}$, and $\rho_{ST_j-PR} = \frac{2^{2R_P-1}}{SNR}$. Substituting (10), (11), and (12) into (9), we can find the OP of the primary system over the Rayleigh fading channel.

B. Nakagami-m Fading

The Nakagami distribution is related to the gamma distribution. Let $m \geq$0.5 and Ω >0 be the shape or fading severity and spread parameter of the Nakagami distribution, respectively, where k >0 and θ >0 are the shape and scale parameter of the gamma distribution. It is possible to obtain a Nakagami random variable from the gamma random variable by setting $k = m$ and $\theta = \frac{\Omega}{m}$ as $X \sim Nakagami(m,\Omega) \approx \sqrt{gamma(m,\frac{\Omega}{m})}$.

Thus, $|h_{i,j}|^2$ follows the gamma distribution with shape parameter $\hat{m}$ >0 and scale parameter $\hat{\theta_{i,j}}$ >0. These parameters are related to the Nakagami-m distribution parameters as $\hat{m} = m$ and $\hat{\theta_{i,j}} = \frac{\Omega_{i,j}}{m}$. Therefore, the OP of the corresponding link in the Nakagami fading channel can be formulated as

$$\begin{aligned} &P_r\{R_{PT-PR}^{DT} < R_P\} \\ &= P_r\{|h_{PT-PR}|^2 < \rho_{PT-PR}\} \\ &= \frac{1}{(\hat{\theta}_{PT-PR})^m \Gamma(m)} \int_0^{\rho_{PT-PR}} t^{m-1} exp(\frac{-t}{\hat{\theta}_{PT-PR}})dt \end{aligned} \tag{13}$$

$$\begin{aligned} &P_r\{R_{PT-ST_j^k}^{ST_j^k} < R_P\} \\ &= P_r\{|h_{PT-ST_j^k}^{k}|^2 < \rho_{PT-ST_j^k}\} \\ &= \frac{1}{(\hat{\theta}_{PT-ST_j^k})^m \Gamma(m)} \int_0^{\rho_{PT-ST_j^k}} t^{m-1} exp(\frac{-t}{\hat{\theta}_{PT-ST_j^k}})dt \end{aligned} \tag{14}$$

$$\begin{aligned} &P_r\{R_{ST_j-PR}^{ST_j} < R_P\} \\ &= P_r\{|h_{ST_j-PR}|^2 < \rho_{ST_j-PR}\} \\ &= \frac{1}{(\hat{\theta}_{ST_j-PR})^m \Gamma(m)} \int_0^{\rho_{ST_j-PR}} t^{m-1} exp(\frac{-t}{\hat{\theta}_{ST_j-PR}})dt \end{aligned} \tag{15}$$

where $\rho_{PT-PR} = \frac{2^{R_P-1}}{SNR}$, $\rho_{PT-ST_j^k} = \frac{2^{2R_P-1}}{SNR}$, $\rho_{ST_j-PR} = \frac{2^{2R_P-1}}{SNR}$, and $\Gamma(.)$ is the gamma function. However, (13), (14), and (15) can be easily solved using popular software such as MATLAB or Mathematica. Therefore, by substituting (13), (14), and (15) into (9), we can obtain the OP of the primary system with cooperative relaying over the Nakagami-m fading channel.

IV. Results and Discussion

This section presents the theoretical results validated by the Monte Carlo simulation results with the help of MATLAB to study the performance of our proposed multi-antenna based cooperative relaying scheme. We evaluate the OP of the proposed system (BR$_n$) and compared with the DT, random relay (RR) selection and best relay selection with single antenna (BR$_1$) [4] schemes.

In Fig. 2 and 3, we show the OP of the primary system with cooperative relaying (BR$_n$) as well as compared with DT, RR and BR$_1$ over Rayleigh and Nakagami-m fading

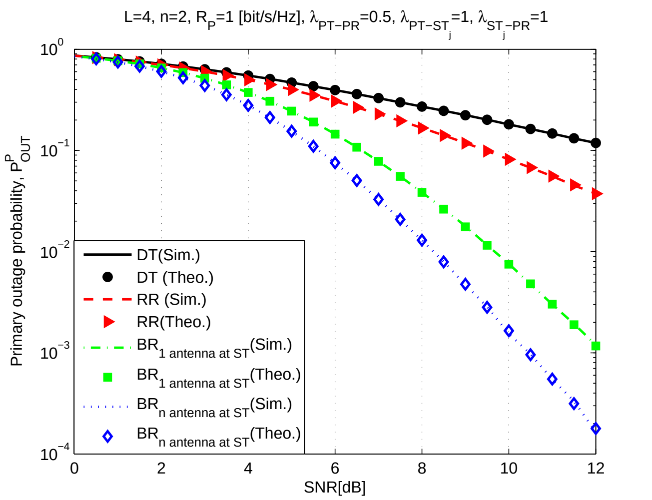

Fig. 2: Performance of DT, RR, BR with one and multi-antenna over Rayleigh fading channel.

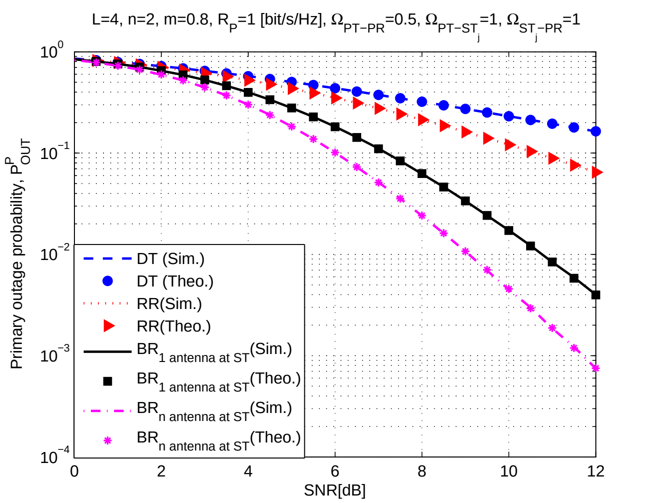

Fig. 3: Performance of DT, RR, BR with one and multi-antenna over Nakagami-m fading channel.

channels respectively as a function of SNR. SNR varies from 0 to 12 dB in each case. We let, L=4, n=2, R_P= 1 bit/s/Hz, λ_{PT-PR}=0.5, $\lambda_{PT-ST_j} = \lambda_{ST_j-PR}$=1 in Fig. 2 and L=4, n=2, m=0.8, R_P= 1 bit/s/Hz, Ω_{PT-PR}=0.5, $\Omega_{PT-ST_j} = \Omega_{ST_j-PR}$=1 in Fig. 3 respectively. From Fig 2 and 3, it can be observed that OP decreases with increasing SNR as expected. It is also shown that the proposed scheme shows improved outage performance than other schemes. Each cooperative scheme also performs better than non-cooperative (DT) scheme. We can say that when L=1, the RR and BR_1 shows the same outage performance. This is because when L=1, BR_1 shows the same diversity as in RR. But our proposed BR_n shows better outage performance because of using multi-antenna in each cooperative node. In each case, theoretical results are well matched with the simulation results. This results further suggests the importance of cooperation to achieve better outage performance.

In Fig. 4, we show the OP of the proposed system over

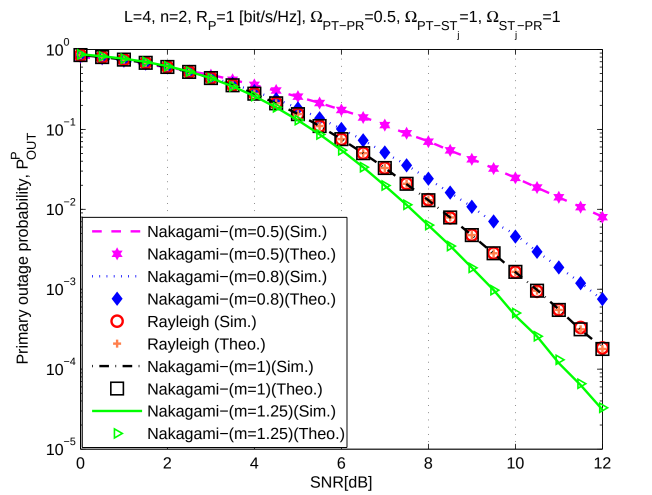

Fig. 4: Comparison of Nakagami-m with Rayleigh fading channels. When m=1, Nakagami fading corresponds to Rayleigh fading.

Nakagami-m fading channel for different values of m and SNR. Same values for L, n, R_P and $\Omega_{i,j}$ as in Fig. 3 are considered. Four cases of m, where m=0.5, 0.8, 1 and 1.25 representing different level of fading severity are also considered. When m=1, Nakagami-m corresponds to Rayleigh channel which is shown in Fig. 4. It is clear from Fig. 4 that P^P_{OUT} decreases with increasing SNR for each cases. It is also observed that P^P_{OUT} increases with decreasing m. This is because, higher value of m represents less severe channel whereas lower value of m represents more severe channels. When $m \longrightarrow \infty$, it corresponds to a situation of a no fading. In each case, theoretical results are validated by the simulation results.

Fig. 5 shows the OP of the BR_n with different values of R_P and L over Rayleigh and Nakagami-m (m=1) fading channel. We consider two cases of R_P where R_P=1 bit/sec/Hz and R_P=1.25 bits/sec/Hz respectively. Two cases of L where L=4 and L=5 as well as same values for n, $\lambda_{i,j}$ and $\Omega_{i,j}$ as in Fig. 2 and 3 are considered. It is clear that, the OP decreases with increasing SNR as well as L as expected. Increasing L represents that more number of nodes participate in the cooperation process. As a result, the number of independent paths between PT and PR increases. It is also shown that OP increases with increasing R_P. This means that, for the same set of parameters if the target rate increases, there is a high probability that the primary system will be in a deep fade or outage.

V. Conclusion

The OP of a primary system exploiting multiple antennas at cooperative STs in Rayleigh and Nakagami-m fading channels is investigated. Our proposed method outperforms other schemes in all cases. OP improves with increasing SNR and when a larger number of STs participates in the cooperation procedure. We also found that, at the cost of overall complexity, the cooperative link outperforms the DT in terms of OP. However, other factors such as secondary

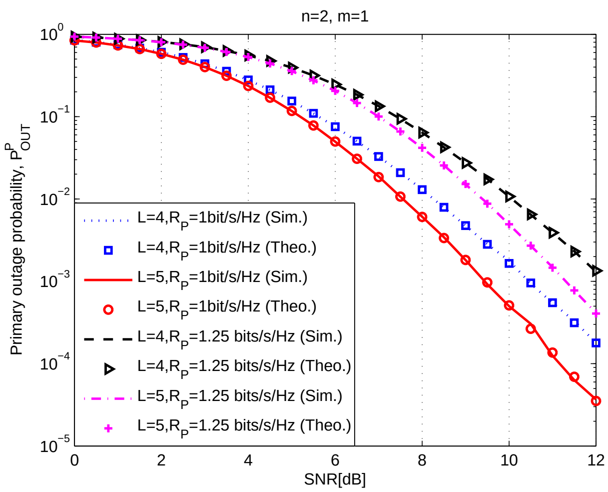

Fig. 5: Primary outage probability of the BR_n over Rayleigh and Nakagami-m (m=1) fading channel for varying L and R_P.

spectrum access policies and bit error rates will be considered in future work.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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