Optimized Two-piecewise Companding Transform for Peak-to-average Power Ratio Reduction in OFDM Systems

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Abstract

In orthogonal frequency division multiplexing (OFDM), large peak-to-average power ratio of the signal leads to major design issues in the analog front-end of the transmitter. Companding is a widely known PAPR reduction technique that involves transforming the signal amplitude using a deterministic function. Design of piecewise linear transforms has recently attracted great attention because of their very low complexity and better control over the design specifications. In this paper, we optimize the design of a two-piecewise linear function by formulating its design equations such that the companding noise is minimized for a pre-determined amount of PAPR reduction. This will be done while keeping the average power unchanged. The designed function has been found to outperform the existing two-piecewise linear transform with regard to error performance and spectral spreading.

1. Introduction

Orthogonal frequency division multiplexing (OFDM) has been widely accepted as the technology of choice in high speed wireless communications. It has been incorporated in many standards of broadband internet access, like IEEE 802.11a/g, IEEE 802.16, 3GPP LTE, LTE-A and ETSI HIPERLAN/2, and in broadcast systems, like digital audio/video broadcasting (DAB/DVB). It offers the advantages of high spectral efficiency and immunity to the effects of multi-path fading. However, being a multi-carrier modulation technique, OFDM signal envelope has large fluctuations and hence a large dynamic range. This causes performance degradation resulting from non-linear distortion introduced by the high power amplifier (HPA) at the transmitter.

Peak-to-average power ratio (PAPR) is commonly used as a metric to quantify the signal's dynamic range. Several PAPR reduction techniques have been proposed [1], including partial transmit sequences (PTS), selective mapping (SLM), tone injection (TI), tone reservation (TR), iterative clipping and filtering (ICF), peak windowing and companding [1]. Due to their low implementation complexity, design of companding transforms has attracted great attention.

Several companding transforms have been proposed. Nonlinear transforms have been designed by amplitude distribution modification [2, 4, 5, 5]. Linear and piecewise linear transforms have been presented in [3, 8, 9]. Due to their low complexity and flexibility of design, piecewise linear companders are more applicable to practical systems. Recently, two-piecewise linear compander (TPWC), which is a piecewise linear transform proposed in [8], has also been used in iterative companding transform and filtering (ICTF), presented in [7]. But the transform presented in [8] is not optimized. Rather, some of its parameters have been chosen heuristically, based on observations from simulations. As a result, noise amplification at the receiver causes severe degradation in bit error rate (BER). Moreover, it does not provide the flexibility to design the transform for any desired PAPR.

In order to make the TPWC optimal and scalable, in this paper, we propose a new design framework to evaluate its parameters such that companding noise is minimized while keeping the average power constant. Instead of choosing the compander parameters on the basis of observations from simulations, as done in [8], we find the best possible set of parameters by formally defining design specifications and constraints. The new optimized two-piecewise linear compander (OpTPWC) can be tuned to any desired PAPR. It also has the capability to provide efficient trade-offs among PAPR, BER and out-of-band interference (OBI). Moreover, the design criteria is such that it aims at achieving the desired PAPR while preserving the signal as much as possible in terms of its spectral characteristics and error performance.

The remainder of this paper has been organized as follows. In Section 2, a typical OFDM system model with a companding transform has been described. The design criteria and procedure for OpTPWC is given in Section 3. Performance Evaluation is presented in Section 4. Finally, conclusion is drawn in Section 5.

2. System Model

A typical baseband equivalent of an OFDM system with a companding transform is shown in Fig. 1. The discrete-time complex envelope of an upsampled OFDM symbol is given as follows:

$$x_n = \frac{1}{\sqrt{NL}} \sum_{k=0}^{NL-1} X_k \exp\left(\frac{j2\pi kn}{NL}\right) \tag{1}$$

for $0 \le n \le NL-1$. X_k are data symbols derived from QPSK or QAM constellations. N is the number of sub-carriers, including data carriers, pilot carriers and null carriers (guard band). L is the oversampling factor. PAPR of an OFDM symbol is defined as follows:

$$\text{PAPR (dB)} = 10\log_{10}\left(\frac{\max\limits_{0\le n\le NL-1} |x_n|^2}{\frac{1}{NL}\sum\limits_{n=0}^{NL-1} |x_n|^2}\right) \tag{2}$$

PAPR reduction performance of any PAPR reduction scheme is evaluated using empirical curves of complementary cumulative

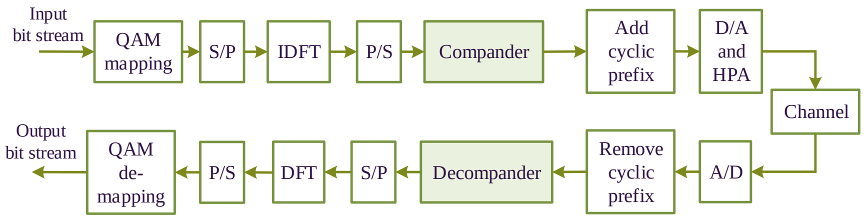

Figure 1. OFDM system model with a companding transform

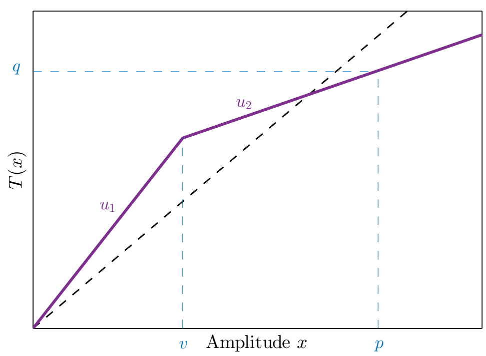

Figure 2. Profile of OpTPWC

distribution function(CCDFs) of PAPR, defined as follows:

$$CCDF_{PAPR}(PAPR_0) = \Pr\left[PAPR > PAPR_0\right] \quad (3)$$

According to Equation (1), OFDM symbol is constructed by the addition of large number of sub-carriers with random amplitude and phases. So by the central limit theorem (CLT) approximation, OFDM signal is a complex Gaussian random process. The amplitude of a complex Gaussian random process is a Rayleigh random process. The probability density function (PDF) of amplitude of OFDM signal is given as follows.

$$f_{|x_n|}(x) = \frac{2x}{\sigma_x^2}\exp\left(-\frac{x^2}{\sigma_x^2}\right) \quad (4)$$

where σ_x^2 is the average power of the OFDM signal.

3. Proposed Design

In this section, we present the design criteria for the OpTPWC. The main aims of the design are: to provide the capability of obtaining any desired PAPR performance and to achieve that desired PAPR while keeping the companding noise as small as possible.

The general profile of OpTPWC is shown in Fig. 2. The parameters to calculate are slopes, u_1 and u_2, and the cut-off point v [8]. The transform $T(x)$ is given as follows:

$$T(x) = \begin{cases} u_1 x, & 0 \le x \le v \\ u_2 x + v(u_1 - u_2), & x > v \end{cases} \quad (5)$$

where $u_1 > 1$ and $u_2 < 1$. Also $T(x)$ only transforms the amplitude of the signal, so that the output of compander can be expressed as follows:

$$y_n = T\left(|x_n|\right)\operatorname{sgn}\left(x_n\right) \quad (6)$$

3.1. Specifying Desired PAPR

Let P be the value of PAPR in the original OFDM signal, such that $CCDF_{PAPR}(P) = \rho$ is known for original OFDM. The peak amplitude at this PAPR is p. If this p is transformed into q, then the CCDF of the transformed signal will be equal to ρ at $PAPR = Q$, where $Q = 10\log_{10}\left(q^2/\sigma_x^2\right)$. So $P - Q$ (dB) is the amount of reduction in PAPR at $CCDF = \rho$. So the straight line with slope u_2 must pass through the point (p, q). Hence, u_1 and u_2 are related as follows:

$$u_2 = \frac{q - u_1 v}{p - v} \quad (7)$$

So by specifying p and q, amount of PAPR reduction can be specified.

3.2. Preservation of Average Power

In order to keep the average power of the signal unchanged after companding operation, following constraint has been used:

$$\sigma_x^2 = \int_0^\infty x^2 f_{|x_n|}(x)dx = \int_0^\infty T^2(x) f_{|x_n|}(x)dx$$
$$\int_0^v u_1^2 x^2 f_{|x_n|}(x)dx + \int_v^\infty \left(u_2 x + v\left(u_1 - u_2\right)\right)^2 f_{|x_n|}(x)dx \quad (8)$$

Substituting the value of u_2 from Equation (7) into Equation (8) and separating the integrals of powers of x, we get

$$\begin{aligned}\sigma_x^2 =& u_1^2 I_2(0,v) + \left(k_1 - u_1 k_2\right)^2 I_2(v,\infty) + \\ & v^2\left(u_1(1+k_2) - k_1\right)^2 I_0(v,\infty) + \\ & 2v\left(k_1 - k_2 u_1\right)\left(u_1(1+k_2) - k_1\right) I_1(v,\infty)\end{aligned} \quad (9)$$

where $k_1 = q/(p-v)$, $k_2 = v/(p-v)$, $I_0(a,b) = \int_a^b f_{|x_n|}(x)dx$, $I_1(a,b) = \int_a^b x f_{|x_n|}(x)dx$ and $I_2(a,b) = \int_a^b x^2 f_{|x_n|}(x)dx$. Considering Equation (9) as a quadratic equation with respect to variable u_1, the co-efficients of u_1^2, u_1 and

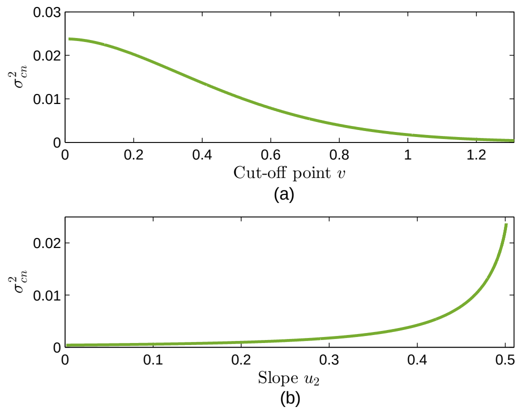

Figure 3. Relationships between companding noise and (a) cut-off point v and (b) slope u_2

u_1^0 are given by c_2, c_1 and c_0, respectively, as follows:

$$c_2 = I_2(0,v) + k_2^2 I_2(v,\infty) + v^2(1+k_2)^2 I_0(v,\infty) - 2vk_2(1+k_2)I_1(v,\infty) \quad (10a)$$

$$c_1 = -2k_1k_2I_2(v,\infty) - 2v^2k_1(1+k_2)I_0(v,\infty) + 2vk_1(1+k_2)I_1(v,\infty) + 2vk_1k_2I_1(v,\infty) \quad (10b)$$

$$c_0 = k_1^2 I_2(v,\infty) + v^2k_1^2 I_0(v,\infty) - 2vk_1^2 I_1(v,\infty) - \sigma_x^2 \quad (10c)$$

There are many values of parameters u_1, u_2 and v that satisfy the Equations (9) and (7) for given constraints on p and q. Among these solutions, the optimum solution will be the one that minimizes the companding noise, described in the next sub-section.

3.3. Minimizing Companding Noise

Companding noise can be defined as the difference between companded and original signals. So the companding noise power is given as $E\left[(T(x)-x)^2\right]$. For the transform under consideration, it can be evaluated as follows.

$$\sigma_{cn}^2 = E\left[(T(x)-x)^2\right] = \int_0^\infty (T(x)-x)^2 f_{|x_n|}(x)dx$$
$$= (u_1-1)^2 I_2(0,v) + (u_2-1)^2 I_2(v,\infty) + 2v(u_2-1)(u_1-u_2)I_1(v,\infty) + v^2(u_1-u_2)^2 I_0(v,\infty) \quad (11)$$

The parameters u_1, u_2 and v will be calculated such that σ_{cn}^2 is minimized while satisfying Equation (9).

3.4. Optimization Algorithm

The solution for (v, u_1, u_2) is obtained using the Algorithm 1. Algorithm 1 first calculates all the possible solutions of Equation (9) that satisfy all given constraints. Then the solution that yields minimum companding noise is selected as the best final solution.

3.5. Relationships between Companding Noise and Compander parameters

Fig. 3 shows how companding noise is related with the parameters u_2 and v. The parameters are such that they satisfy Equation (9). It can be observed that companding distortion is minimum when v approaches p and u_2 approaches zero.

Algorithm 1 Parameters of OpTPWC

Input p, q, σ_x.
Set $v_{array} = 0 : inc : p$, where inc is a small increment.
Initialize Solution Set $S = \{\}$.
for each element v in v_{array} **do**
  Find $k_1 = q/(p-v)$ and $k_2 = v/(p-v)$.
  Find co-efficients c_2, c_1 and c_0 using Equation (10).
  Solve $c_2u_1^2 + c_1u_1 + c_0 = 0$ for u_1 to get (u_{11}, u_{12}).
  Set $u_1 = u_{11}$ or $u_1 = u_{12}$ such that $u_1 > 1$.
  Find u_2 using Equation (7).
  if $u_1 > 1$ and $u_2 < 1$ **then**
    Add (v, u_1, u_2) to S.
  end if
end for
Initialize $\sigma_{cn_{min}} = \infty$.
for each element of S **do**
  Find σ_{cn} using Equation (11).
  if σ_{cn} is less than $\sigma_{cn_{min}}$ **then**
    Accept current element of S as the final solution.
    Set $\sigma_{cn_{min}} = \sigma_{cn}$.
  end if
end for
Output accepted solution for u_1, u_2 and v.

4. Performance Evaluation

In this section, we present simulation results for PAPR, BER and OBI performances for the proposed OpTPWC. The proposed OpTPWC is compared with TPWC in [8] to demonstrate that the transform's performance has been significantly improved.

4.1. Simulation Setup and Parameters

The OFDM signal is simulated according to the physical layer specifications given in IEEE 802.16a, used in Fixed WiMAX. Number of sub-carriers N is 256, which includes 192 data carriers, 8 pilot carriers and null carriers for guard band and DC. 4-QAM and 16-QAM are used as modulation schemes. The oversampling factor L is 4. Perfect synchronization and zero frequency offset are assumed at the receiver. Power spectral density (PSD), evaluated by averaging the periodogram estimates, is used to depict the relative OBI for different profiles of TPWC and OpTPWC. In order to avoid noise amplification at the receiver, the decompander function is selected such that only that part of the companded signal is recovered that results in noise attenuation.

4.2. Simulation Results

4.2.1. PAPR Reduction

CCDF curves for TPWC, presented in [8], are shown in Fig. 4. Modulation scheme used is 4-QAM. It can be seen that while TPWC can only yield two operating conditions according to [8], OpTPWC can be configured to any desired PAPR.

4.2.2. BER Performance

BER performance comparison is given in Fig. 5 for 4-QAM based OFDM and in Fig. 6 for 16-QAM based OFDM. It can be

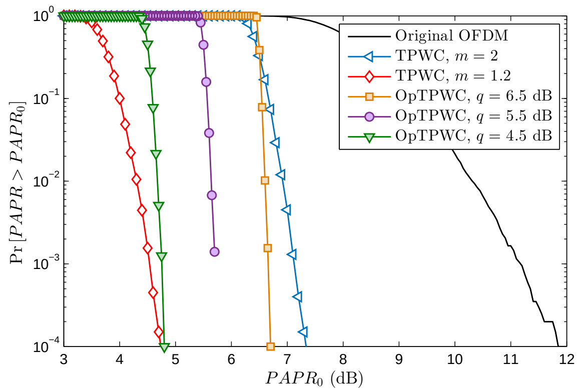

Figure 4. PAPR reduction performance comparison of TPWC and OpTPWC. $\rho = 10^{-2}$, $P = 10$ dB

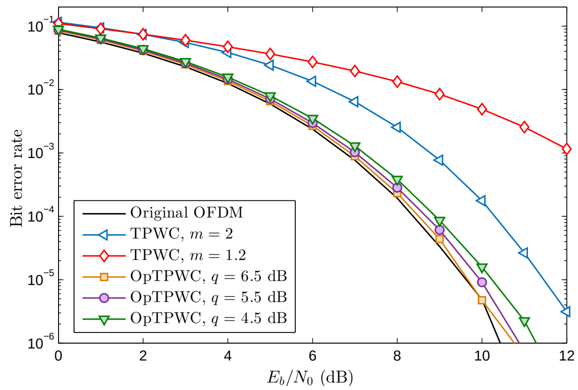

Figure 5. BER performance comparison of TPWC and OpTPWC over AWGN channel using 4-QAM based OFDM. $\rho = 10^{-2}$, $P = 10$ dB

seen that the OpTPWC significantly outperforms TPWC. It can efficiently trade BER with PAPR. BER performance degrades gracefully as PAPR is reduced by setting different values of q.

4.2.3. OBI Performance

OBI performance is compared in Fig. 7. It can be seen that OBI in OpTPWC degrades in small amounts as PAPR is reduced, whereas in TPWC, performance is not optimal for the amount of PAPR reduction shown in Fig. 4. In [7], ICTF scheme has been proposed using TPWC to reduced the OBI. Using OpTPWC, number of iterations required in ICTF to achieve the desired OBI performance will be further reduced. So using OpTPWC, ICTF's performance can also be improved.

4.2.4. Computational Complexity

Piecewise linear transforms are generally preferred because of their low complexity of operation. Complexities of TPWC and OpTPWC are similar, i.e., one floating point operation is required per sample.

5. Conclusion

In this paper, we have optimized the performance of the two-piecewise linear companding transform with respect to the

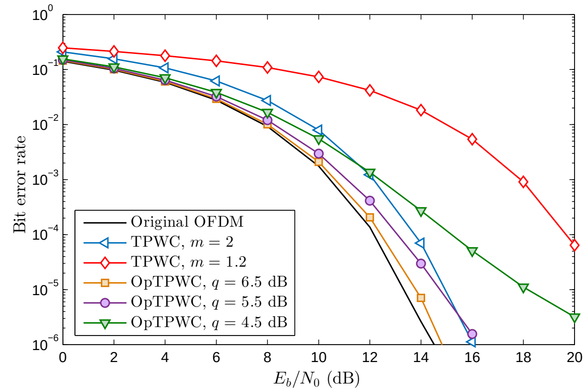

Figure 6. BER performance comparison of TPWC and OpTPWC over AWGN channel using 16-QAM based OFDM. $\rho = 10^{-2}$, $P = 10$ dB

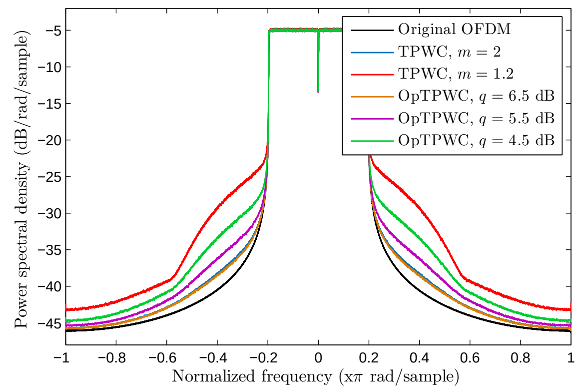

Figure 7. PSDs of TPWC and OpTPWC using 4-QAM based OFDM. $\rho = 10^{-2}$, $P = 10$ dB

companding noise for a given amount of PAPR reduction. Simulation results show that the new transform performs significantly better with regard to BER and OBI. Also it can be tuned to any value of desired PAPR by setting its parameters. Hence, it provides efficient trade-offs between PAPR, BER and OBI.

6. References

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