Cooperative Output Regulation for Higher-order Linear Multi-agent Systems with Fixed and Switching Topologies

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Abstract

The paper presents the cooperative output regulation for heterogeneous linear multi-agent systems (MAS) with fixed and switching topologies. In this approach, each agent dynamics are higher-order with different constrained and state dimension. The leader system of considered heterogeneous agents provides tracking signal and disturbance. A systematic distributed design approach for output regulation problem is proposed via measurement output feedback. Under this approach, it is demonstrated that the heterogeneous multi agent system can reach the output regulation in finite time. For immeasurable internal states of agents and relative states between agent and its neighbors, Luenberger observers and dynamic compensators are employed. The theoretical results are verified through numerical simulations for a multi-agent system of four airplanes.

1. Introduction

Networked multi-agent system (MAS) in control theory has attracted extensive attention of researchers. MAS is very effective model to describe dynamic agents which can exchange information through communication. In recent years, more research has focused on the cooperative control of MAS because of broad applications in many practical problems such as flying control of UAV [1], formation of mobile robots [2], attitude synchronization of spacecraft formation [3] and its advantages such as scalability and strong robustness. According to different control objectives problems of flocking, consensus, formation and tracking have been widely studied.

Consensus problem is to follow a common reference signal generated by the agents themselves and it is the most relevant topic to cooperative output regulation. In [4,5] basic problem frame work formed for consensus problem. Consensus problem for first and second order integrator was investigated in [6, 7, 8].Adaptive consensus problem for linear and nonlinear MAS studied in [9, 10].

In modern control theory output regulation of MAS has been largely studied for tracking and disturbance rejection problems. In consensus problem most of the time homogeneous agents without any leader is considered but output regulation overcome this drawback. In [11] author discussed Full information feedback control and measurement output feedback control for cooperative linear output regulation of homogenous MAS. Full information heterogeneous output regulation problem discussed in [12]. Cooperative output regulation of heterogeneous MAS with internal model principal presented in [13]. Cooperative output regulation of MAS under switching topology proposed in [14].

In this paper we will discuss the linear output regulation problem with higher-order heterogeneous MAS with different switching topologies. The novelty of this paper is that the assumptions of stable nodes and fixed undirected topology.Also the agents have higher order dynamics compared to the double integrator system regulation considered in the literature. Another feature of the approach is that all subsystems have different constraints with different number of states.

An outline of this paper is as follows. In section 2, some preliminaries related to graph theory and problem statement are introduced .In section 3, we give detailed description of output regulation of MAS with measurement output feedback. In section 4, Numerical example is provided to demonstrate our result. In section 5, we present our conclusion.

Following notations will be used in this paper.$\mathcal{R}$ be the set of real numbers.$\mathcal{R}^{m\times n}$ is a matrix space of dimension $m \times n$. $0_{m\times n}$ is a matrix with zero elements and m rows and n colums. $1_N$is a column vector with all the elements 1. $\otimes$ represent the kronecker product and its properties are discussed in [15]. $\sigma(t)$ describe the switching signal which is piecewise constant and $\sigma: [0, +\infty) \rightarrow \mathcal{P} = \{1,2, \dots, \rho\}$ with switching instants $t_0 = 0$, t_1 , t_2 , ... where switching index set is $\mathcal{P}$ and ρ is a nonnegative integer. We assume that switching instants of signal σ satisfy $t_{k+1} - t_k \geq \tau > 0$ for any $k \geq 0$ and for constant τ, and the constant τ is called dwell time.

2. Preliminaries

2.1 Graph theory

In this section, preliminary knowledge is introduced.

First, we introduce some basic terminologies and concepts from graph theory. The relationship among a system of agents can be described by a graph. A digraph or directed graph is denoted as $\mathcal{G} = (\mathcal{O}, \mathcal{E})$ where N node set is $\mathcal{O} = \{\mathcal{O}_1, \dots, \mathcal{O}_N\}$ and $\mathcal{E} \subseteq \mathcal{O} \times \mathcal{O}$ is the set of edges. If graph $\mathcal{G}$ is directed and (i, j) denotes an edge from parent node i to child node j then i is called neighbor of j . For an undirected graph $\mathcal{G}, (i, j) \in \mathcal{E} \Leftrightarrow (j, i) \in \mathcal{E}$ and i , j are neighbors. $\mathcal{N}_i = \{j \mid (j, i) \in \mathcal{E}\}$ denotes the neighbor set of node i. A path in directed graph $\mathcal{G}$ is a finite sequence of edges in the form $(i_1, i_2), (i_2, i_3), \dots, (i_k, i_{k+1})$, with distinct edges, and we say that node i_{k+1} is reachable from node i_1. A globally reachable node $\mathfrak{S}$ of $\mathcal{G}$ is reachable from every other node of $\mathcal{G}$. A directed graph $\mathcal{G}_S = (\mathcal{O}_S, \mathcal{E}_S)$ is a subgraph of $\mathcal{G} = (\mathcal{O}, \mathcal{E})$ if $\mathcal{O}_S \subseteq \mathcal{O}$ and $\mathcal{E}_S \subseteq \mathcal{E} \cap (\mathcal{O}_S \times \mathcal{O}_S)$ .Given a set of n directed graphs where $\{\mathcal{G}_i = (\mathcal{O}, \mathcal{E}_i), i = 1, \dots, n\}$, the directed graph $\mathcal{G} = (\mathcal{O}, \mathcal{E})$ where $\mathcal{E} = \cup_{i=1}^{r} \mathcal{E}_i$ is union of directed graph $\mathcal{G}_i$, denoted by $\mathcal{G} = \cup_{i=1}^{r} \mathcal{G}_i$ .The nonnegative and weighted adjacency matrix of a directed graph $\mathcal{G}$ is denoted as $\mathcal{A} =$

$[a_{ij}] \in \mathcal{R}^{N\times N}$,where $a_{ij} \geq 0$ and $a_{ii} = 0$ (if edge start from agent i and end at agent j then $a_{ij} > 0$). In the case of undirected graph $\mathcal{G}$, $a_{ij} = a_{ji}$ .Its Degree matrix $D = diag\{\beta_1, \dots, \beta_N\} \in \mathbb{R}^{N\times N}$ is a matrix with only diagonal non-zero elements, where diagonal elements are $\beta_i = \Sigma_{j=1}^{N} a_{ij}$ for $i = 1,2 \dots, N$ .The Laplacian matrix of a directed graph $\mathcal{G}$ is defined as $\mathcal{L} = D - \mathcal{A}$ .Set $\Delta = diag(a_{10,}, \dots, a_{N0})$, which is also a diagonal matrix with $N \times N$ dimension. Define $H = \mathcal{L} + \Delta$, which tells us about the connectivity of whole directed graph $\mathcal{G}$. A time-varying directed graph can be defined as $\mathcal{G}_{\sigma(t)} = (\mathcal{O}, \mathcal{E}_{\sigma(t)})$ where for all $t \geq 0$, edges $\mathcal{E}_{\sigma(t)} \subseteq \mathcal{O} \times \mathcal{O}$. Besides this, we have an adjacency matrix $\mathcal{A}_{\sigma(t)} = [a_{ij}(t)] \in \mathcal{R}^{N\times N}$ satisfying, for all $t \geq 0, a_{ii}(t) = 0$ wher $i = 1,2,,3, \dots, N$ and $a_{ij}(t) \geq 0, where\ i,\ j = 1,2,3, \dots, N$. A dynamic directed graph $\mathcal{G}_{\sigma(t)}$ can be defined such that $\mathcal{A}_{\sigma(t)}$ is the nonnegative matrix of directed graph $\mathcal{G}_{\sigma(t)}$.We can say $\mathcal{G}_{\sigma(t)}$ is the directed graph of $\mathcal{A}_{\sigma(t)}$.

2.2 Problem statement

Consider the cooperative output regulation of heterogeneous linear MAS with N nodes, which is expected to reject the disturbance d while tracking a common reference r at different nodes. The nodes dynamics are described by following equations.

$$\begin{aligned} \dot{x}_i &= A_i x_i + B_i u_i + E_i\ \omega \\ y_{mi} &= C_{mi} x_i + D_{mi} u_i + F_{mi} \omega \\ e_i &= C_i x_i + D_i u_i + F_i \omega \qquad i = 1, \dots, N \end{aligned} \tag{1}$$

Where $x_i \in \mathcal{R}^{n_i}$ is the state vector , $y_{mi} \in \mathcal{R}^{p_{mi}}$ is the measurement output, $e_i \in \mathcal{R}^{p_i}$ is the error output, and $u_i \in \mathcal{R}^{m_i}$ is the ith subsystems control input and A_i , B_i , E_i $C_{mi}, D_{mi}, F_{mi}, C_i, D_i, F_i$ are the matrices of appropriate dimension. $\omega \in \mathcal{R}^{q}$ is the reference and disturbance signal which is generated by following differential equations respectively :

$$\dot{r} = A_r r \ \ ,\ \ r(0) = r_0 \ \ ,\ \ \dot{d} = A_d d \ \ ,\ \ d(0) = d_0$$

Where $r \in \mathcal{R}^h$ and $d \in \mathcal{R}^g$. Lumping the disturbance d and reference input r together generates the exogenous signal of node i denoted by $\omega = [r^T \ \ d^T]^T$, which is governed by

$$\dot{\omega} = S\omega \tag{2}$$

Where $S = \begin{bmatrix} A_r & \\ & A_d \end{bmatrix}$ is appropriate dimension matrix.

Equation (2) is called exosystem (or leader system) and equation (1) describes the dynamics of agents (or follower system) .We can represent a switching adjacency matrix $\mathcal{A}_{\sigma(t)} = [a_{ij}(t)]$, $where\ \ j, i = 0,1, \dots, N$. If the control input of agent i $(or\ u_i)$ can access the reference signal ω of exosystem then $a_{i0}(t) > 0$, for $\{i = 1, \dots N\}$ and remaining elements of $\mathcal{A}_{\sigma(t)}$ are satisfying $a_{ii}(t) = 0$ for any $t \geq 0$, $i = 0,1, \dots, N$ .Let $\bar{\mathcal{G}}_{\sigma(t)} = (\bar{\mathcal{O}}, \bar{\mathcal{E}}_{\sigma(t)})$ be a dynamic directed graph of $\mathcal{A}_{\sigma(t)}$.Then node set $\bar{\mathcal{O}} = \{0,1, \dots, N\}$ with 0 associated to the exosystem and all other nodes associated to N subsystems, and $(j,\ i) \in \bar{\mathcal{E}}_{\sigma(t)}$ if and only if $a_{ij}(t) > 0$ at time instant t.

A dynamic compensator having states $\eta_i \in \mathcal{R}^q, i = 1, \dots, N$ is described by following equation.

$$\dot{\eta}_i = S_i\eta_i + \gamma\left(\textstyle\sum_{j\in N_i(t)} a_{ij}(t)(\eta_j - \eta_i)\right) + a_{i0}(t)(\omega - \eta_i) \tag{3}$$

Where γ is an arbitrary constant. $a_{i0}(t) > 0$ if and only if node i access the exogenous signal otherwise $a_{i0}(t) = 0$.Equation (3) also tells us about the dependency of η_j and η_i on each other, $j \in \mathcal{N}_i(t)$,we call equation (3) a distributed observer .

We will give a dynamic measurement output feedback control as

$$\begin{cases} u_i = \mathcal{K}_{1i}\xi_i + \mathcal{K}_{2i}\eta_i \\ \dot{\eta}_i = S_i\eta_i + \gamma\left(\displaystyle\sum_{j\in N_i(t)} a_{ij}(t)(\eta_j - \eta_i)\right) + a_{i0}(t)(\omega - \eta_i) \\ \dot{\xi}_i = A_i\xi_i + B_i u_i + E_i\eta_i + L_i(C_{mi}\xi_i + D_{mi}u_i + F_{mi}\eta_i - y_{mi}) \end{cases} \tag{4}$$

Where $\mathcal{K}_{1i} \in \mathcal{R}^{m_i\times n_i}$, $\mathcal{K}_{2i} \in \mathcal{R}^{m_i\times q}$ are controller gains and $L_i \in \mathcal{R}^{m_i\times p_i}$ is observer gain.The follower system (1), feedback control law (4) with exogenous system (2) can be put into following closed loop system form:

$$\begin{aligned} &\dot{x}_c = A_{c,\sigma(t)}x_c + B_{c,\sigma(t)}\omega \ \ , \qquad x_c(0) = x_{c0} \\ &\dot{\omega} = S\omega \\ &e = C_{c,\sigma(t)}x_c + D_{c,\sigma(t)}\omega \end{aligned} \tag{5}$$

Detail of $A_{c,\sigma(t)}$, $B_{c,\sigma(t)}$, $C_{c,\sigma(t)}$ and $D_{c,\sigma(t)}$ matrices which expressed in above equation (5) is shown in theorem 1.

Now we are ready to describe the cooperative output regulation of heterogeneous MAS.

Definition 1: Given the MAS (1), the exosystem (2), and corresponding $\bar{\mathcal{G}}_{\sigma(t)}$ digraph, find the measurement output feedback control law (4) such that the following properties hold.

(1) Origin of closed loop system $\dot{x}_c$ is exponentially stable when $\omega = 0$.

(2) For $x_i(0)$,$\eta_i(0), and\ \omega(0)$

$$\lim_{t\to\infty} e_i(t) = 0, \qquad i = 1,2 \dots, N$$

Where $x_i(0)$,$\eta_i(0), and\ \omega(0)$ are initial conditions.

Let us make few assumptions related to classical linear output regulation problem and regulation problem of switched systems.

A 1: All the eigenvalues of S do not have negative real part.

A 2: The pairs$(A_i\ ,\ B_i), i = 1,2 \dots, N$ are stabilizable.

A 3: The pairs$(C_{mi}\ ,\ A_i), i = 1,2 \dots, N$ are detectable.

A 4: The linear matrix equations

$$\begin{aligned} X_i S &= A_i X_i + B_i U_i + E_i \\ 0 &= C_i X_i + D_i U_i + F_i \quad ,\ i = 1,2 \dots, N \end{aligned} \tag{6}$$

Where X_i and U_i are the solution of linear matrix equation.

A 5: There exists a subsequence $\{i_k\}$ of $\{i: i = 0,1, \dots\}$ with $t_{ik+1} - t_{ik} < v$ for nonnegative v in such a way that leader node or exosystem in the union graph $\bigcup_{j=i_k}^{i_{k+1}-1} \bar{G}_{\sigma(t)}$ can access all other nodes.

Remark 1: Assumptions from ***A1*** to ***A4*** are standard assumptions and also used by [12], [14].Our results will discuss the directed graph with fixed and variable switching topologies and follower nodes are higher order so we need extra assumption ***A 5*** for switching network topologies. Assumptions similar to ***A 5*** are also discussed in [14]

3. Solvability of the problem

In this section, we will discuss two lemmas related to linear switched systems.

***Lemma* 1 (Huang [14])** Suppose that origin of unforced part of closed-loop linear switched system (5)

$$\dot{x}_c = A_{c,\sigma(t)} x_c$$

is exponentially stable. If, the following equations satisfies with a constant matrix X_c for all positive values of time.

$$\begin{aligned} X_c S &= A_{c,\sigma(t)} X_c + B_{c,\sigma(t)} \\ 0 &= C_{c,\sigma(t)} X_c + D_{c,\sigma(t)} \end{aligned} \tag{7}$$

Then $$\lim_{t\to\infty} e(t) = 0 \tag{8}$$

Remark 2: This ***Lemma 1*** also discussed in (Y. Su [16]) for homogeneous agents.

Let $\Delta_{\sigma(t)} = block\ diag(a_{10}, \dots, a_{N0})$ with $a_{i0} > 0$, then $\bar{\mathcal{L}}_{\sigma(t)}$ partitioned as

$$\bar{\mathcal{L}}_{\sigma(t)} = \left(\begin{array}{c|c} \sum_{j=1}^{N} a_{0j}(t) & -[a_{01}(t), \dots, a_{0N}(t)] \\ \hline -\Delta_{\sigma(t)} \mathbb{1}_N & H_{\sigma(t)} \end{array}\right)$$

Where $a_{0j} > 0$ for all $j = 1, \dots, N$ if $(j, 0) \in \bar{\varepsilon}_{\sigma(t)}$ .then $\Delta_{\sigma(t)} 1_N = H_{\sigma(t)} 1_N$ Since $\bar{\mathcal{L}}_{\sigma(t)} 1_{N+1} = 0$

Lemma* 2 (Huang [14])** Under ***A 1 and ***A 5*** the origin of linear switched system

$$\begin{pmatrix} \dot{\zeta} \\ \dot{\xi} \end{pmatrix} = \begin{pmatrix} M & N \\ 0 & I_N \otimes S - \gamma(H_{\sigma(t)} \otimes I_q) \end{pmatrix} \begin{pmatrix} \zeta \\ \xi \end{pmatrix} \tag{9}$$

Of equation (9) is exponentially stable with a Hurwitz matrix M and random positive constant γ .

Remark 3: When $\rho = 1$ then switching network topology becomes fixed network topology .In this special case ***A 5*** and $H \triangleq H_{\sigma(t)}$ are specialized to the following.

A 6: $\Delta \triangleq \Delta_{\sigma(t)} \neq 0$

To remove dependency of Assumption ***A 1*** for the case $\rho = 1$, we can sharpen the ***Lemma* 2** by having sufficiently large positive constant γ .

Corollary* 1 (Huang [14]),** Under ***A 6, the origin of the linear system

$$\begin{pmatrix} \dot{\zeta} \\ \dot{\xi} \end{pmatrix} = \begin{pmatrix} M & N \\ 0 & I_N \otimes S - \gamma(H_{\sigma(t)} \otimes I_q) \end{pmatrix} \begin{pmatrix} \zeta \\ \xi \end{pmatrix}$$

is Hurwitz with a Hurwitz matrix M and random large positive constant γ .

Under ***A 2,*** there exists $\mathcal{K}_{1i}, i = 1, \dots, N,$ such that $A_i + B_i \mathcal{K}_{1i}$, is Hurwitz. Let $\mathcal{K}_{2i}$ be as follows:

$$\mathcal{K}_{2i} = U_i - \mathcal{K}_{1i} X_i \,, \ i = 1,2 \dots, N \tag{10}$$

X_i *and* U_i are the solution of equation (6).

Let we have following diagonal matrices:

$A = \mathcal{B}\ell ock\ diag(A_1, \dots, A_N)$, $F = \mathcal{B}\ell ock\ diag(F_1, \dots, F_N)$,
$B = \mathcal{B}\ell ock\ diag(B_1, \dots, B_N)$, $X = \mathcal{B}\ell ock\ diag(X_1, \dots, X_N)$,
$C = \mathcal{B}\ell ock\ diag(C_1, \dots, C_N)$, $U = \mathcal{B}\ell ock\ diag(U_1, \dots, U_N)$,
$D = \mathcal{B}\ell ock\ diag(D_1, \dots, D_N)$, $\mathcal{K}_1 = \mathcal{B}\ell ock\ diag(\mathcal{K}_{11}, \dots, \mathcal{K}_{1N})$,
$E = \mathcal{B}\ell ock\ diag(E_1, \dots, E_N)$, $\mathcal{K}_2 = \mathcal{B}\ell ock\ diag(\mathcal{K}_{21}, \dots, \mathcal{K}_{2N})$,

Then equation (6) and equation (10) imply

$$\begin{aligned} X(I_N \otimes S) &= AX + B\mathcal{K}_1 X + B\mathcal{K}_2 + E \\ 0 &= CX + D\mathcal{K}_1 X + D\mathcal{K}_2 + F \end{aligned} \tag{11}$$

Theorem* 1 *: According to ***Definition 1*** and assumptions ***A 1 to A 5*** cooperative output regulation of heterogeneous multi-agent problem can be solved by output feedback control law (4) if origin of closed loop system (5) is exponentially stable, where $\mathcal{K}_{1i}$ such that $A_i + B_i \mathcal{K}_{1i}$,is Hurwitz. $\mathcal{K}_{2i}$ defined in (10).Under ***A 3*** **,**there exits $L_i, i = 1, \dots, N$ such that $A_i + L_i C_{mi}$ is Hurwitz . Where γ is random nonnegative constant.

Proof: The ith agent closed loop system under control law (4) is

$$\begin{aligned} \dot{x}_i &= A_i x_i + B_i(\mathcal{K}_{1i}\xi_i + \mathcal{K}_{2i}\eta_i) + E_i\, \omega \\ \dot{\xi}_i &= (A_i + B_i \mathcal{K}_{1i} + L_i C_{mi})\xi_i + (B_i \mathcal{K}_{2i} + E_i + L_i F_{mi})\eta_i \\ &\quad - L_i C_{mi} x_i - L_i F_{mi} \omega \\ \dot{\eta}_i &= S_i \eta_i + \gamma\left(\sum_{j\in N_i(t)} a_{ij}(t)(\eta_j - \eta_i)\right) + a_{i0}(t)(\omega - \eta_i) \\ e_i &= C_i x_i + D_i(\mathcal{K}_{1i}\xi_i + \mathcal{K}_{2i}\eta_i) + F_i \omega \,, \qquad \dot{\omega} = S\omega \end{aligned} \tag{12}$$

Let $x = [x_1^T, \dots, x_N^T]$, $\eta = [\eta_1^T, \dots, \eta_N^T]$, $\xi = [\xi_1^T, \dots, \xi_N^T]$
$L = block\ diag[L_1, \dots, L_N], F_m = block\ diag[F_{m1}, \dots, F_{mN}]$,
$C_m = block\ diag[C_{m1}, \dots, C_{mN}]$, $\hat{\omega} = 1_N \otimes \omega$

The equation (12) can be written in the form of (5)

$$\begin{aligned} \dot{x} &= Ax + B(\mathcal{K}_1 \xi + \mathcal{K}_2 \eta) + E\, \omega \\ \dot{\xi} &= (A + B\mathcal{K}_1 + LC_m)\xi + (B\mathcal{K}_2 + E + LF_m)\eta - LC_m - LF_m \omega \\ \dot{\eta} &= ((I_N \otimes S) - \gamma(H_{\sigma(t)} \otimes I_q))\eta + \gamma(H_{\sigma(t)} \otimes I_q)\hat{\omega} \\ e &= Cx + D(\mathcal{K}_1 \xi + \mathcal{K}_2 \eta) + F\omega \quad , \quad \dot{\hat{\omega}} = (1_N \otimes S)\hat{\omega} \end{aligned} \tag{13}$$

With $[x^T \ \ \xi^T \ \ \eta^T\,]^T$ and

$$A_{c,\sigma(t)} = \begin{pmatrix} A & B\mathcal{K}_1 & B\mathcal{K}_2 \\ -LC_m & A + B\mathcal{K}_1 + LC_m & B\mathcal{K}_2 + E + LF_m \\ 0 & 0 & (I_N \otimes S) - \gamma(H_{\sigma(t)} \otimes I_q) \end{pmatrix}$$

$$B_{c,\sigma(t)} = \begin{pmatrix} E \\ -LF_m \\ \gamma(H_{\sigma(t)} \otimes I_q) \end{pmatrix} \,, \ C_{c,\sigma(t)} = (C \quad D\mathcal{K}_1 \quad D\mathcal{K}_2)$$

$D_{c,\sigma(t)} = F$ Let $M = \begin{pmatrix} A & B\mathcal{K}_1 \\ -LC_m & A + B\mathcal{K}_1 + LC_m \end{pmatrix}$

By row and column operation on M we get Hurwitz matrix so first property in ***Definition 1*** holds. For verification of second property:

Let $X_c \triangleq (X, X, I_{qN})^T$ then, by (11)

$$A_{c,\sigma(t)} X_c + B_{c,\sigma(t)} = \begin{pmatrix} AX + B\mathcal{K}_1 X + B\mathcal{K}_2 + E \\ AX + B\mathcal{K}_1 X + B\mathcal{K}_2 + E \\ (I_N \otimes S) \end{pmatrix}$$

$$((X(I_N \otimes S) \quad X(I_N \otimes S) \quad (I_N \otimes S))^T = X_c(I_N \otimes S)$$

And $C_{c,\sigma(t)} X_c + D_{c,\sigma(t)} = 0$

X_c is the solution of (7) .By ***Lemma* 1** $\lim_{t\to\infty} e(t) = 0$.

Remark 4: First select $\mathcal{K}_{1i}$ in such a way that eigenvalues of $A_i + B_i \mathcal{K}_{1i}$,are in left half plane (or Hurwitz) .Secondly, solve the linear matrix equation (6) for the solution of (X_i, U_i) after that solve equation (10) for $\mathcal{K}_{2i}$. Lastly, find a gain matrix L_i such that $A_i + L_i C_{mi}$ is Hurwitz and also select γ having arbitrary constant.

4. Simulation results

Taking the aircraft model given in [17] as an example. Consider a team of three aircrafts linear models named B747, Jetstar and MuPAL-α to track a given reference signal. First there is an exosystem (2) generating the disturbance and reference signals. Exosystem states are $\omega = (r_v, r_{\phi_1}, r_{\phi_2}, d_v, d_\phi)^T$, where $r_v, r_{\phi_1}, r_{\phi_2}$, are the reference signal states and d_v, d_ϕ are the side way velocity and roll angle disturbances respectively. Hence the exosystem (2) is

$$S = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2/9 & 0 & 0 \\ 0 & 1/2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & 0 \end{pmatrix}$$

A constant sideway velocity r_v Can represent with the first state of exosystem. r_{ϕ_1}, r_{ϕ_2} are sinusoidal roll angle states, and d_v, d_ϕ are represent sinusoidal sideway velocity/roll angle disturbance. Then for $i = 1, \ldots, 4$ (1) is used to describe the lateral-direction motion of ith aircrafts, whose input, outputs, and states are set as $u_i = (\delta_{a_c}, \delta_{r_c})^T, y_{mi} = (v_i, \phi_i)^T$, and $x_i = (V_i, P_i, \phi_i, r_i)^T$, wher V_i is sideway velocity, P_i, is roll rate, ϕ_i, is roll angle, r_i, is yaw rate, δ_{a_c}, is aileron deflection command and δ_{r_c} is rudder deflection command.

Case 1. Fixed network topology is shown in **Fig .1**. Leader node (exosystem) is represented with 0 and all other nodes are followers. In **Fig.2** and **Fig.3** fixed network topology results are shown.

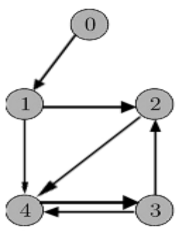

Fig.1. Fixed network topology (0 is leader agent)

The models of aircrafts can be represented by (1).

(Linear model of B747)

$$A_1 = A_2 = \begin{pmatrix} -0.0890 & 9.9566 & 9.6989 & -66.6209 \\ -0.0197 & -0.9750 & 0 & 0.3270 \\ 0 & 1 & 0 & 0.1495 \\ 0.0025 & -0.1660 & 0 & -0.2170 \end{pmatrix}$$

$$B_1 = B_2 = \begin{pmatrix} 0 & 0.9969 \\ -0.2270 & 0.0636 \\ 0 & 0 \\ -0.0264 & -0.1510 \end{pmatrix}, \quad C_1 = C_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

(Linear model of MuPAL-α)

$$A_3 = \begin{pmatrix} -0.1781 & 6.0791 & 9.7633 & -65.6230 & 0 & 2.8900 \\ -0.0575 & -3.8100 & 0 & 1.3430 & -10.7500 & 1.1870 \\ 0 & 1 & 0 & 0.0944 & 0 & 0 \\ 0.0253 & -0.0628 & 0 & -0.4750 & 0.3450 & -2.2200 \\ 0 & 0 & 0 & 0 & -11.1111 & 0 \\ 0 & 0 & 0 & 0 & 0 & -11.1111 \end{pmatrix}$$

$$B_3 = \begin{pmatrix} 0 & -2.8900 \\ 10.7500 & -1.1870 \\ 0 & 0 \\ -0.3450 & 2.2200 \\ 22.2222 & 0 \\ 0 & 22.2222 \end{pmatrix}, \quad C_3 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

(Linear model of Jetstar)

$$A_4 = \begin{pmatrix} -0.1750 & 9.5040 & 9.7828 & -135.9137 \\ -0.0387 & -1.3000 & 0 & 0.1810 \\ 0 & 1 & 0 & 0.0699 \\ 0.0242 & -0.1640 & 0 & -0.2610 \end{pmatrix}$$

$$B_4 = \begin{pmatrix} 0 & 5.7768 \\ -3.1400 & 1.6100 \\ 0 & 0 \\ 0.7670 & -1.8100 \end{pmatrix}, \quad C_4 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Other matrices are valued as:

$$C_{m1} = C_{m2} = C_{m4} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}, C_{m3} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

$$E_1 = E_2 = \begin{pmatrix} 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 10 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, E_3 = \begin{pmatrix} 0 & 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 10 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$E_4 = \begin{pmatrix} 0 & 0 & 0 & 25 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 50 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad Di = Dmi = 0, \quad Fmi = 0$$

$$Fi = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \end{pmatrix}, \quad i = 1, \ldots 4, \quad \gamma = 1$$

By solving regulator equation (6), such that

$$X_1 = X_2 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0.0001 & 0.4982 & -0.0050 & 0.0182 & -49.8086 \\ 0 & 0 & 1 & 0 & 0 \\ -0.0008 & 0.0264 & 0.0722 & -0.2605 & -2.7379 \end{pmatrix}$$

$$U_1 = U_2 = \begin{pmatrix} -0.0070 & -0.3057 & 0.0486 & -21.2621 & 29.4459 \\ 0.0105 & -0.1984 & 0.0139 & -10.4867 & 17.5297 \end{pmatrix}$$

$$X_3 = \begin{pmatrix} 1.0000 & 0 & 0 & 0 & 0 \\ 0.0002 & 0.4960 & -0.0138 & -0.0119 & -9.9173 \\ 0 & 0 & 1 & 0 & 0 \\ -0.0022 & 0.0423 & 0.1465 & 0.1261 & -0.8766 \\ -0.0089 & -0.3552 & 0.0692 & -2.5429 & 6.7954 \\ 0.0223 & -0.1696 & -0.0404 & -1.3048 & 1.7967 \end{pmatrix}$$

$$U_3 = \begin{pmatrix} -0.0045 & -0.1760 & 0.0381 & -1.5773 & 3.2833 \\ 0.0112 & -0.0857 & -0.0185 & -0.7333 & 0.8397 \end{pmatrix}$$

$$X_4 = \begin{pmatrix} 1.0000 & 0 & 0 & 0 & 0 \\ 0.0001 & 0.4906 & -0.0208 & -0.0074 & -9.7928 \\ 0 & 0 & 1.0000 & 0 & 0 \\ -0.0008 & 0.0627 & 0.1393 & 0.0494 & -1.3862 \end{pmatrix}$$

$$U_4 = \begin{pmatrix} -0.0797 & -2.1705 & 0.7118 & -43.4961 & 41.5447 \\ 0.0316 & -0.7113 & -0.2095 & -1.6386 & 5.1669 \end{pmatrix}$$

Controller gains $\mathcal{K}_{1i}$, $\mathcal{K}_{2i}$ and observer gains L_i are

$$\mathcal{K}_{11} = \mathcal{K}_{12} = \begin{pmatrix} 0.0603 & 3.1329 & 2.9505 & -2.3714 \\ -1.8951 & -7.6450 & -6.2055 & 42.3486 \end{pmatrix}$$

$$\mathcal{K}_{21} = \mathcal{K}_{22} = \begin{pmatrix} -0.1424 & -3.5589 & -1.8430 & -43.3557 & 68.9369 \\ 1.9634 & 0.3851 & -0.0640 & -3.7883 & -10.9941 \end{pmatrix}$$

$$\mathcal{K}_{31} = \begin{pmatrix} 0.6317 & 1.8551 & 3.7856 & -11.1149 & -2.8502 & 1.6721 \\ -1.6257 & -2.3377 & -4.3630 & 31.5838 & 2.0016 & -5.7057 \end{pmatrix}$$

$$\mathcal{K}_{32} = \begin{pmatrix} -0.7238 & -1.3551 & -1.8284 & -5.2195 & 28.3015 \\ 1.8523 & -0.5178 & -0.6850 & -7.0990 & 1.9908 \end{pmatrix}$$

$$\mathcal{K}_{41} = \begin{pmatrix} 0.5656 & 2.0465 & 2.2831 & -4.9442 \\ -1.8777 & -0.7789 & -0.6502 & 17.0345 \end{pmatrix}$$

$$\mathcal{K}_{42} = \begin{pmatrix} -0.5768 & -1.1946 & -1.8670 & -22.5874 & 117.8418 \\ 1.9025 & -0.2601 & -0.5700 & -6.0349 & 25.3734 \end{pmatrix}$$

$$L_1 = L_2 = \begin{pmatrix} -22.6999 & -0.4071 \\ -0.2810 & -0.4831 \\ -0.4071 & -2.4446 \\ 1.8483 & -0.3508 \end{pmatrix}, L_3 = \begin{pmatrix} -21.5673 & -0.3613 \\ 0.2035 & -0.1845 \\ -0.3613 & -2.1700 \\ 1.7786 & -0.2679 \\ 0.0236 & 0.0050 \\ -0.0523 & 0.0004 \end{pmatrix}$$

$$L_4 = \begin{pmatrix} -31.8164 & -0.2173 \\ -0.1120 & -0.3121 \\ -0.2173 & -2.2903 \\ 1.8722 & -0.1590 \end{pmatrix}$$

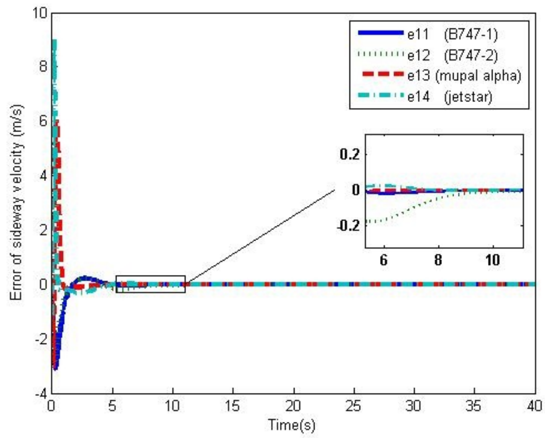

Fig.2. Trajectories of velocity errors e1 in Case 1

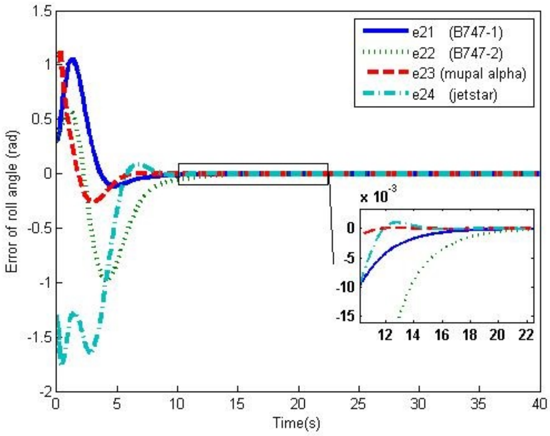

Fig.3. Trajectories of Roll angle errors e2 in Case 1

Case 2(a). Now we assume that the switching topologies can be change with following switching signal:

$$\sigma(t) = \begin{cases} 1 & if\ \mathrm{K}T \leq t \leq (\mathrm{K}+0.25)T \\ 2 & if\ (\mathrm{K}+0.25)T \leq t \leq (\mathrm{K}+0.5)T \\ 3 & if\ (\mathrm{K}+0.5)T \leq t \leq (\mathrm{K}+0.75)T \\ 4 & if\ (\mathrm{K}+0.75)T \leq t \leq (\mathrm{K}+1)T \end{cases} \quad \mathrm{K} = 0,1,2, \ldots$$

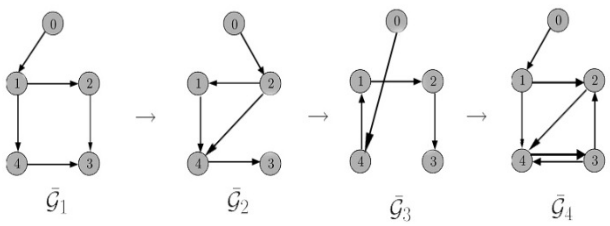

Fig.4.Fully connected switching topologies

Fig.4 shows fully connected switching topology $\bar{\mathcal{G}}_{\mathcal{P}}$ with $\mathcal{P} = \{1, \ldots, 4\}$ .We applying the measurement output feedback controller on fully connected switching topologies with T=2s. Results are given in **Fig.5** and **Fig.6**. ***A 5*** is essential condition to solve the switching network topologies for cooperative output regulation of MAS .From the Results of fixed and fully connected switching topologies it can be seen that roll angle and sideway velocity of the agents converge to their center.

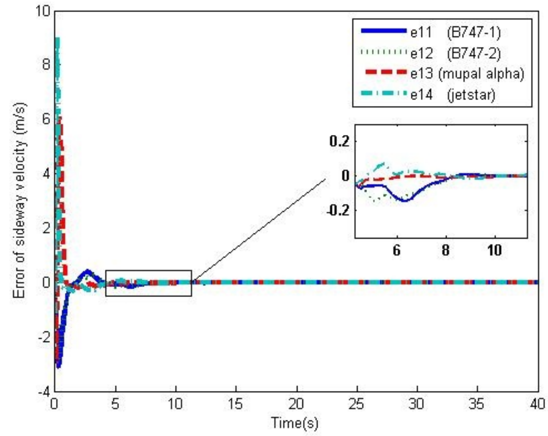

Fig.5. Trajectories of velocity errors e1 in Case 2(a)

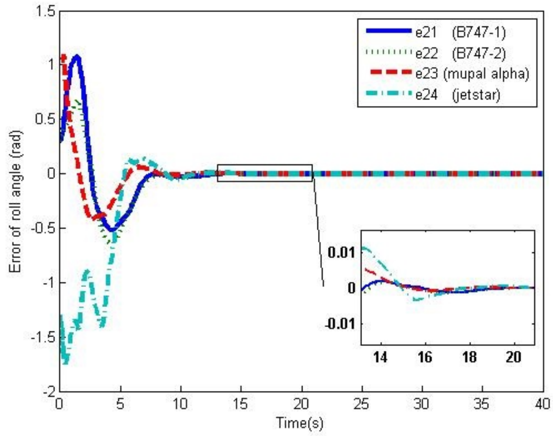

Fig.6. Trajectories of Roll angle errors e2 in Case 2(a)

Case 2(b). We discuss the case of partially connected switching topology when communication between leader and follower or between two neighbors is lost. All the parameters are same as in case 2 (a) and initial values are randomly chosen between [0, 1]. We obtain partially connected switching topology in **Fig.7** and its Simulation results in **Fig.8** and **Fig.9**. It is observed that sideway velocity and roll angle errors of all agents converge to zero.

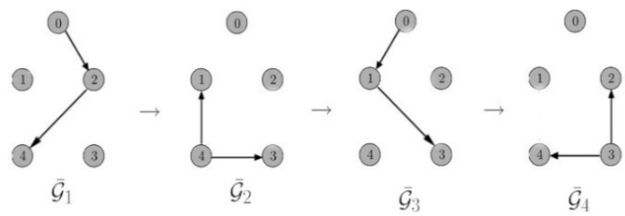

Fig.7. Partially connected switching topologies

Graph can switch periodically from $\bar{\mathcal{G}}_1\ to\ \bar{\mathcal{G}}_4$ and as long as $\bar{\mathcal{G}}_{\sigma(t)}$ satisfies ***A 5*** cooperative output regulation can be solved with any switching period. On the other hand regulation error of any follower approach to zero, if that follower in the union graph is reachable from the leader.

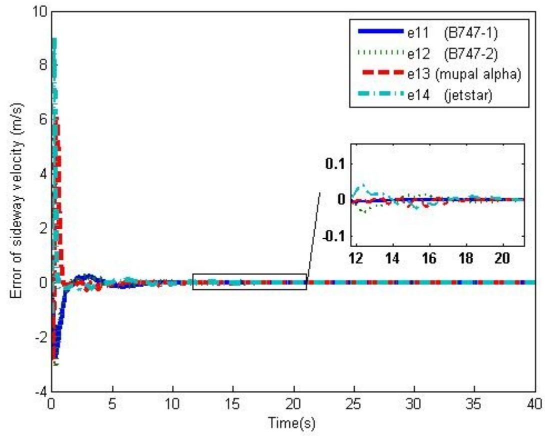

Fig.8. Trajectories of velocity errors e1 in Case 2(b)

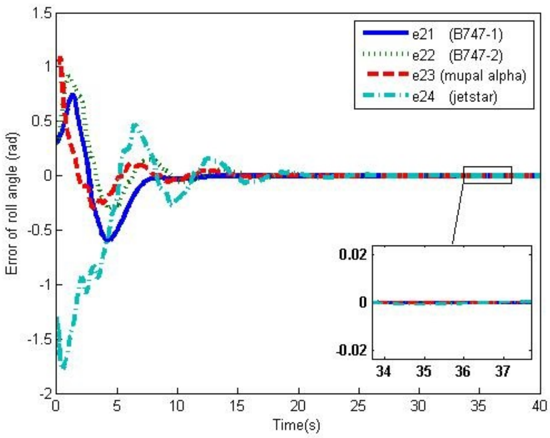

Fig.9. Trajectories of Roll angle errors e2 in Case 2(b)

5. Conclusions

This paper presents a measurement output feedback control method to solve the cooperative output regulation of higher-order MAS under fixed and switching network topologies. Luneburg observer and distributed observer are used to measure the internal states and relative states of agents.The considered output regulation problem is solved under some standard assumptions. Further work will be focused on the cooperative output regulation problem for switching exosystem and linear parameter varying systems.

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