

Inverse Optimal Control Approach to Model Predictive Control for Linear System Models

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Abstract

In this study, we propose an inverse optimal control based model predictive control approach. In inverse optimal control strategy, we firstly construct a stabilizing feedback control law and then search a meaningful cost functional. In that respect, we develop an alternative to solving the Riccati equation in MPC for linear time invariant system models. The control law is established with an appropriate scalar matrix which is found by using Big-Bang Big-Crunch(BB-BC) optimization algorithm. Simulations are done on a liquid level control system and the performance of the proposed method is compared with the performances of the classical model predictive, linear quadratic regulator and classical discrete time PID controller methods. The performance of the proposed controller is much better than the other controllers in respect to various criteria.

1. Introduction

Model Predictive Control (MPC) methods [1] are used in a wide range of applications from chemistry to aerospace [2–4]. The model of the system plays a crucial role in all MPC techniques [4–7] and the models obtained by linearizing the original nonlinear model around an operating point are mostly preferred. The main idea in MPC is to define an optimization problem with the prediction of the future behavior of the system obtained from the system model for a prediction horizon and then find the optimal control signal for a predetermined control horizon. The key difference between MPC and Linear Quadratic Regulator (LQR) is that MPC solves the optimization problem within a moving time horizon window whilst LQR solves the same problem within infinite window. The advantages of using a moving time horizon window include the ability to perform real-time optimization with hard constraints on plant variables [8].

When optimal control problem is considered for infinite horizon and nonlinear systems, the problem requires the solution of Hamilton Jacobi Bellman (HJB) equation which is extremely difficult. Moreover, an analytical solution of HJB does not exist for most nonlinear systems[9-10]. Inverse Optimal Control (IOC) theory views the problem from a different perspective. In this approach, firstly, a stabilizing feedback control law for a given plant is constructed. Then, a meaningful cost functional that depends on the state variables and control inputs is searched [10-11]. In [11], an IOC approach is proposed where the control law is constructed using a Control Lyapunov Function (CLF) and then the IOC problem is transformed into the determination of an appropriate scalar matrix.

In this study, we propose a new strategy for the solution of MPC problem where we employ IOC approach given in [11]. We develop and implement this strategy for linear time invariant system models. Therefore, this procedure provides an alternative to solving the Riccati equation in MPC. The control law is established with an appropriate scalar matrix which is found by using Big-Bang Big-Crunch(BB-BC) optimization algorithm [12] for each step.

This paper is organized in five chapters as follows: the classical MPC method is described in chapter 2. IOC method is covered in chapter 3. Chapter 4 describes the proposed IOC based MPC approach for linear systems. Finally, chapter 5 covers the results and discussions of this research.

2. Model Predictive Control (MPC)

The state space representation of linear time invariant system in discrete-time is represented by the following relations,

$$\begin{aligned} \mathbf{x}_{k+1} &= \mathbf{A}\mathbf{x}_k + \mathbf{B}u_k \\ y_k &= \mathbf{C}\mathbf{x}_k \end{aligned} \quad (1)$$

When the classical MPC methods employ the above state space model, the system equation can be rewritten as in (2) involving prediction horizon denoted by k_p .

$$\begin{aligned} \hat{\mathbf{x}}_{k+k_p+1} &= \mathbf{A}\hat{\mathbf{x}}_{k+k_p} + \mathbf{B}\hat{u}_k \\ \hat{y}_{k+k_p} &= \mathbf{C}\hat{\mathbf{x}}_{k+k_p} \end{aligned} \quad (2)$$

Here, $\mathbf{A} \in \mathbb{R}^{N \times N}$, $\mathbf{B} \in \mathbb{R}^{N \times l}$ and $\mathbf{C} \in \mathbb{R}^{l \times N}$ are the system, the input and the output matrices, respectively. $\hat{\mathbf{x}}_{k+k_p}$ refers to the prediction state in k_p steps. In addition, prediction state is equal to system state while $k_p=0$ which means $\hat{\mathbf{x}}_k = \mathbf{x}_k$. In this study, we assume that there is no constraint on system. Relying on [8] the following definitions can be made

$$\begin{aligned} \hat{\mathbf{x}}_{k+k_p+1}^{ag} &= \begin{bmatrix} \Delta\hat{\mathbf{x}}_{k+k_p+1} \\ \hat{y}_{k+k_p+1} \end{bmatrix}^T, \quad \mathbf{A}_g = \begin{bmatrix} \mathbf{A} & o_m^T \\ \mathbf{C}\mathbf{A} & 1 \end{bmatrix}, \quad \mathbf{B}_g = \begin{bmatrix} \mathbf{B} \\ \mathbf{C}\mathbf{B} \end{bmatrix}, \\ \mathbf{C}_g &= \begin{bmatrix} o_m & 1 \end{bmatrix} \\ \Delta\hat{\mathbf{x}}_{k+k_p+1} &= \hat{\mathbf{x}}_{k+k_p+1} - \hat{\mathbf{x}}_{k+k_p}, \\ \Delta\hat{u}_{k+k_p} &= \hat{u}_{k+k_p} - u_{k+k_p-1}, \quad o_m = [0 \quad \dots \quad 0] \end{aligned} \quad (3)$$

Using the definitions given in (3) and system model (2) we finally obtain the following augmented system dynamic representation

$$\begin{aligned}\hat{\mathbf{x}}_{k+k_p+1}^{ag} &= \mathbf{A}_g \hat{\mathbf{x}}_{k+k_p}^{ag} + \mathbf{B}_g \Delta \hat{u}_{k+k_p} \\ \hat{y}_{k+k_p} &= \mathbf{C}_g \hat{\mathbf{x}}_{k+k_p}^{ag}\end{aligned}\quad (4)$$

Denoting the prediction horizon as K_y and the control horizon as K_u , we can define the following vectors.

$$\begin{aligned}\mathbf{Y} &= [\hat{y}_{k+1} \quad \hat{y}_{k+2} \quad \dots \quad \hat{y}_{k+K_y}]^T \\ \Delta \mathbf{U} &= [\Delta u_k \quad \Delta u_{k+1} \quad \dots \quad \Delta u_{k+K_u}]^T\end{aligned}\quad (5)$$

From (3) and (5), the system output can be represented in the following compact form,

$$\mathbf{Y} = \mathbf{F} \hat{\mathbf{x}}_k^{ag} + \boldsymbol{\varphi} \Delta \mathbf{U} \quad (6)$$

where

$$\mathbf{F} = \begin{bmatrix} \mathbf{C}_g \mathbf{A}_g \\ \mathbf{C}_g \mathbf{A}_g^2 \\ \vdots \\ \mathbf{C}_g \mathbf{A}_g^{K_y} \end{bmatrix}, \quad \boldsymbol{\varphi} = \begin{bmatrix} \mathbf{C}_g \mathbf{B}_g & 0 & \dots & 0 \\ \mathbf{C}_g \mathbf{A} \mathbf{B}_g & \mathbf{C}_g \mathbf{B}_g & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{C}_g \mathbf{A}^{K_y-1} \mathbf{B}_g & \mathbf{C}_g \mathbf{A}^{K_y-2} \mathbf{B}_g & \dots & \mathbf{C}_g \mathbf{A}^{K_y-K_u} \mathbf{B}_g \end{bmatrix} \quad (7)$$

Relying on the output trajectory error and the control effort the cost function can be defined as follows:

$$F(\mathbf{u}) = \sum_{kp=1}^{K_y} (y_{k+k_p}^d - \hat{y}_{k+k_p})^2 + \sum_{kp=0}^{K_u} R (\Delta \hat{u}_{k+k_p})^2 \quad (8)$$

Here, R is the penalty term of the control effort. In order to construct the problem as the tracking of a reference signal, the following definition is made on the reference signal,

$$\mathbf{Y}^d = [y_k^d \quad y_k^d \quad \dots \quad y_k^d]^T \quad \mathbf{Y}^d \in \mathbb{R}^{K_y \times 1} \quad (9)$$

Then, the cost function (8) is rewritten in a compact form as follows:

$$F(\mathbf{u}) = (\mathbf{Y}^d - \mathbf{Y})^T (\mathbf{Y}^d - \mathbf{Y}) + \Delta \mathbf{U}^T \mathbf{R}_{ag} \Delta \mathbf{U} \quad (10)$$

where $\mathbf{I} \in \mathbb{R}^{K_u \times K_u}$ is an identity matrix, $\mathbf{R}_{ag} = R \mathbf{I}$ is a diagonal matrix. Since the evolved cost function is quadratic the optimal control problem can be solved by using a gradient based approach and the control increment can be obtained as follows

$$\Delta \mathbf{U} = (\boldsymbol{\varphi}^T \boldsymbol{\varphi} + \mathbf{R}_{ag})^{-1} \boldsymbol{\varphi}^T (\mathbf{Y}^d - \mathbf{F} \hat{\mathbf{x}}_k) \quad (11)$$

Therefore, the following control signal (u_k) is employed to the system

$$u_k = u_{k-1} + \Delta \hat{u}_k \quad (12)$$

Here ($\Delta \hat{u}_k$) is the first element of $\Delta \mathbf{U}$ obtained in every step.

2.2. Inverse Optimal Control Approach

A discrete-time affine-in-input nonlinear system model is represented as

$$\begin{aligned}\mathbf{x}_{k+1} &= f(\mathbf{x}_k) + g(\mathbf{x}_k) \mathbf{u}_k \\ \mathbf{y}_k &= h(\mathbf{x}_k)\end{aligned}\quad (13)$$

where $\mathbf{x}(t) \in \mathbb{R}^n$ represents the state of system, $\mathbf{u}(t) \in \mathbb{R}^m$ refers to input, and $f : \mathbb{R}^n \rightarrow \mathbb{R}^n$, $g : \mathbb{R}^n \rightarrow \mathbb{R}^{n \times m}$ are nonlinear functions ($f(0)=0$ and $g(\mathbf{x}_k) \neq 0 \quad \forall \mathbf{x}(t) \neq 0$). The cost function that has to be minimized for the regulator problem case is

$$V(\mathbf{x}_k) = \sum_{n=k}^{\infty} (l(\mathbf{x}_n) + \mathbf{u}_n^T \mathbf{R} \mathbf{u}_n) \quad (14)$$

where $l : \mathbb{R}^n \rightarrow \mathbb{R}^+$ is a positive semi-definite function, \mathbf{x}_k is the state vector and $\mathbf{R} \in \mathbb{R}^{m \times m}$ is a symmetric positive weighting matrix. This equation can be rewritten as

$$\begin{aligned}V(\mathbf{x}_k) &= l(\mathbf{x}_k) + \mathbf{u}_k^T \mathbf{R} \mathbf{u}_k + \sum_{n=k+1}^{\infty} (l(\mathbf{x}_n) + \mathbf{u}_n^T \mathbf{R} \mathbf{u}_n) \\ V(\mathbf{x}_k) &= l(\mathbf{x}_k) + \mathbf{u}_k^T \mathbf{R} \mathbf{u}_k + V(\mathbf{x}_{k+1})\end{aligned}\quad (15)$$

Using the principle of optimality, we obtain the following relation

$$V^*(\mathbf{x}_k) = \min_{\mathbf{u}_k} \{l(\mathbf{x}_k) + \mathbf{u}_k^T \mathbf{R} \mathbf{u}_k + V^*(\mathbf{x}_{k+1})\} \quad (16)$$

Then, the control rule that minimizes the cost function is obtained as

$$\mathbf{u}_k^* = \bar{u}(\mathbf{x}_k) = -\frac{1}{2} \mathbf{R}^{-1} g^T(\mathbf{x}_k) \frac{\partial V^*(\mathbf{x}_{k+1})}{\partial \mathbf{x}_{k+1}} \quad (17)$$

$V^*(\mathbf{x}_k)$ can be found by solving the following Hamiltonian-Jacobi-Bellman (HJB) partial differential equation in discrete time.

$$\begin{aligned}l(\mathbf{x}_k) + V^*(\mathbf{x}_{k+1}) - V^*(\mathbf{x}_k) + \dots \\ + \frac{1}{4} \left[\frac{\partial V^*(\mathbf{x}_{k+1})}{\partial \mathbf{x}_{k+1}} \right]^T g(\mathbf{x}_k) \mathbf{R}^{-1} g^T(\mathbf{x}_k) \frac{\partial V^*(\mathbf{x}_{k+1})}{\partial \mathbf{x}_{k+1}} = 0\end{aligned}\quad (18)$$

It is well known that solving HJB equation is a very cumbersome problem. For linear regulator problem case, this equation is reduced to the Riccati equation.

Instead of solving HJB equation this optimal control problem can be solved by “Inverse Optimal Control (IOC)” method. In literature there are many approaches to IOC problem [9, 10, 13–16]. One of these approaches rely on Definition 1 [17, 18].

Definition 1:[10]

Control Law \mathbf{u}_k^* (17) can be assumed to be inverse optimal control if

- i. It achieves global exponential stability of the equilibrium point $\mathbf{x}_k = 0$ for System (13)
- ii. It minimizes a cost functional defined as (14) for (13) where $l(\mathbf{x}_k) = -\bar{V}|_{u_k^*}$ and

$$\bar{V} := V(\mathbf{x}_{k+1}) - V(\mathbf{x}_k) + \mathbf{u}_k^T \mathbf{R} \mathbf{u}_k \leq 0 \quad (19)$$

Therefore, IOC requires the knowledge of $V(\mathbf{x}_k)$. Then, for this purpose a candidate quadratic CLF $V(\mathbf{x}_k)$ given in (20) that meets the conditions dictated in Definition 1 can be searched.

$$V^*(\mathbf{x}_k) = \frac{1}{2} \mathbf{x}_k^T \mathbf{P} \mathbf{x}_k \quad \mathbf{P} = \mathbf{P}^T > 0 \quad (20)$$

Here, $\mathbf{P} \in \Re^{n \times n}$ is assumed to be positive definite and symmetric, i.e. $\mathbf{P} = \mathbf{P}^T > 0$. The inverse optimal control law which satisfies stability and minimize the cost function (14) can be found by selecting an appropriate matrix \mathbf{P} . Then, the state feedback control law can be written as

$$\mathbf{u}_k^* = -\frac{1}{2}(\mathbf{R} + \mathbf{P}_2(\mathbf{x}_k))^{-1}\mathbf{P}_1(\mathbf{x}_k) \quad (21)$$

where $\mathbf{P}_1(\mathbf{x}_k) = g^T(\mathbf{x}_k)\mathbf{P}f(\mathbf{x}_k)$ and $\mathbf{P}_2(\mathbf{x}_k) = \frac{1}{2}g^T(\mathbf{x}_k)\mathbf{P}g(\mathbf{x}_k)$.

The following theorem gives the necessary condition for matrix \mathbf{P} to satisfy the requirements of Definition 1.

Theorem 1 [10]:

Consider the affine-in-input discrete-time nonlinear system (13).

If there exists a matrix $\mathbf{P} = \mathbf{P}^T > 0$ such that the following inequality holds

$$\begin{aligned} V_f(\mathbf{x}_k) - \frac{1}{4}\mathbf{P}_1^T(\mathbf{x}_k)(\mathbf{R} + \mathbf{P}_2(\mathbf{x}_k))^{-1}\mathbf{P}_1(\mathbf{x}_k) &\leq \zeta_Q \|\mathbf{x}_k\|^2 \\ V_f(\mathbf{x}_k) = V(f(\mathbf{x}_k)) - V(\mathbf{x}_k), \quad (22) \\ V(f(\mathbf{x}_k)) = \frac{1}{2}f^T(\mathbf{x}_k)\mathbf{P}f(\mathbf{x}_k) \text{ and } \zeta_Q > 0 \end{aligned}$$

Then, the equilibrium point ($\mathbf{x}_k = 0$) of the system (13) is globally exponential stabilized by the control law (21) with the CLF (20). Moreover, this control law will minimize the cost functional given (14), with $l(\mathbf{x}_k) = -\bar{V}|_{u_k^*}$. Hence, the optimal

value function will be equal to $V^*(\mathbf{x}_0) = V(\mathbf{x}_0)$ [10].

By this theorem the IOC problem transforms to the problem of finding an appropriate \mathbf{P} matrix. This approach is still in active research [11]. For linear systems this approach provides an alternative to the Discrete-time Algebraic Riccati Equation (DARE). Considering $f(\mathbf{x}_k) = \mathbf{A}\mathbf{x}_k$ and $g(\mathbf{x}_k) = \mathbf{B}$ the inverse optimal control law (21) can be found as

$$u_k^* = -\frac{1}{2}(\mathbf{R} + \mathbf{B}^T\mathbf{P}\mathbf{B})^{-1}\mathbf{B}^T\mathbf{P}(\mathbf{A}\mathbf{x}_k) \quad (23)$$

When this method is revised for the tracking problem the feedback control law becomes

$$u_k^* = -\frac{1}{2}(\mathbf{R} + \mathbf{B}^T\mathbf{P}\mathbf{B})^{-1}\mathbf{B}^T\mathbf{P}(\mathbf{A}\mathbf{x}_k - \mathbf{x}_{k+1}^d) \quad (24)$$

3. Model Predictive Control using inverse optimality approach

In this study we propose Model Predictive Control using Inverse Optimality approach (IO-MPC) presented in the previous section.

The augmented prediction states $\hat{\mathbf{x}}_{k+kp+1}^{ag}$ of the system can be found by using discrete-time linear augmented model (14). For any k^{th} step, the cost function (8) is reformulated as

$$V(\hat{\mathbf{z}}_{k+kp}) = \sum_{n=k+kp}^{k+K_y} (l(\hat{\mathbf{z}}_n) + \Delta\hat{u}_n^T R \Delta\hat{u}_n) \quad (25)$$

where $\hat{\mathbf{z}}_n = \hat{\mathbf{x}}_n^{ag} - \mathbf{x}_n^{agd}$. Here, \mathbf{x}_n^{agd} is the augmented desired trajectory. The control increment value that minimizes (25) along the prediction horizon is obtained as below

$$\Delta\hat{u}_{k+kp} = \bar{u}(\hat{\mathbf{z}}_{k+kp}) = -\frac{1}{2}R^{-1}\mathbf{B}_g^T \frac{\partial V^*(\hat{\mathbf{z}}_{k+kp+1})}{\partial \hat{\mathbf{z}}_{k+kp+1}} \quad (26)$$

The following definition is the revised version of Definition 1 where infinite time is limited to the prediction horizon K_y .

Definition 2:

$\Delta\hat{u}_{k+kp}$ in (26) is the control increment that

- i. It achieves (global) asymptotic stability of $\mathbf{x}_k^{ag} = 0$ for system (3) along reference \mathbf{x}_k^{agd}
 - ii. It minimizes the cost functional defined $V(\hat{\mathbf{z}}_{k+kp})$ where $I(\hat{\mathbf{z}}_{k+kp}) = -\bar{V}|_{\Delta\hat{u}_{k+kp}}$ and
- $$\bar{V} := V(\hat{\mathbf{z}}_{k+kp+1}) - V(\hat{\mathbf{z}}_{k+kp}) + (\Delta\hat{u}_{k+kp}^*)^T R \Delta\hat{u}_{k+kp}^* \leq 0$$
- and $V(\hat{\mathbf{z}}_{k+kp})$ is a (radially unbounded) positive definite function.

In this approach a candidate quadratic CLF $V^*(\hat{\mathbf{z}}_{k+kp})$ given in (28) that meets the conditions dictated in Definition 2 can be searched for each k step along prediction horizon.

$$V^*(\hat{\mathbf{z}}_{k+kp}) = \frac{1}{2}\hat{\mathbf{z}}_{k+kp}^T \mathbf{P} \hat{\mathbf{z}}_{k+kp} \quad \mathbf{P} = \mathbf{P}^T > 0 \quad (27)$$

Here, the IOC increment which satisfies Definition 2 and minimize the cost function (25) can be calculated as in (28) by finding an appropriate matrix \mathbf{P} along the prediction horizon.

$$\Delta\hat{u}_{k+kp}^* = -\frac{1}{2}(R + \frac{1}{2}\mathbf{B}_g^T \mathbf{P} \mathbf{B}_g)^{-1} \mathbf{B}_g^T \mathbf{P} (\mathbf{A}_g \hat{\mathbf{x}}_{k+kp}^{ag} - \mathbf{x}_{k+kp+1}^{agd}) \quad (28)$$

Then, the first control increment $\Delta\hat{u}_k^*$ from $\Delta\hat{u}_{k+kp}^*$ which is found by (28) is used in (12). Later on, the cost function (25) is defined again for the next steps by moving the prediction window. The optimality problem for each k step is solved by selecting an appropriate \mathbf{P} matrix and the optimal control increment $\Delta\hat{u}_k^*$ is found by (28). The optimal control input of the system is calculated using (12).

In the proposed IO-MPC method BB-BC algorithm [12] is used to find appropriate \mathbf{P} matrix elements for each k step. Firstly, the ranges of the candidate \mathbf{P} matrix elements are randomly generated. Then, in BB-BC algorithm the crunching point (m_c) is obtained by the candidate \mathbf{P} matrix which is the minimum value of the fitness function f^i defined as follows.

- i. Unless the candidate \mathbf{P} matrix is positive definite, A maximum F_{\max} is assigned to the fitness function f^i
- ii. Unless the conditions in Definition 2 are met when $k_p \leq K_u$, the fitness function should be F_{\max}
- iii. If \mathbf{P} matrix is positive definite and the conditions in definition 2 are met when $k_p \leq K_u$, the fitness function is determined as

$$f^i = F_{\mathbf{P}_{matrix}} = \sum_{kp=0}^{K_y} \left(\mathbf{e}_{k+kp+1}^T Q_{iompc} \mathbf{e}_{k+kp+1} \right) \quad (29)$$

where Q_{iompc} is a diagonal weighting matrix and

$$\begin{aligned}\mathbf{e}_{k+kp+1} &= \hat{\mathbf{x}}_{k+kp+1}^{ag} - \hat{\mathbf{x}}_{k+kp+1}^{agd} \\ \hat{\mathbf{x}}_{k+kp+1} &= f(\hat{\mathbf{x}}_{k+kp}) + g(\hat{\mathbf{x}}_{k+kp})\hat{u}_{k+kp} \\ \hat{u}_{k+kp} &= \begin{cases} \hat{u}_{k+kp-1} + \Delta\hat{u}_{k+kp} & kp \leq K_u \\ \hat{u}_{k+kp-1} & K_u \leq kp \leq K_y \end{cases}\end{aligned}$$

4. Simulation Results

In this study, simulations are done on the liquid level control system given in Figure 3.1. The performance of the proposed IO-MPC method is compared with the performances of the Classical Model Predictive Control (CMPC) [8], optimal LQR [19] and classical discrete time PID methods.

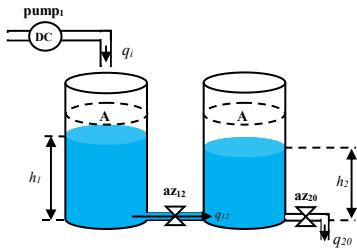


Fig. 1. The schematic structure of the liquid level system

The nonlinear mathematical model of the system is given as follows:

$$\begin{aligned}h_1(t) &= \frac{1}{A}[q_1(t) - q_{12}(t)] \\ h_2(t) &= \frac{1}{A}[q_{12}(t) - q_{20}(t)] \\ y(t) &= h_2(t)\end{aligned}\quad (30)$$

where

$$\begin{aligned}q_{12}(t) &= az_{12}S_n \operatorname{sgn}(h_1(t) - h_2(t))\sqrt{2g|h_1(t) - h_2(t)|} \\ q_{20}(t) &= az_{20}S_n\sqrt{2gh_2(t)}\end{aligned}\quad (31)$$

The input flow rate q_1 [m^3/s] is taken in the range of [0; 0.0001]. The liquid level heights h_1 [m] and h_2 [m] of tanks are both in the range of [0.001; 0.95]. The related parameters are given in Table 1.

Table 1. The parameters of liquid-level system

Parameters	Value/Definitions
$h_1(t)$: liquid level of tank ₁	(m)
$h_2(t)$: liquid level of tank ₂	output (m)
$q_1(t)$: supplying flow rate of pump ₁	input (m^3/s)
az_{12} : outflow coefficient between tank ₁ and tank ₂	0.3
az_{20} : outflow coefficient from tank ₂ to reservoir	0.27
A : Cross section of the cylinders	0.0154 (m^2)
S_n : section of connection pipe n	5×10^{-5} (m^2)
g : gravitation coefficient	9.81 (m/s^2)

The states, the input and output variables are defined as $\mathbf{x}(t) = [x_1(t), x_2(t)] = [h_1(t), h_2(t)]$, $u(t) = q_1(t)$ and $y(t) = x_2(t)$, respectively. Then, the discrete linear prediction model of the system is found by linearization at the equilibrium point $\mathbf{x}^* = [0.5; 0.276243]$ and the matrices in the augmented system model (3) become

$$\mathbf{A}_g = \begin{bmatrix} 0.9955 & 0.0045 & 0 \\ 0.0045 & 0.9918 & 0 \\ 0.0045 & 0.9918 & 1 \end{bmatrix}, \mathbf{B}_g = \begin{bmatrix} 64.7874 \\ 0.1474 \\ 0.1474 \end{bmatrix}, \mathbf{C}_g = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

The sampling period is $T_s = 1s$ and the desired (reference) system trajectory is $\mathbf{x}_k^{agd} = [0 \ 0 \ x_{2,k}^d]$ regarding \mathbf{C}_g .

The symmetrical \mathbf{P} matrix ($\mathbf{P} \in \mathbb{R}^{3 \times 3}$) for the proposed IO-MPC is shown as

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{12} = p_{21} & p_{13} = p_{31} \\ p_{21} = p_{12} & p_{22} & p_{23} = p_{32} \\ p_{13} = p_{31} & p_{32} = p_{23} & p_{33} \end{bmatrix} \quad (32)$$

The ranges of the diagonal and off-diagonal parameters are defined as $[10^{-6}; 1000]$ and $[-1000; 1000]$, respectively. R in (25) is chosen as 40000 for all control techniques. The prediction and the control horizon are both selected as $K_y = K_u = 50$. Q_{iomp} in (30) is chosen as

$$Q_{iomp} = \begin{bmatrix} 0.001 & 0 & 0 \\ 0 & 0.001 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (33)$$

Since CMPC and LQR structures need the linear model of the system, the augmented linear model (3) is used in the design of these controllers. The parameters of CMPC are chosen as

$$Q_{mpc} = Q_{iomp}, K_y^{cmpe} = 50 \text{ and } K_u^{cmpe} = 5.$$

The LQR parameters are chosen as $R = 40000$ and $Q_{lqr} = Q_{iomp}$ and the feedback control gain is found as $\mathbf{K}_{lqr} = [0.00315 \ 0.0732 \ 0.00449]$ by using MATLAB function “dlqr.m”.

In this study, digital PID is applied to trajectory tracking by the following structure,

$$u_k = u_{k-1} + [K_P \ K_I \ K_D] \begin{bmatrix} e_k - e_{k-1} \\ e_k \\ e_k - 2e_{k-1} + e_{k-2} \end{bmatrix} \quad (34)$$

The PID parameters are also found by using BB-BC method to minimize the fitness function given below

$$f_{PID}^i = F_{PID} = \sum_{k=1}^{K_{\max}} \left(\mathbf{e}_k^T Q_{pid} \mathbf{e}_k \right) \quad (35)$$

where Q_{pid} is the weighing matrix.

In simulations, all of the controllers are tested on nonlinear system model. The initial and reference liquid level values are taken as $h_{10} = h_{20} = 10^{-3}$ and $h_{1f} = 0.35, h_{2f} = 0.1973$,

respectively. The states of the system are shown as Fig. 2 for each controller. The performance measures are given in Table 2. It is clearly seen that the performance of the proposed controller is better than the others in all respects. The control inputs are given at Fig.3.

Table 2. Performance measures for controllers

Controller	%OS	t_{set} (s) (<i>Set. T.</i>)	SSE
adapted LQR	20.08	357.44	2.57
Digital PID	16.90	346.07	2.55
CMPC	18.09	349.97	2.53
IOMPC	3.434	282.75	2.41

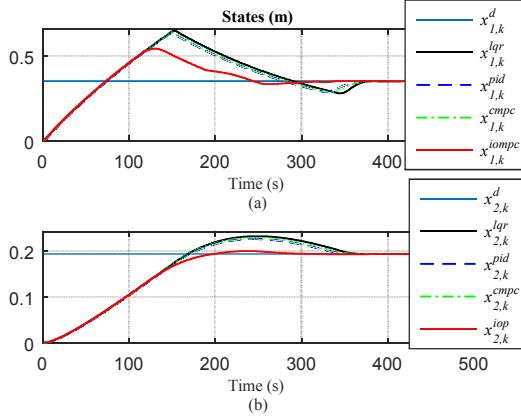


Fig. 2 Illustration of system states (x_k) for all control methods

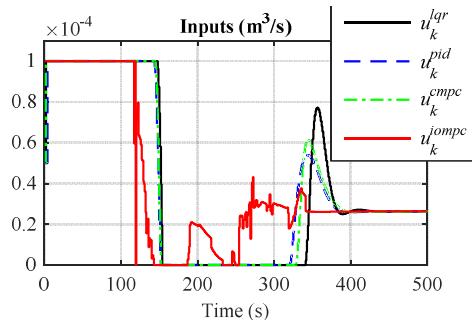


Fig. 3. Illustration of system inputs (u_k) for all control methods

6. Conclusions

In this study, a solution to MPC problem is proposed based on IOC approach. In that respect, we develop an alternative to solving the Riccati equation in MPC for linear time invariant system models. In this approach, the control law is obtained by searching an appropriate scalar matrix via Big-Bang Big-Crunch(BB-BC) optimization algorithm. Simulations are done on a liquid level control system. The proposed method is compared with the classical model predictive, linear quadratic regulator and classical discrete time PID controller methods on various performance criteria. It is observed that the performance of the proposed controller is much better than the other controllers.

7. References

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Acknowledgement

This study is supported within context of Namık Kemal University Support program on Participation in Scientific Activities