One Continuous and Digital Chaotic Attractor

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Abstract

Implementation of continuous chaotic systems in not easy task, because of strong sensitivity on parts parameters values, e.g. temperature dependence etc. On the other hand the digital systems can be easily simulated and inexpensive constructed with almost the same behavior as continuous systems. Digital chaotic attractor also enables simply changing parameters of system. This paper describes a new four-dimensional digital autonomous chaotic system derived from continuous chaotic system. Analysis of the results shows that this system has complex dynamics with some interesting characteristics. The digital chaotic system was also realized by means of microcontroller.

1. Introduction

Nonlinear systems able to develop chaotic dynamics have a great potential in a large class of applications, because of the complexity of the behavior they exhibit and of the sensitivity of their initial condition or operation to the process parameters. While these behavior have been seen as disadvantages, now such systems in some applications can be used, e.g. random number generators [1, 2] cryptographic methods based on chaotic systems [3 - 7], bio-medical engineering [8] and chaotic sensors [9 - 15].

In the investigation of chaos theory and applications, it is very important to find new chaotic systems or to enhance complex dynamics and topological structure based on the existing chaotic attractors. In 1999, Chen constructed a 3-D chaotic system via a simple state feedback to the second equation in the Lorenz system, yielding [16 - 22]

$$\begin{aligned} \dot{x}_1 &= a\left(x_2 - x_1\right) \\ \dot{x}_2 &= \left(c - a\right)x_1 + cx_2 - x_1x_2 \\ \dot{x}_3 &= x_1x_2 - bx_3 \end{aligned} \qquad (1)$$

In 2002 the so-called Lorenz system family was constructed as a connection of the Lorenz, Lü and Chen systems; it is described by

$$\begin{aligned} \dot{x}_1 &= \left(25\alpha + 10\right)\left(x_2 - x_1\right) \\ \dot{x}_2 &= \left(28 - 35\alpha\right)x_1 + \left(29\alpha - 1\right)x_2 - x_1x_3 \\ \dot{x}_3 &= x_1x_2 - \frac{\left(\alpha + 8\right)x_3}{3} \end{aligned} \qquad (2)$$

where α is a real parameter, and for all $\alpha \in [0, 1]$ the system is chaotic.

Qi et al. [23] proposed a 4-D autonomous system with the cubic nonlinearities which is described by

$$\begin{aligned} \dot{x}_1 &= a\left(x_2 - x_1\right) + x_2x_3x_4 \\ \dot{x}_2 &= b\left(x_1 + x_2\right) - x_1x_3x_4 \\ \dot{x}_3 &= -cx_3 + x_1x_2x_4 \\ \dot{x}_4 &= -dx_4 + x_1x_2x_3 \end{aligned} \qquad (3)$$

Here, x_i (i=1, 2, 3, 4) are the state variables and *a, b, c, d* are positive real constants, but such system cannot produce a four-wing chaotic attractor. By further study, it was found that the principle of generation of four-wing chaotic attractor is to destroy the exchangeable and symmetric properties of system (3) between the third or fourth equation, therefore modified system is

$$\begin{aligned} \dot{x}_1 &= a\left(x_2 - x_1\right) + x_2x_3x_4 \\ \dot{x}_2 &= b\left(x_1 + x_2\right) - x_1x_3x_4 \\ \dot{x}_3 &= -cx_3 + ex_2 + x_1x_2x_4 \\ \dot{x}_4 &= -dx_4 + x_1x_2x_3 \end{aligned} \qquad (4)$$

where e is a constant parameter.

When a =50, b = 4.3, c =13, d =20, e =6, system (4) is in a chaotic mode with the largest Lyapunov exponent (LE) l_1 = 2.016. With further changes of parameters a =50, b = 10, c =13, d =20, e =6, increase of the first positive LE, up to l_1 = 7.9812. The new derived continuous chaotic attractor is described in next part.

2. New Continuous Chaotic Attractor

After investigation was found new simplified (the term $x_1x_2x_3$ in first equation was deleted and ex_2 was placed in 4th equation). The new continuous chaotic system described by

$$\begin{aligned} \dot{x}_1 &= a\left(x_2 - x_1\right) \\ \dot{x}_2 &= b\left(x_1 + x_2\right) - x_1x_3x_4 \\ \dot{x}_3 &= -cx_3 + x_1x_2x_4 \\ \dot{x}_4 &= ex_2 - dx_4 + x_1x_2x_3 \end{aligned} \qquad (5)$$

where parameters are a =50, b = 10, c =26, d =15, e =5. Initial values are: x_{01}=0.1; x_{02}=0.02; x_{03}=0; x_{04}=0. The system (5) is dissipative, because

$$\nabla V = \frac{\partial \dot{x}_1}{\partial x_1} + \frac{\partial \dot{x}_2}{\partial x_2} + \frac{\partial \dot{x}_3}{\partial x_3} + \frac{\partial \dot{x}_4}{\partial x_4}$$
$$= -a + b - c - d = -50 + 10 - 26 - 15 < 0 \tag{6}$$

The system has only one real equilibrium $S_0 = [0 \quad 0 \quad 0 \quad 0]$. By linearizing system (5) at S_0, one obtains the Jacobian

$$J = \begin{bmatrix} a & -a & 0 & 0 \\ b & b & 0 & 0 \\ 0 & 0 & -c & 0 \\ 0 & e & 0 & d \end{bmatrix} \tag{7}$$

The eigenvaluse of matrix J are

$$\lambda_{1,2} = \frac{1}{2}\left[b - a \pm \sqrt{a^2 + 6ab + b^2} \right]$$
$$\lambda_3 = -c; \qquad \lambda_4 = -d \tag{8}$$

Therefore numerical values of $\lambda_{1\text{-}4}$ are [-15; 17.42; -57.42; -26]. Because $\lambda_2>0$ implying that the equilibrium S_0 is unstable. The simulation results of system described by eq. (5) are shown in Fig. 1-6, where are time evolutions of signals, phase portraits, frequency spectra, histograms and autocorrelations.

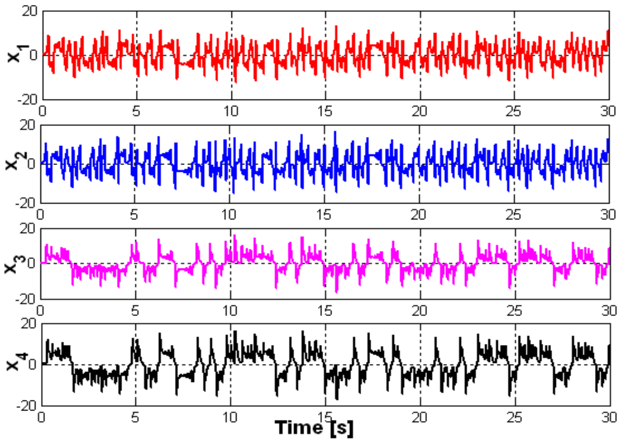

Fig. 1. Time evolution of state space variables, continuous system

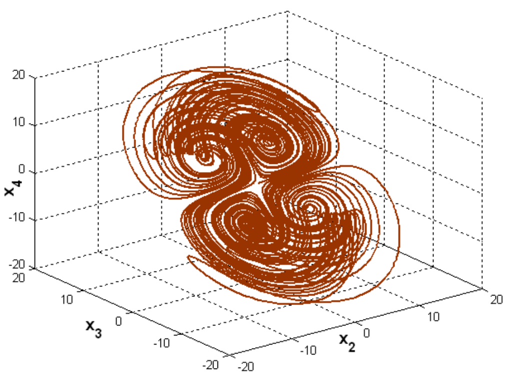

Fig. 2. 3D phase portrait of continuous system according (5)

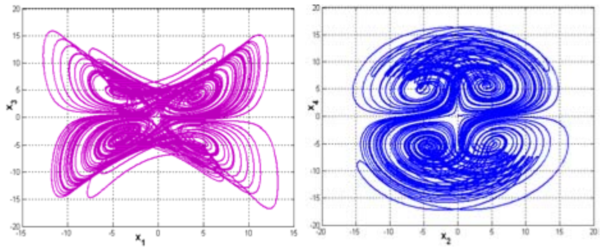

Fig. 3. Phase portrait of x_1 versus x_3 (left) and x_2 versus x_4 (right), continuous system

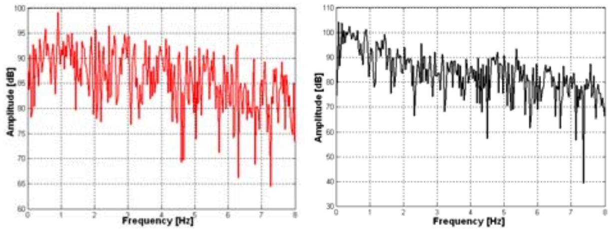

Fig. 4. Frequency spectrum of x_1 (left) and x_4 (right), continuous system

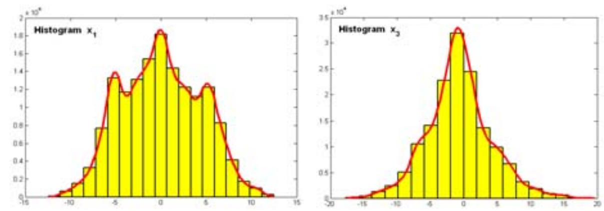

Fig. 5. Histogram of x_1 (left) and x_3 (right), continuous system

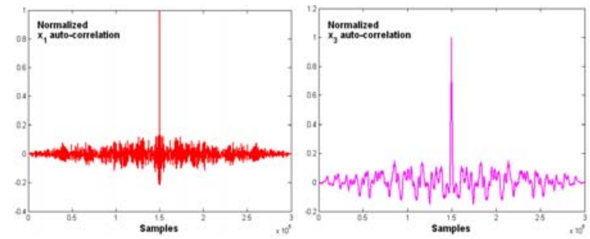

Fig. 6. Autocorrelation of x_1 (left) and x_3 (right), continuous system

3. Discrete Chaotic System

Because of problems with realization of continuous chaotic systems the discrete chaotic system was derived and tested. Such system is described by equations

$$\begin{aligned} x_1(k+1) &= x_1(k) + q\left[a\left(x_2(k) - x_1(k)\right)\right] \\ x_2(k+1) &= x_2(k) + q[b\left(x_1(k) + x_2(k)\right) \\ &\quad - x_1(k)x_3(k)x_4(k) \\ x_3(k+1) &= x_3(k) + q[-cx_3(k) \\ &\quad + x_1(k)x_2(k)x_4(k) \\ x_4(k+1) &= x_4(k) + q[ex(k)_2 - dx_4(k) \\ &\quad + x_1(k)x_2(k)x_3(k)] \end{aligned} \tag{9}$$

where a = 50, b = 10, c = 26, d = 15, e = 5 and q = 0.00015.

Results of discrete system simulations are illustrated in Fig. 7- time evolution of state space variables, Fig. 8- 3D phase portrait and Fig. 9- 2D phase portraits. Signals shown in this figures are similar as signals of continuous system. From this figures can be seen that discrete system behavior is similar as continuous system. Such discrete system can be easily constructed by microcontroller and digital/analog (D/A) converters.

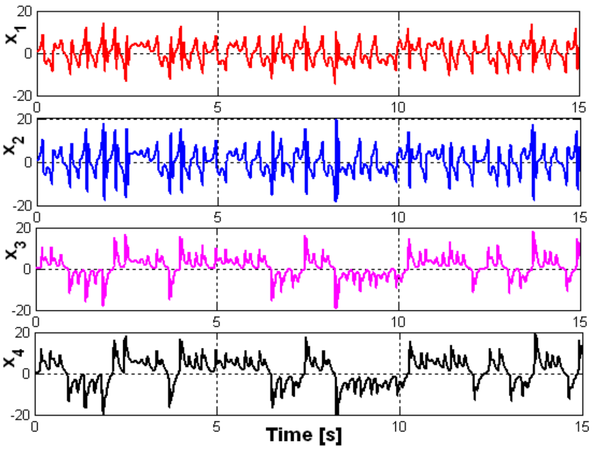

Fig. 7. Time evolution of state space variables, discrete system according eq. (9)

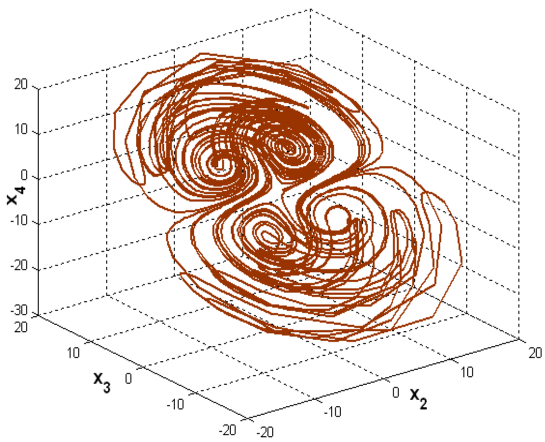

Fig. 8. 3D phase portrait of discrete system

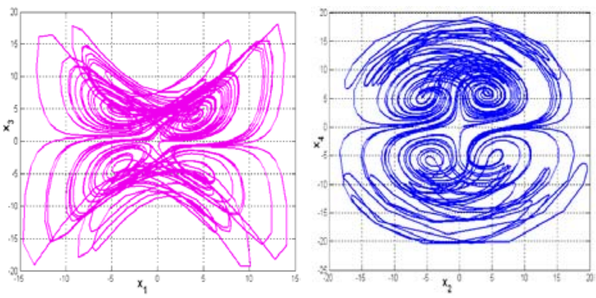

Fig. 9. Phase portrait of x_1 versus x_3 (left) and x_2 versus x_4, discrete system

4. Construction Results

The previous discrete system described by eq. (9) was implemented in AT91SAM 32-bit ARM-Cortex-M3 RISC processor running at 84 MHz, 96 KBytes of SRAM, 512 KBytes of Flash memory for code, with 2-channel 12-bit 1 Msps D/A converters. A 32-bit core allows operations on 4 bytes wide data within a single CPU clock.

Simplified block diagram of digital chaotic system is shown in Fig. 10. Discrete equations (9) were programmed and loaded in processor. Scope signals on the outputs of D/A converters (without lowpass filters on outputs) are shown in Fig. 11 and 12. Phase portraits generated from this signals are illustrated in Fig. 13 (of x_1 versus x_3) and Fig. 14 (x_2 versus x_4). Measuring results presents good agreement with theory, simulations and construction.

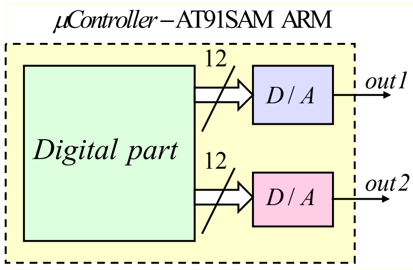

Fig. 10. Block diagram of digital system

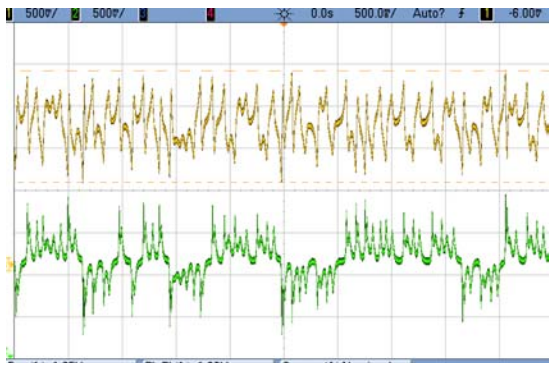

Fig. 11. Time evolution of x_1 and x_3, constructed system

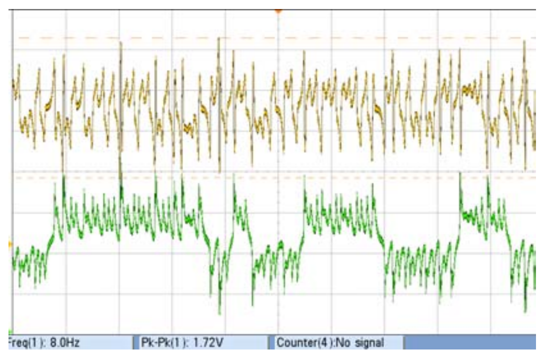

Fig. 12. Time evolution of x_2 and x_4, constructed system

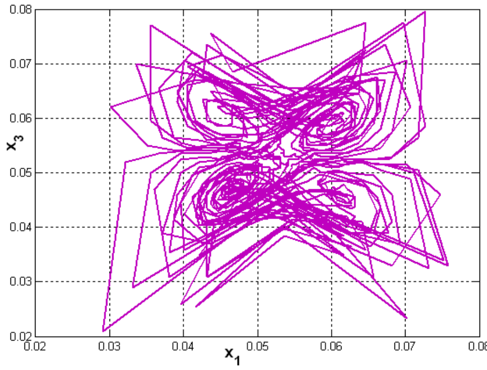

Fig. 13. Phase portrait of x_1 versus x_3, constructed system

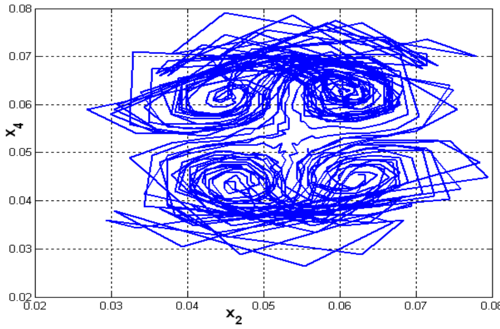

Fig. 14. Phase portrait of x_2 versus x_4, constructed system

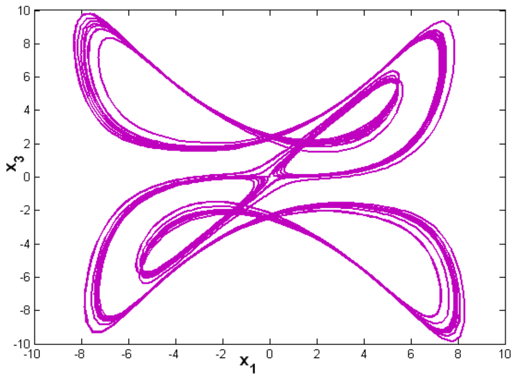

Fig. 15. Phase portrait of x_1 versus x_3, modified discrete system

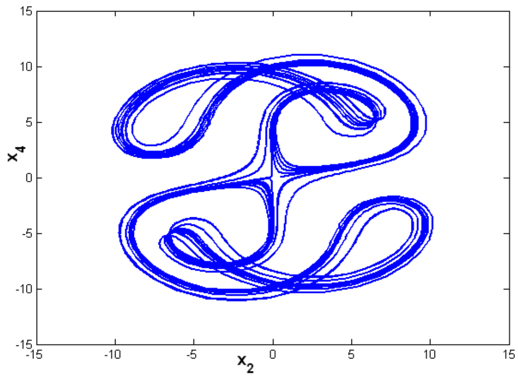

Fig. 16. Phase portrait of x_2 versus x_4, modified discrete system

5. System modification

Behavior of discrete system described by eq. (9) can be easily modified by means of parameters changing. Results of simulation for parameters a = 30, b = 8, c = 26, d = 15, e = 6 and q = 0.00015 are illustrated in Fig. 15 and 16 (phase portraits). Signals of the system are not chaotic but periodic. Example of parameters changing influence, when the system is running is shown in time evolution of x_1, see Fig. 17. Until *time*<=7.5 the parameters are a = 50, b = 10, c = 26, d = 15, e = 5 (chaotic behavior), for *time*>7.5 value of parameters a and b are changed, therefore after changing parameters are a = 30, b = 8, c = 26, d = 15, e = 6 (periodic behavior).

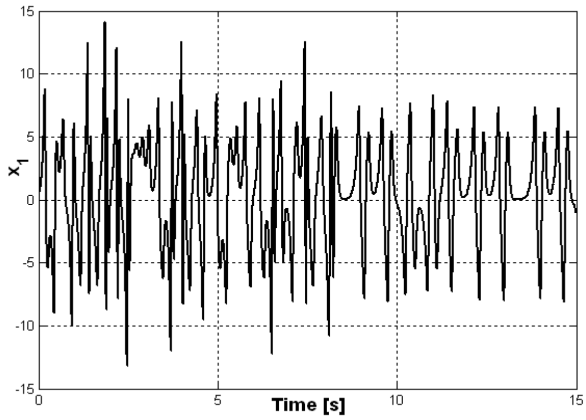

Fig. 17. Time evolution of state space variable x_1, discrete system, where in *time*>7.5 the system is switched from chaotic behavior to periodic behavior by means of parameters changing

6. Conclusions

In this work the new chaotic system was described. This system is modified version of four-wing attractor. On the beginning the continuous system is solved and simulated. In next part the discrete system is presented and used for realization by means of microcontroller. On the end also example of system modification is illustrated. Among main advantage of discrete approach are simple construction and modification and stable function even if the surrounding conditions are changing. From the simulation of continuous, discrete and real measuring on constructed system can be seen good agreement. Example of changing the parameters of the discrete system while system is running is also included.

The presented systems are very convenient to investigate the dynamical behavior of multi-wing chaotic attractors.

7. Acknowledgment

This work was supported by Department of Applied Electronics and Telecommunications, University of West Bohemia, Plzen, Czech Republic and by the European Regional Development Fund and the Ministry of Education, Youth and Sports of the Czech Republic under the Regional Innovation Centre for Electrical Engineering (RICE), project No. LO1607, the Internal Grant Agency of University of West Bohemia in Plzen and the project SGS-2015-002.

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