Random Walk with Variable Step Size as Mutation Operator of Genetic Algorithm for Solving Combined Heat and Power Economic Dispatch Problem

A. Haghrah[1], M. A. Nekoui[1], B. Mohammadi-ivatloo[2]

[1] Faculty of Electrical Engineering, K.N.Toosi University of technology, Tehran, Iran
arslan.haghrah@email.kntu.ac.ir, manekoui@eetd.kntu.ac.ir
[2] Faculty of Electrical Engineering, University of Tabriz, Tabriz, Iran
bmohammadi@tabrizu.ac.ir

Abstract

In this paper, a mutation operator based on the random walk with variable step size utilized by genetic algorithm (GA) is under study. Proposed method is used to solve combined heat and power economic dispatch (CHPED) problem. Combined heat and power economic dispatch problem is a complex and complicated optimization problem, which has non-convex, nonlinear and non-smooth objective function and many equality and inequality constraints. Normally, the step size of the random walk is obtained by probability distributions including exponential family and heavy-tailed distributions, which are both used in implementations of this paper. The proposed method achieved from different distributions, as determiner of step size in random walk based mutation, are implemented on the problem. The results are compared with numerous heuristic algorithms available in the literature. Also, convergence characteristics obtained from different mutation operators are compared. The analysis of achieved results and their improvements shows the capability of the random walk as mutation operator of GA.

1. Introduction

Considering the growing trend of heat and power demand, combined heat and power (CHP) generation has gained a special attention due to its economic and environmental characteristics. Unlike the combined cycle (CC) generation plants which have at most 60% of efficiency, combined heat and power generation units achieve energy efficiency as much as 90% [1]. Besides that, greenhouse gas emission in CHP units is reduced by 13-18% compared with their CC counterparts [2]. According to the aforementioned features of combined heat and power generation, this model can be a reasonable alternative for conventional generation plants [3, 4]. Combined heat and power economic dispatch problem is the task to assign generation of plants in the scheduling horizon to achieve minimum operation cost beside satisfying heat and power demands and operational constraints [2]. Heat and power interdependence, valve point effect consideration, transmission losses and prohibited operating zones make solving the CHPED problem a complex and difficult task [5] which have widely gained the attention of researchers in recent years. Reviewing prior approaches from early days up to now, demonstrates a vast variety of methods proposed to solve CHPED problem. A method based on separability of the objective function is proposed in [6] which introduces a two-step iterative process. Also non-linear optimization methods, such as dual and quadratic programming are applied to the problem. In [7] problem is decomposed to two subproblems, power dispatch and heat dispatch which are interconnected by heat and power feasible region constraints. Also Lagrange multipliers are used to solve subproblems. As one of the first applications of the metaheuristic methods to solve CHPED problem in [8] Genetic Algorithm based penalty function method is proposed. An evolutionary programming based algorithm is proposed in [9]. Differential evolution algorithm improved with the Gaussian mutation is implemented on the CHPED problem in [10] assuming valve-point loading effect and prohibited operating zone for conventional thermal generators which has caused a significant improvement in results. A self-adaptive real-coded genetic algorithm is introduced in [11] for solving the CHPED problem. Tournament selection and simulated binary crossover are used in order to augment self-adaptation capability to the genetic algorithm, which has decreased computational effort and enhanced convergence characteristics. In [2] harmony search (HS) is used to solve the problem. Comparisons provided by this paper shows the ability of HS for solving combined heat and power economic dispatch problem and its superiority to formerly proposed methods. In more recent studies like [12] genetic algorithm with improved mühlenbein mutation is implemented to solve CHPED problem, in which reported results show improvement of cost in almost all cases compared with previous works. A modified group search optimization method is proposed in [13]. Reported results in this paper show significant improvements at first glance, however, they are not sufficiently precise. A hybrid method is proposed in [14] based on civilized swarm optimization (CSO) and Powell's pattern search method (PPS) for solving the problem. In this hybrid method, CSO is used due to its global search abilities, while the PPS method has the task of local search. The reported results show improvements and besides that, they are precise and feasible in sense of constraints and cost function.

In this paper random walk with variable step size as mutation operator of the real-coded genetic algorithm (RCGA-RWM) is used for solving combined heat and power economic dispatch problem. Valve-point effect and non-convex cost function for power only units are considered in the model used, which causes more complexity and hardness. The step size in proposed algorithm is obtained by different probabilistic distributions which can be generally classified into two groups, heavy-tailed and exponential family distributions. Convergence characteristics obtained by using distributions from these two groups are compared and a discussion is carried on the results.

The rest of this paper has been organized as follows: Section 2 represents the mathematical formulation of the CHPED problem assuming valve-point effects and transmission losses. Section 3 provides the brief description and basic aspects of GA and description of the proposed RCGA-RWM. Section 4 expresses the implementation of the proposed procedure to two test systems and provides a comparison of the obtained results with the recent researches in the area of the CHPED problem. The paper conclusions are presented in Section 5.

2. Formulation of the CHPED Problem

The objective function of the CHPED problem is the summation of operational costs for power only units, combined heat and power units and heat only units, which is formulated as (1):

$$min \sum_{i=1}^{N_p} C_i(P_i^p)+\sum_{j=1}^{N_c} C_j(P_j^c, H_j^c)+\sum_{k=1}^{N_h} C_k(H_k^h)\ (\$/h) \tag{1}$$

In the formula above C indicates the total production cost for the unit and, N_p, N_c and N_h respectively are the number of conventional thermal units, co-generation units, and heat-only units. Heat and power generation output of units are represented by H and P. i, j and k are indices corresponding with above-mentioned unit types. Production cost formula for conventional thermal units, co-generation units, and heat-only units are as below:

$$C_i(P_i^p) = \alpha_i(P_i^p)^2 + \beta_i P_i^p + \gamma_i\ (\$/h) \tag{2}$$

$$C_j(P_j^c, H_j^c) = a_j(P_j^c)^2 + b_j P_j^c + c_j + d_j(H_j^c)^2 + e_j H_j^c + f_j H_j^c P_j^c\ (\$/h) \tag{3}$$

$$C_k(H_k^h) = a_k(H_k^h)^2 + b_k H_k^h + c_k\ (\$/h) \tag{4}$$

where $C_i\left(P_i^p\right)$ is the cost function corresponding with conventional thermal unit i, over one hour period, producing P_i^p MW. α_i, β_i, and γ_i are the cost coefficients of ith conventional thermal unit. The cost function of conventional thermal units are modelled using quadratic function approximation (2)[15, 16, 17]. $C_j\left(P_j^c, H_j^c\right)$ is the cost function corresponding with co-generation unit j and a_j, b_j, c_j, d_j, e_j and f_j are the cost coefficients of this unit. As it can be seen from (3) the cost function of the co-generation unit is convex in both power output P^c and heat output H^c. The cost of heat-only unit k is defined by $C_k\left(H_k^h\right)$ producing H^h MWth heat. a_k, b_k, and c_k are the cost coefficients of kth heat-only unit.

Constraints which are involved in this optimization problem can be listed as below:

- Power production and demand balance

$$\sum_{i=1}^{N_p} P_i^p + \sum_{j=1}^{N_c} P_j^c = P_d \tag{5}$$

  in which P_d indicates the electrical power demand of system.

- Heat production and demand balance

$$\sum_{j=1}^{N_c} H_j^c + \sum_{k=1}^{N_h} H_k^h = H_d \tag{6}$$

  where thermal demand of the system is represented by H_d.

- Capacity limits of conventional units

$$P_i^{pmin} \leq P_i^p \leq P_i^{pmax}\ \ i = 1, \ldots, N_p \tag{7}$$

- Capacity limits of CHP units

$$P_j^{cmin}(H_j^c) \leq P_j^c \leq P_j^{cmax}(H_j^c)\ \ j = 1, \ldots, N_c \tag{8}$$

$$H_j^{cmin}(P_j^c) \leq H_j^c \leq H_j^{cmax}(P_j^c)\ \ j = 1, \ldots, N_c \tag{9}$$

  Minimum and maximum limits of jth CHP unit power generation are represented as functions of generated heat, $P_j^{cmin}(H_j^c)$ and $P_j^{cmax}(H_j^c)$. Similarly, for heat generation of unit, the limits are functions of generated power in the form $H_j^{cmin}(P_j^c)$ and $H_j^{cmax}(P_j^c)$.

- Production limits of heat-only units

$$H_k^{hmin} \leq H_k^h \leq H_k^{hmax}\ \ k = 1, \ldots, N_h \tag{10}$$

2.1. Valve point impact consideration

Quadratic and cubic cost functions are used in most of the reported works.[18] However, the wire drawing impacts cause a ripple in production cost, when steam admission valve starts to open. This impact is modeled by adding a sinusoid term to production cost of the generation units.[19] The sinusoid term, which is taken into account in this work, makes the optimization problem non-convex and non-differentiable. Finally, cost function considering valve point effect is expressed as below:

$$C_i(P_i^p) = \alpha_i(P_i^p)^2 + \beta_i P_i^p + \gamma_i + |\lambda_i sin(\rho_i(P_i^{pmin} - P_i^p))| \tag{11}$$

in which λ_i and ρ_i are the valve-point effect's cost coefficients.

3. Real Coded Genetic Algorithm with Random Walk based mutation

The algorithm proposed in this paper is based on the conventional genetic algorithm, improved with random walk based mutation. Talking about one dimension, assume that a particle starts from any initial state, moving one unit left or right with probability 0.5. In the next step again, regardless of its position, it moves right or left with equal probability. The pattern achieved by repeating this procedure is called a random walk[20]. As a mutation operator, the random walk would act more effective if the step size is a variable, achieved from a probabilistic distribution. Among the existing probability distributions, exponential family and heavy-tailed distributions are on the attention of this paper. Probability density functions studied in this paper are illustrated in Fig. 1 and listed as below:

- Normal distribution (RWM1)

$$f(x|\sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}} \tag{12}$$

- Exponential distribution (RWM2)

$$f(x|\lambda) = \lambda e^{-\lambda x} \tag{13}$$

- Lévy distribution (RWM3)

$$f(x|c) = \sqrt{\frac{c}{2\pi}} \frac{e^{-\frac{c}{2x}}}{x^{\frac{3}{2}}} \tag{14}$$

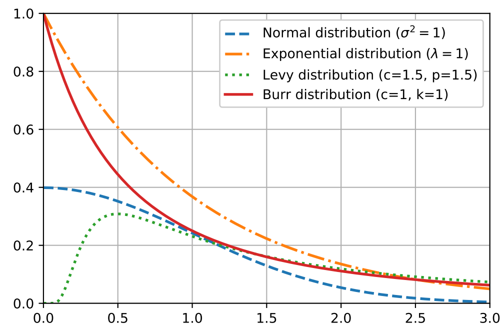

Figure 1. Probability density functions which are under study

Table 1. Algorithm parameters used for mutation operators.

Method	Parameters
RWM1	$\sigma^2 = rand()$
RWM2	$\lambda = rand()$
RWM3	$p = 1.5 \times rand()$
RWM4	$c = 5 \times rand(), k = rand()$

- Burr distribution (RWM4)

$$f(x|c,k) = ck\frac{x^{c-1}}{(1+x^c)^{k+1}} \qquad (15)$$

All details of the algorithm proposed in this paper are adopted from Ref. [12] except mutation function which is introduced in this paper. The probability distribution function parameters used in this paper are represented in Table 1 in which the function $rand()$ returns a random number in range $[0, 1)$ with uniform distribution. The parameters are achieved by testing numerous numbers over many runs.

4. Case studies

Genetic algorithm with random walk based mutation is applied to two test systems. A comparison is done between proposed method and some other recently developed methods. Also, the effects of the probability distribution, on convergence characteristics of proposed method is studied. The results reported in this paper are achieved by running the algorithm for 100 times on each test system. It should be noted that all numbers reported are rounded up to 2 digits after decimal point.

4.1. Test system 1

The first test system which is studied contains 13 power only units, 6 CHP units and 5 heat only units. The power and heat demand considered is 2350 MW and 1250 MWth respectively. Unit data in this test system is adopted from Ref. [21]. The results achieved by implementation of proposed algorithm are represented in Table 2. Also Table 3 represents a comparison with results achieved by formerly developed algorithms. Convergence characteristics of proposed methods are represented in Fig. 2.

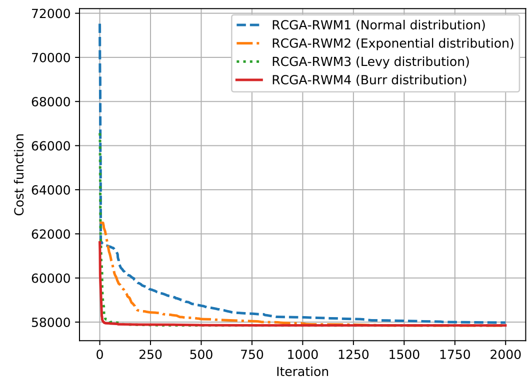

Figure 2. Convergence characteristics of the proposed algorithm with different probabiliy distribution functions for first test system.

Table 2. Optimal dispatch results for test system 1 using proposed methods.

Output	RWM1	RWM2	RWM3	RWM4
P_1	538.57	538.57	538.61	538.56
P_2	224.78	299.37	299.23	299.66
P_3	299.64	224.68	299.25	299.61
P_4	109.87	109.90	109.95	109.87
P_5	109.91	159.74	109.93	109.87
P_6	109.96	109.91	109.89	109.87
P_7	110.05	159.81	110.03	109.87
P_8	109.98	159.91	110.01	109.87
P_9	109.90	109.88	109.87	109.87
P_{10}	77.47	40.10	77.52	77.40
P_{11}	77.49	40.08	77.46	40.16
P_{12}	92.39	55.18	55.15	92.40
P_{13}	92.60	55.25	55.06	55.00
P_{14}	81.03	81.08	81.04	81.02
P_{15}	40.00	40.02	40.03	40.27
P_{16}	81.12	81.11	81.06	81.06
P_{17}	40.00	40.00	40.02	40.05
P_{18}	10.02	10.00	10.01	10.05
P_{19}	35.20	35.39	35.88	35.56
H_{14}	104.81	104.84	104.82	104.81
H_{15}	74.99	75.02	75.03	75.23
H_{16}	104.85	104.86	104.82	104.83
H_{17}	75.00	75.00	75.02	75.05
H_{18}	40.01	40.00	40.00	40.02
H_{19}	20.09	20.17	20.40	20.25
H_{20}	473.87	470.15	469.96	469.80
H_{21}	59.99	59.99	60.00	60.00
H_{22}	59.99	60.00	60.00	60.00
H_{23}	116.45	120.00	119.96	120.00
H_{24}	119.96	119.99	120.00	120.00
Total Cost	57979.45	57842.20	57847.51	57849.43

4.2. Test system 2

This test system is large scale equivalent of the first test system which is obtained by duplicating its data. The power and heat demand considered is 4700 MW and 2500 MWth respectively.[21]. Proposed algorithm with four types of prob-

Table 3. Comparison of proposed methods with formerly reported results for test system 1.

Method	Mean	Max	Min
TVAC-PSO[21]	58498.31	58359.55	58122.75
RCGA-IMM[12]	58066.64	58301.90	57927.69
COA[22]	58010.29	58175.13	57938.33
NCSO[23]	57908.32	57911.95	57907.12
RWM1	58556.01	59429.71	57979.45
RWM2	58003.16	58305.33	57842.20
RWM3	57921.94	58001.83	57847.51
RWM4	57922.06	58000.71	57849.43

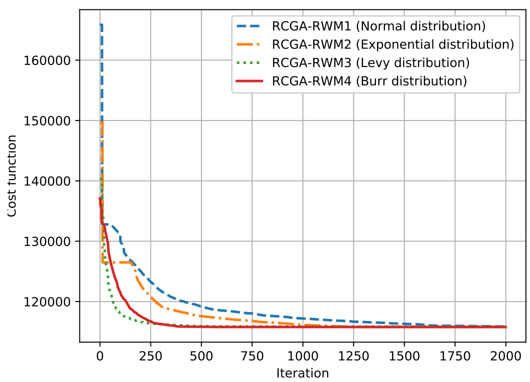

Figure 3. Convergence characteristics of the proposed algorithm with different probabiliy distribution functions for second test system.

ability distribution used in mutation operator for determining step size of the random walk is also applied to this test system. The results achieved are presented in Table 4 and compared with recently developed algorithms in Table 5. Convergence characteristics of proposed methods are represented in Fig. 3.

5. Conclusion

Combined heat and power economic dispatch problem, is solved by the genetic algorithm with random walk based mutation with variable step size, which step size is obtained from probabilistic variables. Four probability distributions are studied and results are compared with some recently proposed algorithms. Also, convergence characteristics of achieved GAs using different probability distributions are compared. The simulation results prove the ability of RCGA-RWM for solving medium and large size CHPED problems. Comparison of results achieved by using different probability distributions shows that heavy-tail distributions are faster in sense of convergence, in contrast with exponential family distributions. However, exponential distributions avoid premature convergence, which eventually can cause better global search. In the first case exponential distribution obtained better result but in the second test system, Burr distribution is dominant.

6. References

[1] M. Alipour, B. Mohammadi-Ivatloo, and K. Zare, "Stochastic risk-constrained short-term scheduling of industrial cogeneration systems in the presence of demand response programs," *Applied Energy*, vol. 136, pp. 393–404, 2014.

[2] A. Vasebi, M. Fesanghary, and S. Bathaee, "Combined heat and power economic dispatch by harmony search algorithm," *International Journal of Electrical Power & Energy Systems*, vol. 29, no. 10, pp. 713–719, 2007.

[3] L. Dong, H. Liu, and S. Riffat, "Development of small-scale and micro-scale biomass-fuelled chp systems–a literature review," *Applied thermal engineering*, vol. 29, no. 11, pp. 2119–2126, 2009.

[4] H. Wang, R. Lahdelma, X. Wang, W. Jiao, C. Zhu, and P. Zou, "Analysis of the location for peak heating in chp based combined district heating systems," *Applied Thermal Engineering*, vol. 87, pp. 402–411, 2015.

[5] M. Alipour, K. Zare, and B. Mohammadi-Ivatloo, "Short-term scheduling of combined heat and power generation units in the presence of demand response programs," *Energy*, vol. 71, pp. 289–301, 2014.

[6] F. J. Rooijers and R. A. van Amerongen, "Static economic dispatch for co-generation systems," *IEEE Transactions on Power Systems*, vol. 9, no. 3, pp. 1392–1398, 1994.

[7] T. Guo, M. I. Henwood, and M. Van Ooijen, "An algorithm for combined heat and power economic dispatch," *IEEE Transactions on Power Systems*, vol. 11, no. 4, pp. 1778–1784, 1996.

[8] Y. Song and Q. Xuan, "Combined heat and power economic dispatch using genetic algorithm based penalty function method," *Electric machines and power systems*, vol. 26, no. 4, pp. 363–372, 1998.

[9] K. P. Wong and C. Algie, "Evolutionary programming approach for combined heat and power dispatch," *Electric Power Systems Research*, vol. 61, no. 3, pp. 227–232, 2002.

[10] C. Jena, M. Basu, and C. Panigrahi, "Differential evolution with gaussian mutation for combined heat and power economic dispatch," *Soft Computing*, pp. 1–8, 2014.

[11] P. Subbaraj, R. Rengaraj, and S. Salivahanan, "Enhancement of combined heat and power economic dispatch using self adaptive real-coded genetic algorithm," *Applied Energy*, vol. 86, no. 6, pp. 915–921, 2009.

[12] A. Haghrah, M. Nazari-Heris, and B. Mohammadi-ivatloo, "Solving combined heat and power economic dispatch problem using real coded genetic algorithm with improved mühlenbein mutation," *Applied Thermal Engineering*, vol. 99, pp. 465–475, 2016.

[13] E. Davoodi, K. Zare, and E. Babaei, "A gso-based algorithm for combined heat and power dispatch problem with modified scrounger and ranger operators," *Applied Thermal Engineering*, vol. 120, pp. 36–48, 2017.

[14] N. Narang, E. Sharma, and J. Dhillon, "Combined heat and power economic dispatch using integrated civilized swarm optimization and powell's pattern search method," *Applied Soft Computing*, 2016.

[15] E. Khorram and M. Jaberipour, "Harmony search algorithm for solving combined heat and power economic dispatch problems," *Energy Conversion and Management*, vol. 52, no. 2, pp. 1550–1554, 2011.

Table 4. Optimal dispatch results for test system 2 using proposed methods.

Output	RWM1	RWM2	RWM3	RWM4	Output	RWM1	RWM2	RWM3	RWM4
P_1	538.65	538.70	538.69	538.56	P_{31}	10.02	10.01	10.04	10.01
P_2	224.48	299.42	224.32	225.15	P_{32}	35.03	35.04	35.29	35.03
P_3	225.07	224.72	224.42	224.84	P_{33}	81.09	81.02	81.15	81.01
P_4	109.93	159.83	110.01	109.87	P_{34}	40.02	40.03	40.60	40.02
P_5	159.84	109.95	159.74	159.73	P_{35}	81.05	81.08	81.06	81.00
P_6	110.04	159.82	109.63	159.74	P_{36}	40.01	40.01	40.23	40.00
P_7	110.06	110.05	109.88	109.88	P_{37}	10.02	10.01	10.03	10.00
P_8	109.88	110.04	109.87	109.87	P_{38}	35.10	35.10	35.59	35.00
P_9	159.88	109.93	109.96	109.87	H_{27}	104.78	104.82	104.69	104.80
P_{10}	77.48	77.48	77.33	77.56	H_{28}	75.01	75.00	75.33	75.00
P_{11}	77.41	77.70	77.94	77.48	H_{29}	104.80	104.81	104.90	104.80
P_{12}	92.42	92.42	92.40	55.00	H_{30}	75.01	75.02	75.00	75.00
P_{13}	92.61	92.45	55.19	55.06	H_{31}	40.00	40.00	40.01	40.00
P_{14}	538.68	538.59	538.59	538.57	H_{32}	19.99	20.02	20.00	20.01
P_{15}	224.58	224.70	299.31	299.99	H_{33}	104.84	104.81	104.86	104.81
P_{16}	224.98	224.77	224.22	300.62	H_{34}	74.98	75.02	75.50	75.01
P_{17}	109.86	159.78	109.76	109.87	H_{35}	104.81	104.84	104.57	104.80
P_{18}	109.87	109.90	159.92	109.87	H_{36}	74.98	75.01	75.14	75.00
P_{19}	159.69	109.99	110.05	159.73	H_{37}	40.00	40.00	40.01	40.00
P_{20}	109.90	109.91	159.70	109.87	H_{38}	20.02	20.05	20.27	20.00
P_{21}	109.96	109.91	109.80	109.87	H_{39}	487.49	482.80	477.63	480.57
P_{22}	110.10	109.94	110.01	109.87	H_{40}	60.00	59.99	59.98	60.00
P_{23}	77.58	77.64	77.42	40.19	H_{41}	59.99	60.00	59.89	60.00
P_{24}	77.53	40.09	40.02	77.40	H_{42}	119.97	119.98	119.96	119.99
P_{25}	92.51	55.20	92.49	55.03	H_{43}	119.80	119.97	119.77	119.98
P_{26}	92.61	92.61	92.41	92.43	H_{44}	453.80	457.89	462.74	460.23
P_{27}	81.01	81.08	81.22	81.00	H_{45}	59.98	60.00	59.97	60.00
P_{28}	40.01	40.01	40.43	40.00	H_{46}	59.98	59.99	59.96	60.00
P_{29}	81.01	81.05	81.21	81.00	H_{47}	119.80	120.00	119.90	120.00
P_{30}	40.02	40.02	40.04	40.00	H_{48}	119.97	119.99	119.91	120.00
Method		RWM1		RWM2		RWM3		RWM4	
Total Cost		115869.53		115800.95		115828.19		115747.39	

Table 5. Comparison of proposed methods with formerly reported results for test system 2.

Method	Mean	Max	Min
TVAC-PSO[21]	-	-	117824.90
COA[22]	116835.55	117068.27	116789.915
CSA[24]	117245.6	117636.1	116843.30
NCSO[23]	115995.88	116047.22	115967.72
RWM1	116436.08	119532.65	115869.53
RWM2	116217.50	117714.26	115800.95
RWM3	116025.45	116872.28	115828.19
RWM4	115939.02	118244.80	115747.39

[16] Y. Song, C. Chou, and T. Stonham, "Combined heat and power economic dispatch by improved ant colony search algorithm," *Electric Power Systems Research*, vol. 52, no. 2, pp. 115–121, 1999.

[17] L. Wang and C. Singh, "Stochastic combined heat and power dispatch based on multi-objective particle swarm optimization," *International Journal of Electrical Power & Energy Systems*, vol. 30, no. 3, pp. 226–234, 2008.

[18] C.-T. Su and C.-L. Chiang, "An incorporated algorithm for combined heat and power economic dispatch," *Electric Power Systems Research*, vol. 69, no. 2, pp. 187–195, 2004.

[19] B. Mohammadi-Ivatloo, A. Rabiee, and A. Soroudi, "Nonconvex dynamic economic power dispatch problems solution using hybrid immune-genetic algorithm," *IEEE Systems Journal*, vol. 7, no. 4, pp. 777–785, 2013.

[20] P. Révész, *Random walk in random and non-random environments*. World Scientific, 2005.

[21] B. Mohammadi-Ivatloo, M. Moradi-Dalvand, and A. Rabiee, "Combined heat and power economic dispatch problem solution using particle swarm optimization with time varying acceleration coefficients," *Electric Power Systems Research*, vol. 95, pp. 9–18, 2013.

[22] M. Mehdinejad, B. Mohammadi-Ivatloo, and R. Dadashzadeh-Bonab, "Energy production cost minimization in a combined heat and power generation systems using cuckoo optimization algorithm," *Energy Efficiency*, vol. 10, no. 1, pp. 81–96, 2017.

[23] A. Meng, P. Mei, H. Yin, X. Peng, and Z. Guo, "Crisscross optimization algorithm for solving combined heat and power economic dispatch problem," *Energy Conversion and Management*, vol. 105, pp. 1303–1317, 2015.

[24] T. T. Nguyen, D. N. Vo, and B. H. Dinh, "Cuckoo search algorithm for combined heat and power economic dispatch," *International Journal of Electrical Power & Energy Systems*, vol. 81, pp. 204–214, 2016.