

# Model Reference Adaptive Control of a Quadruple Tank Process with Actuator Faults

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## Abstract

**PID controllers are still the most popular controllers used in process industry. Most of plants in industry contain multiple PID loops where interaction between control channels is highly possible. System instability and catastrophic consequences are inevitable for safety critical plants in the case of actuator and sensor faults/failures. In such case, a fault tolerant control (FTC) system may be a solution. Adaptive control method includes a class of methods which is appropriate for active FTC since they have the ability of adaptation to variations in the system parameters. In this study we utilize a prototype that mimics complex, industrial processes. Our aim is to develop a model reference adaptive control system for chosen plant. A model that covers all possible actuator faults is proposed and integrated to the plant simulation. Results are compared with PI-controlled system.**

## 1. Introduction

Many real world systems, especially industrial processes, are composed of multiple interacting variables. They can contain multiple control loops that work simultaneously. In addition, due to increasing demands on production quality and efficiency, these multivariable plants are getting more complicated. Such a complex system is very sensitive to faults in system components especially in actuators and sensors. Minor faults or total failures in sensors and actuators that occur in process can result in catastrophic consequences. In this case, control system for the multivariable plant must have the ability to accommodate faults and maintain system stability and performance.

Failures are difficult to predict in time and can be abrupt, incipient and intermittent [1]. Another classification can be made about their location of occurrence. Faults can cause a reduction in the efficiency even further could lead to overall system failure and instability. Therefore, in the design phase of compensators one must construct an actuator failure model that contains all possible cases is critical. Then, the failure model can be integrated to control system model or could be used for robustification of controller. This control is generally referred to as fault tolerant control (FTC) which has become popular in the last decades. There are so many approaches in FTC area and this make difficult to classify the methods or standardize terminology [2, 3]. In the literature, the methods are generally classified as active FTC and passive FTC. In passive approach, possible faults/failures are assumed to be known and all fault assumptions and normal operating conditions are considered at

the design stage. Therefore all robust control methods for FTC belong to this class. This type of FTC system is also known as reliable control systems or control systems with integrity. On the other hand, an active FTC reacts to system component malfunctions (including actuators, system itself, and sensors) by changing its parameters or structure based on real-time identification which is carried out by Fault Detection Diagnosis (FDD). Since adaptation and reconfiguration mechanisms are a basic requirement, generally adaptive control schemes belong to this class. Furthermore, due to their adaptation property in the case of system parameter changes, adaptive methods may not require the FDD components. This implies that low computational effort is possible if an adaptive scheme is chosen as main control mechanism.

In this study, we utilize a prototype that mimics complex industrial processes. The system under study is a nonlinear quadruple tank system which is a benchmark experimental facility developed for research purposes for the processing and aerospace industries [4]. It consist of nonlinear MIMO interconnected liquid tanks. Different from the original benchmark, we consider valve actuated configuration. A general fault/failure model that covers all possible malfunctions is given and integrated to simulations. A conventional PI controller is chosen to track reference water level. Then, in random time and mode, various actuator fault/failure scenarios are taken into account. Conventional PI controller cannot compensate the failure effects since it has not a reconfiguration mechanism. To solve this problem a model reference adaptive controller is designed and fault compensation property is compared with PI controller via computer simulations.

## 2. Problem Formulation and System Descriptions

### 2.1. Actuator Failure Model

In a general fault model, including only a reduction of efficiency is not enough. A comprehensive fault model for valve actuator must contain other cases such as stuck, hard-over, leakage. These faults are encountered frequently in the process control industry. To represent valve actuator faults, one can use the following mathematical model [5]:

$$u_{fi} = \rho_i u_i(t) + \sigma_i(t)(u_{si} - \rho_i u_i(t)) + \delta_i(t) \quad (1)$$

where  $u_{fi} \in R$ ,  $i = 1, \dots, m$  represents system control inputs and  $\sigma_i(t)$  implies that a total fault (i.e. a complete failure) of i-th actuator of system,  $\rho_i$  is a multiplier that represent the loss of

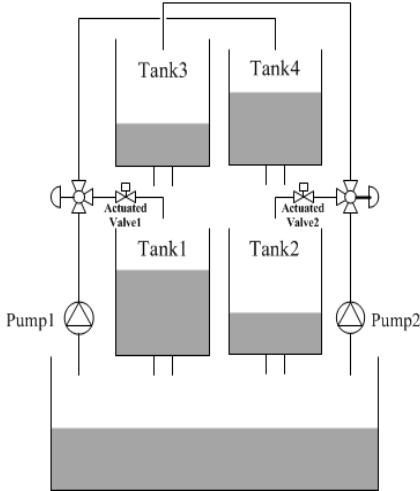
effectiveness of actuator,  $u_{si}$  is a bounded generally constant signal that corresponds to hard-over and stuck type uncontrollable failures and  $\delta_i(t)$  is again a bounded signal and can be considered as input disturbance such as valve leakage faults. A linear MIMO system model with actuator faults are integrated can be written as :

$$y(t) = G_f \rho u_f + G_f \sigma(u_s - \rho u_f) + \bar{y}_{\delta(t)} \quad (2)$$

where  $y(t) \in \mathbb{R}^m$  is system output,  $G_f = C(sI - A)^{-1}B$  is a square MIMO system transfer matrix with actuator faults/failures.

## 2.2. System Model

The process consists of four interconnected water tanks and two pumps. The system inputs are voltages to the two valve actuator and the outputs the water levels in the lower two tanks. The system is depicted in Figure 1



**Fig. 1.** Schematic diagram of the quadruple-tank process

Our goal is to control the level of the two lower tanks with two actuated valves. The output of each pump is split into two using three-way valves. Here we will give first the mathematical model of the original system and then valve dynamics will be added. The nonlinear dynamics of the system is given by:

$$\begin{aligned} A_1 \frac{dh_1}{dt} &= -a_1 \sqrt{2gh_1} + a_3 \sqrt{2gh_3} + \gamma_1 k_1 v_1 \\ A_2 \frac{dh_2}{dt} &= -a_2 \sqrt{2gh_2} + a_4 \sqrt{2gh_4} + \gamma_2 k_2 v_2 \\ A_3 \frac{dh_3}{dt} &= -a_3 \sqrt{2gh_3} + (1 - \gamma_2) k_2 v_2 \\ A_4 \frac{dh_4}{dt} &= -a_4 \sqrt{2gh_4} + (1 - \gamma_1) k_1 v_1 \end{aligned} \quad (3)$$

where  $k_i$  is the pump constant,  $\gamma_1, \gamma_2 \in (0,1)$  are main pipe valve position settings. By choosing an operating point and introducing the variables  $x_i := h_i - h_{io}$  and  $u_i := v_i - v_{io}$ , the system can be represented in state space form as follows:

$$\begin{aligned} \frac{dx}{dt} &= \begin{bmatrix} -\frac{1}{T_1} & 0 & \frac{A_3}{A_1 T_3} & 0 \\ 0 & -\frac{1}{T_2} & 0 & \frac{A_4}{A_2 T_4} \\ 0 & 0 & -\frac{1}{T_3} & 0 \\ 0 & 0 & 0 & -\frac{1}{T_4} \end{bmatrix} x + \begin{bmatrix} \frac{\gamma_1 k_1}{A_1} & 0 \\ 0 & \frac{\gamma_1 k_1}{A_1} \\ 0 & \frac{(1 - \gamma_2) k_2}{A_3} \\ \frac{(1 - \gamma_1) k_1}{A_4} & 0 \end{bmatrix} u \\ y &= \begin{bmatrix} k_c & 0 & 0 & 0 \\ 0 & k_c & 0 & 0 \end{bmatrix} x \end{aligned} \quad (4)$$

where the time constants are

$$T_i = \frac{A_i}{a_i} \sqrt{\frac{2h_{io}}{g}}, \quad i=1,\dots,4 \quad (5)$$

where  $h_{io}$  is water level of each tank at operating point. Transfer function matrix of the linearized system can be written as:

$$G(s) = \begin{bmatrix} \frac{\gamma_1 c_1}{1 + T_1} & \frac{(1 - \gamma_2) c_1}{(1 + T_3 s)(1 + s T_1)} \\ \frac{(1 - \gamma_1) c_2}{(1 + s T_4)(1 + s T_2)} & \frac{\gamma_2 c_2}{1 + s T_2} \end{bmatrix} \quad (6)$$

where  $c_1 = T_1 k_1 k_c / A_1$  and  $c_2 = T_2 k_2 k_c / A_2$

**Table 1.** Process parameters

Parameter	Value
Height of tanks, $h_{max}$	20 cm
Bottom area, Tank1, Tank3, $A_1, A_3$	$28 \text{ cm}^2$
Bottom area, Tank1, Tank3, $A_2, A_4$	$32 \text{ cm}^2$
Out pipe cross-sections, $a_1, a_3$	$0.071 \text{ cm}^2$
Out pipe cross-sections, $a_2, a_4$	$0.057$
Level measurement device constant, $k_c$	$0.500 \text{ V/cm}$
Gravity $g$	$981 \text{ cm/s}^2$

**Table 2.** System operating point

Parameters	Value
$(h_{1o}, h_{2o}) \text{ cm}$	$(12.26, 12.78)$
$(h_{3o}, h_{4o}) \text{ cm}$	$(1.63, 1.41)$
$(v_1, v_2) \text{ V}$	$(3.00, 3.00)$
$(k_1, k_2) \text{ cm}^3/\text{Vs}$	$(3.33, 3.35)$
$(\gamma_1, \gamma_2)$	$(0.70, 0.60)$

By using the real system parameters given in Table 1, a realistic model is obtained. Operating point parameters given in Table 2 is used to linearize system model. Since we consider the case that flow to the tanks are supplied by a servo valve actuator, the valve dynamics must be added to transfer matrix. The valves are frequently represented by a first order transfer function between controller output  $U(s)$  and system input  $M(s)$ :

$$G_v = \frac{M(s)}{U(s)} = \frac{K_v}{T_v s + 1} \quad (7)$$

In simulations, valve actuator parameters are chosen as  $K_v = 1$  and  $T_v = 0.1 \text{ s}$ . The transfer matrices of the plant with valve dynamics:

$$G(s) = \begin{bmatrix} \frac{1.56}{(1+62.3s)(1+0.1s)} & \frac{0.89}{(1+22.8s)(1+62.3)(1+0.1s)} \\ \frac{0.85}{(1+30s)(1+90.6s)(1+0.1s)} & \frac{1.70}{(1+90.6s)(1+0.1s)} \end{bmatrix} \quad (8)$$

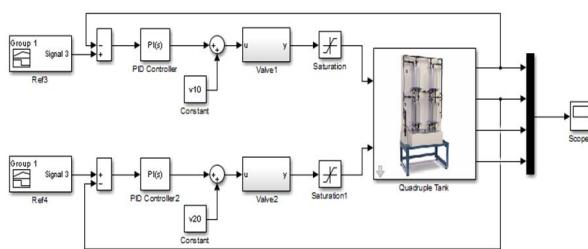
### 3. Fault Tolerant Control Design

There are two ways of dealing with MIMO linear system control problems. One way is full multivariable control design that incorporates the interaction dynamics rigorously and explicitly. Another way is to treat interactions as a form of disturbance. Latter approach is a completely decentralized control and it constitutes the main structure of the controller given in this paper. Since we use minimum phase system model, the interaction between tanks are small. It can be seen from diagonal elements of relative gain array (RGA) matrix of system [6]. RGA matrix in this configuration is:

$$\Lambda = \begin{bmatrix} 1.4 & -0.4 \\ -0.4 & 1.4 \end{bmatrix} \quad (9)$$

Main diagonal elements are positive and bigger in magnitude hence we can design a stable controller in a decentralized manner. Here we are aiming to design a decentralized model reference adaptive controller (MRAC) to compensate valve actuator failures and force the system to track reference levels of Tank1 and Tank2. In order to obtain a desired reference model, valve dynamics and system parameters of minimum-phase plant are assumed to be unknown and in normal operation condition of the system which block structure given in Figure 2 is considered as reference model of adaptive controller. The interaction between tanks is accepted as disturbances. Decentralized PI controlled closed loop system is subjected to system identification process by using reference inputs and output response data. In this case, for Tank1 and Tank2, the closed loop transfer functions that have one zero and three poles with unknown coefficients with known signs can be written as follows :

$$G_{CLi}(s) = \frac{Y_i(s)}{R_i(s)} = \frac{b_{1i}s + b_{0i}}{s^3 + a_{1i}s^2 + a_{2i}s + a_{3i}}, \quad i = 1, 2 \quad (10)$$



**Fig. 2.** Valve actuated closed loop system in normal mode

In order to meet MRAC objective with a control law that is free of differentiators and uses only measurable signals, plant and reference model must satisfy some requirements such as plant numerator polynomial is monic, plant degree is known, plant relative degree is known; reference model numerator and

denominator are monic Hurwitz polynomials and relative degree is the same as that of plant. In other words, reference model must be chosen to be strictly positive definite (SPR) to obtain a stable controller. Following the certainty equivalence principle, the controller given below can be used [7]:

$$u_i = \theta_{i1}^{*T} \frac{\alpha_i(s)}{\Lambda_i(s)} u_i + \theta_{i2}^{*T} \frac{\alpha_i(s)}{\Lambda_i(s)} y_{ip} + \theta_{i3}^{*T} y_{ip} + c_{i0}^* r_i \quad i = 1, 2 \quad (11)$$

where  $\alpha(s) \triangleq \alpha_{n-2}(s) = [s^{n-2}, s^{n-3}, \dots, s, 1]$  for  $n \geq 2$ ,  $\alpha(s) \triangleq 0$  for  $n=1$ .  $\theta_1, \theta_2 \in \mathbb{R}^{n-1}$ ;  $c_0, \theta_3 \in \mathbb{R}$  are constant parameters to be designed and  $\Lambda(s)$  is an arbitrary monic Hurwitz polynomial of degree  $n-1$ . The controller given in (11) has known parameters ( $\theta^*$ ) i.e., the controller is designed for a known plant transfer function therefore it is named as model reference controller (MRC). However, by using certainty equivalence, we can define an update law and use the same controller with unknown parameters. We can write the equation in compact form as follows:

$$u = \theta^{*T} \omega \quad (12)$$

Let us define the parameter error as:

$$\tilde{\theta} = \theta(t) - \theta^* \Rightarrow \theta(t) = \tilde{\theta} + \theta^* \quad (13)$$

By using (13), (12) can be written as follows:

$$u = \tilde{\theta}^T \omega + \theta^{*T} \omega \quad (14)$$

In order to choose an adaptation law, we have to first find out how the tracking error is related to the parameter error:

$$e = G_m(s)p^*(u - \theta^{*T} \omega) = G_m(s)(p^* \tilde{\theta}^T \omega) \quad (15)$$

where  $G_m$  is reference model transfer function;  $p$  is the ratio of the plant and reference model high frequency gain. Following Lyapunov function is used to prove system stability:

$$V(\tilde{\theta}, e) = \frac{1}{2} e^T P_m e + \frac{1}{2} |p^*| \tilde{\theta}^T \Gamma^{-1} \tilde{\theta} \quad (16)$$

Here  $P_m = P_m^T > 0$  and  $\Gamma = \Gamma^T > 0$ . By using Kalman–Yakubovich lemma and Lyapunov function above, adaptive law is obtained as follows:

$$\dot{\theta} = -\text{sgn}\left(\frac{k_p}{k_m}\right) \Gamma e \omega \quad (17)$$

In the design procedure of above controller, we assume that system has relative degree 1 which gives the opportunity to design reference model as SPR. The reference model we consider for the plant has relative degree 2. In this case reference model can no longer be SPR. We can solve this problem by using the fact that is identity for some and rewrite the error equation:

$$e = G_m(s)(s + p_0)p^*(u_f - \theta^{*T} \phi) \quad (18)$$

with  $u_f = \frac{1}{s+p_0}u$ ,  $\phi = \frac{1}{s+p_0}\omega$ , which implies that the control law in compact form is:

$$u = \theta^T \omega + \dot{\theta}^T \phi \quad (19)$$

The reference model degree is  $n=3$  hence choosing the degree of  $\Lambda(s)$  as  $n-1=2$ , fixing  $\Lambda(s) = s^2 + s + 1$  and taking  $\alpha(s) = [s \ 1]$  for  $n=3$  designed controller for each output takes the following form:

$$u = \theta^T(t) \begin{bmatrix} s \\ s^2 + s + 1 \end{bmatrix} u + \theta_2^T(t) \begin{bmatrix} s \\ s^2 + s + 1 \end{bmatrix} y_p + \theta_3(t)y_p + \theta_4(t)r + \dot{\theta}^T \phi$$

$$\dot{\theta} = -\Gamma e \phi \text{sgn} \left( \frac{k_p}{k_m} \right)$$

$$\phi = \frac{1}{s+1} \omega \quad (20)$$

#### 4. Simulations

In normal operation with decentralized PI-control an acceptable performance is aimed such as less than 1% overshoot and 40 sec settling time. The controller settings for PI controllers  $G_{ci}(s) = K_i(1 + 1/T_i s)$ ,  $i=1,2$  ( $K_1, T_1 = (15, 4)$  and  $(K_1, T_1) = (14, 5)$ ). After identification process, identified transfer functions for Tank1 and Tank2 are obtained as follows:

$$G_{id1}(s) = \frac{Y_1(s)}{R_1(s)} = \frac{1.56s + 0.39}{6.24s^3 + 62.46s^2 + 2.56s + 0.39} \quad (21)$$

$$G_{id2}(s) = \frac{Y_2(s)}{R_2(s)} = \frac{1.70s + 0.34}{9.06s^3 + 90.74s^2 + 2.70s + 0.34} \quad (22)$$

Before starting the simulations, same actuator failure blocks are constructed for valve actuator so that in any time of simulation,

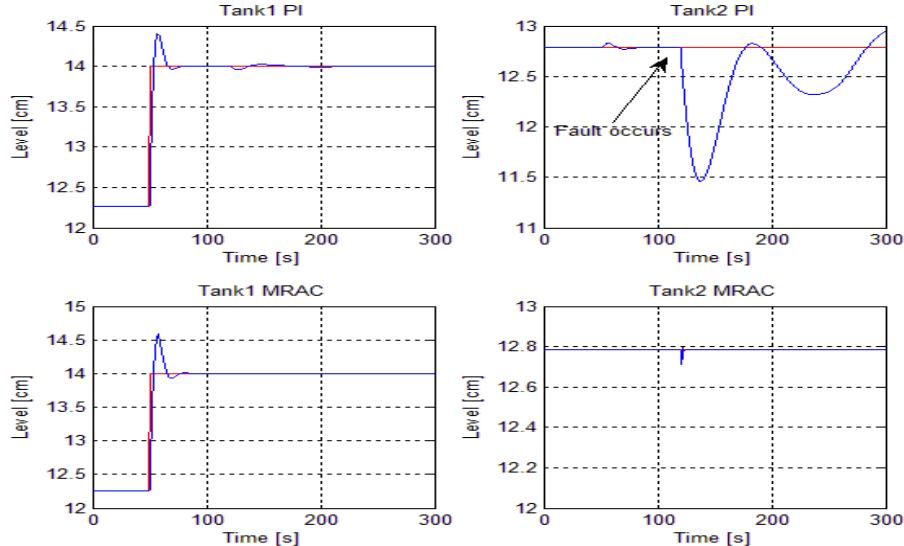


Fig. 4 Loss of effectiveness fault mode for valve2 at time 120 sec (— reference)

random failure mode with unknown size can be applied to controlled system. Various fault cases of valve are considered such as; leakage and loss of effectiveness. Figure 3.5 shows the block structure of failure block integrated water tank model. Faulty actuator block with various failure modes that is integrated to simulation can be seen in Figure 2.8.

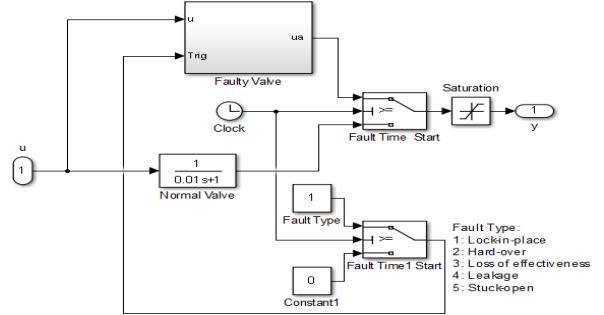
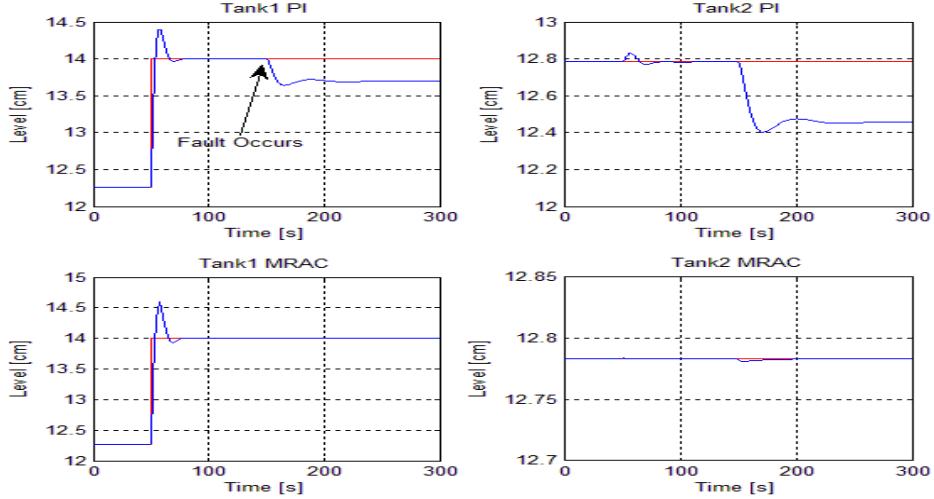
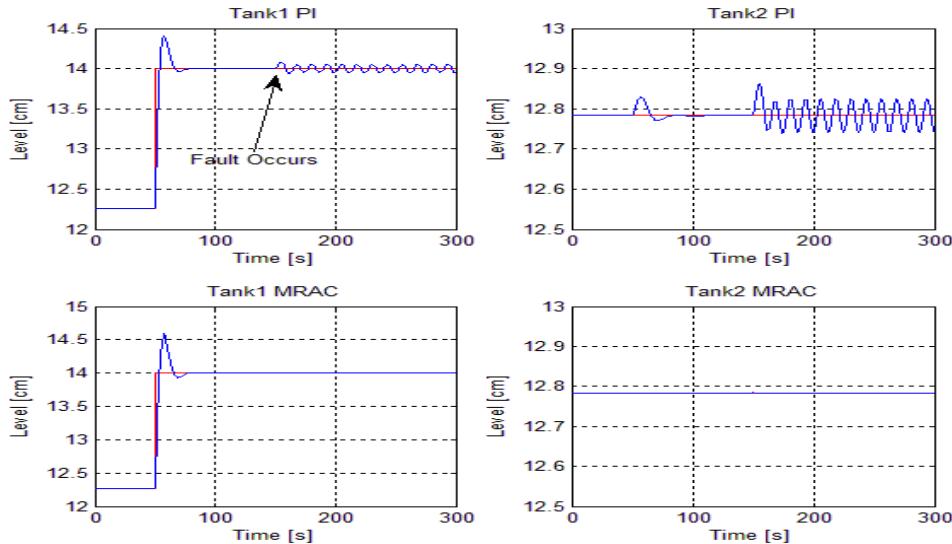


Fig. 3. Integrated fault mode simulation

Simulation durations are 300 sec, sampling time is selected to be 0.1 sec. The failure responses of both adaptive and non-adaptive controllers are examined for various faults scenarios. There are possibilities such that two valves can jump to faulty mode simultaneously, or one at a time. Plant is subjected to leakage and loss of effectiveness type failures by considering these cases. Following figures shows the performance of decentralized PI-controlled system and MRAC system in the case of partly failures. In Figure 4, the fault occurs in valve2 at time 120s and PI controlled system exhibits high amplitude oscillations in Tank2 level compared to adaptive controlled system. In Figure 5, both valves jump to loss of effectiveness fault mode at 150s of simulation. In normal controller response, it can be seen that Tank1 and Tank2 tracking errors increases while adaptive controller compensates the fault effect. In Figure 6, adaptive controller tracks the reference signal with zero steady state error but PI controlled system levels oscillates around reference level signal.



**Fig. 5.** Both valves jump to loss effectiveness fault mode at time 150s (— reference)



**Fig. 6.** Both valves jump to leakage fault mode at time 150 sec(— reference)

## 6. Conclusions

In this study, a simulation of the quadruple tank system is constructed. An adaptive fault tolerant control system is designed and compared with a non-adaptive decentralized control system. In simulations, process is subjected to valve faults with different scenarios. Simulation results show that adaptive controller can compensate any partial failures and disturbances with good transient behavior while PI-controlled system exhibits undesired high amplitude oscillations and bad transient behavior.

## Acknowledgment

This work is supported by The Scientific and Technological Research Council of Turkey (TUBITAK), through project 116E020.

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