Optimal Analytical PI and PID Tuning Rules for Controlling Stable Processes with Inverse Response

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Abstract

In industrial applications, it is possible to encounter processes that show an initial response in opposite to the final steady-state. Such processes involve difficulties to control them. Hence, researchers have recently started paying great attention to the control of such processes. This paper reports on deriving optimal tuning rules for controlling open loop stable processes with inverse response. Simple and optimal analytical tuning rules for PI and PID controllers have been derived based on integral performance criteria. Simulation examples have been given to show the usefulness of the proposed controller design method.

1. Introduction

Proportional-Integral-Derivative (PID) controllers have three terms to be calculated in order to control a process. Additionally, they perform quite satisfactorily for a wide range of operation conditions. Moreover, they are well-known by engineers. Therefore, despite great progress in engineering, PID controllers continue to be used extensively in the industry.

Processes exhibiting an inverse response before reaching the steady state can commonly be found in chemical process. Examples of such processes are the level of drum boiler in a distillation column, the exit temperature of a tubular exothermic reactor or outlet concentration of an autocatalytic CSTR [1], [2]. So far, very little attention has been paid to the control of inverse response processes. Luyben [3] suggested using a PI (Proportional-Integral) controller for controlling stable processes with inverse response characteristics. The limitation of this method is that the dead time must not be larger than 3.2. Luyben [4] proposed an identification and control of integrating processes with inverse response. Tuning of PID controllers for controlling stable processes including a zero was given by Chien et al. [5]. Camacho et al. [6] gave application of variable structure control to chemical processes with inverse response. Identification of inverse response processes using second order model based on transient step responses was studied by Balaguer et al. [7]. More recently, Jeng and Lin [8] proposed a PID controller to control stable/integrating processes with inverse response and time delay.

Integral performance criterion is a well-known approach to determine tuning parameters of PID controllers. Analytical tuning rules based on time weighted integral of error for a first-order plus dead time plant model were provided by Zhuang and Atherton [9]. Kaya [10] used open loop stable first or second order plus dead time plant transfer function models and gave simple tuning rules for a PI/PID controller in a Smith predictor structure based on time weighted integral of error. Visioli [11] supplied tuning rules based on the minimization of integral criteria for integrating and unstable first order plus time delay processes. Ali and Majhi [12] obtained analytical expressions correlating controller parameters and plant model parameters based time weighted integral of error performance criteria for integrating processes. Neither of the above cited studies considered processes with inverse response.

Therefore, this paper aims to provide simple, optimal and analytical tuning rules for a PI and PID controller for controlling open loop stable plus dead time process with inverse response. Repeated optimizations on the error signal to minimize it were carried out to obtain relationships between the controller and processes model transfer function parameters. Simulation results have been provided to illustrate the use of the proposed design method.

The paper is organized as follows. Section 2 gives a very short review of inverse response processes, followed by a brief review of integral performance criteria in Section 3. Development of optimum analytical tuning rules for PI and PID controllers for controlling stable processes with inverse response characteristics is provided Section 4. Simulation examples are supplied in section 5. Finally, conclusions are given in section 6.

2. Inverse Response Systems

Inverse response systems give an initial response that is opposite to the reference input applied to system. Consider the block diagram given in Fig. 1. The transfer function between the output and the input of this system given by

$$G(s)=\frac{Y(s)}{U(s)}=\frac{(K_1-K_2)\left[\dfrac{K_1T_2-K_2T_1}{K_1-K_2}s+1\right]}{(T_1s+1)(T_2s+1)} \tag{1}$$

Here, when the conditions $T_1 \ll T_2$ and $K_2 < K_1$ are ensured then the system shown in Fig. 1 will first respond in opposite to the applied input. However, after a very short time, the system starts to follow the reference and reaches the steady state.

If, a process transfer function given by eqn. (1) ensures the following condition

$$\frac{K_1T_2-K_2T_1}{K_1-K_2}<0 \tag{2}$$

then, it is called an inverse response process.

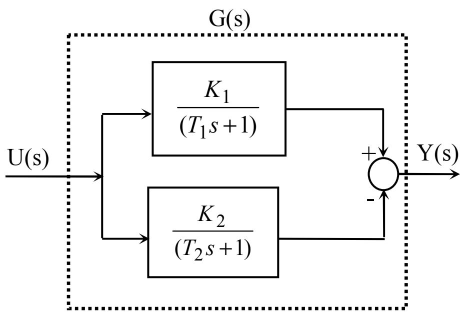

Fig.1. Inverse Response Process

3. Integral Performance Criteria

One of the well-known approach to identify tuning parameters of PID controllers is the integral performance criteria. Many text books, such as [13], [14], include short sections dedicated to the technique.

Åström [15] suggested a recursive algorithm that can be used to calculate the Integral of Squared Error (ISE) on today's digital computers using the s-domain approach. The ISE criterion given by

$$J_0 = \int_0^\infty e^2(t)dt \tag{3}$$

can easily be evaluated in the s-domain:

$$J_0 = \frac{1}{2\pi j}\int_0^\infty E(s)E(-s)ds \tag{4}$$

Here, *E(s)* is the error signal and assumed to be given by *E(s)=B(s)/A(s)*. *A(s)* and *B(s)* are polynomials with real coefficients given by

$$A(s) = a_0 s^m + a_1 s^{m-1} + \ldots + a_{m-1}s + a_m$$

$$B(s) = b_1 s^{m-1} + \ldots + b_{m-1}s + b_m$$

Time weighted version of the ISE criterion given by

$$J_n = \int_0^\infty [t^n e(t)]^2 dt \tag{5}$$

can also be evaluated using this method, due to the fact that $L\{tf(t)\} = -dF(s)/ds$. Here L denotes the Laplace transform and $L\{f(t)\} = F(s)$. In (5), $n=0$, $n=1$, and $n=2$ corresponds to the ISE, ISTE and IST^{2}E criteria. It is well-known that the ISE criterion frequently results in closed loop responses with large overshoots in response to step input changes. Therefore, in this study, optimal PI and PID controller tuning rules will be derived based on the ISTE and IST^{2}E criteria, which have been shown resulting in very satisfactory closed loop performances in the sense of less overshoot and small settling time.

4. Design of Optimum PI and PID Controllers

Derivation analytical and optimal PI and PID tuning rules to control open loop stable processes with inverse response based on minimization of the error signal in sense of ISTE and IST^{2}E integral performance criteria are given in this section.

4.1 Analytical PI Controller Tuning Rules

Let us assume that the actual plant transfer function can be modeled by

$$G(s) = \frac{K(-T_0 s+1)e^{-\theta s}}{(Ts+1)^2} \tag{6}$$

The ideal PI controller transfer function given by

$$G_c(s) = K_c(1+\frac{1}{T_i s}). \tag{7}$$

is considered. Equations (6) and (7) are used in the error function of the classical feedback control loop. Then, repeated optimizations based on integral performance criteria given in the previous section were carried out and relations between the normalized dead time, θ / T , and KK_c and T_i / T for varying values of T_0 / T have been obtained.

Fig. 1 shows the relation between KK_c and θ / T for the ISTE criterion. Similarly, Fig. 2 shows the relation between T_i / T and θ / T again for the ISTE criterion. In the figures, solid lines correspond to values obtained from the optimizations and asterisks correspond to values achieved from curve fitting formulae for KK_c and T_i / T given below:

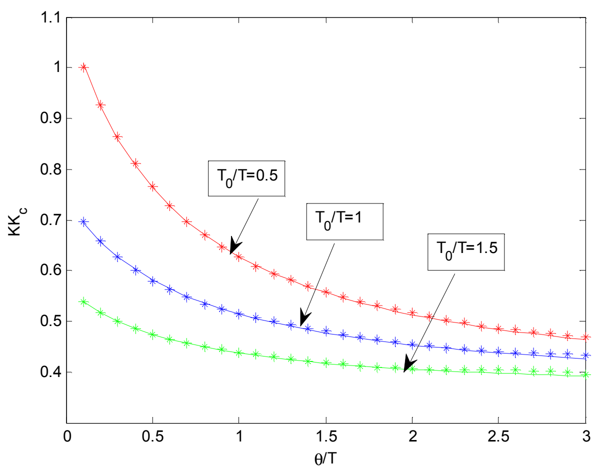

Fig.2. KK_c values for range of $0.1 \le \theta / T \le 3$ and varying T_0 / T values

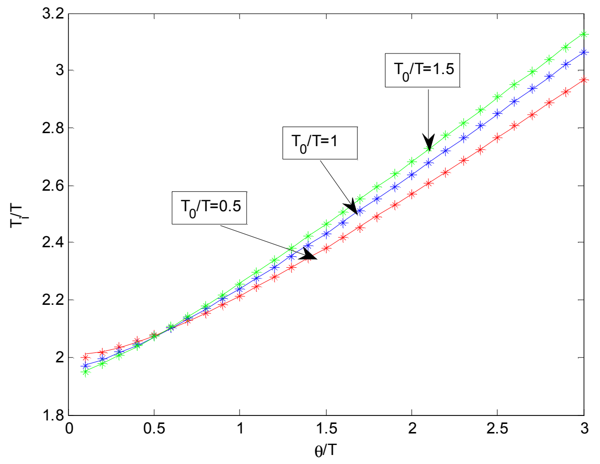

Fig. 3. T_i / T values for range of $0.1 \le \theta / T \le 3$ and varying T_0 / T values

$$KKc = 1.598 - 1.579\left(\frac{\theta}{T}\right) - 1.191\left(\frac{T_0}{T}\right) + 1.095\left(\frac{\theta}{T}\right)^2 + 1.517\left(\frac{\theta}{T}\right)\left(\frac{T_0}{T}\right) + 0.3348\left(\frac{T_0}{T}\right)^2 - 0.4167\left(\frac{\theta}{T}\right)^3 - 0.8458\left(\frac{\theta}{T}\right)^2\left(\frac{T_0}{T}\right) - 0.4383\left(\frac{\theta}{T}\right)\left(\frac{T_0}{T}\right)^2 + 0.08229\left(\frac{\theta}{T}\right)^4 \; 0.2082\left(\frac{\theta}{T}\right)^3\left(\frac{T_0}{T}\right) + 0.2077\left(\frac{\theta}{T}\right)^2\left(\frac{T_0}{T}\right)^2 - 0.006704\left(\frac{\theta}{T}\right)^5 - 0.01769\left(\frac{\theta}{T}\right)^4\left(\frac{T_0}{T}\right) - 0.3312\left(\frac{\theta}{T}\right)^3\left(\frac{T_0}{T}\right)^2 \tag{8}$$

and

$$\frac{Ti}{T} = 2.028 + 0.06344\left(\frac{\theta}{T}\right) - 0.09945\left(\frac{T_0}{T}\right) + 0.1066\left(\frac{\theta}{T}\right)^2 + 0.1793\left(\frac{\theta}{T}\right)\left(\frac{T_0}{T}\right) + 0.01862\left(\frac{T_0}{T}\right)^2 - 0.01275\left(\frac{\theta}{T}\right)^3 - 0.01523\left(\frac{\theta}{T}\right)^2\left(\frac{T_0}{T}\right) - 0.03003\left(\frac{\theta}{T}\right)\left(\frac{T_0}{T}\right)^2 \tag{9}$$

For the IST^{2}E criterion, the same procedure was followed to obtain tuning formulae for KK_c and T_i / T which are given below:

$$KKc = 0.924 - 0.4654\left(\frac{\theta}{T}\right) - 0.475\left(\frac{T_0}{T}\right) + 0.1381\left(\frac{\theta}{T}\right)^2 + 0.2555\left(\frac{\theta}{T}\right)\left(\frac{T_0}{T}\right) + 0.1011\left(\frac{T_0}{T}\right)^2 - 0.01522\left(\frac{\theta}{T}\right)^3 - 0.03446\left(\frac{\theta}{T}\right)^2\left(\frac{T_0}{T}\right) - 0.04056\left(\frac{\theta}{T}\right)\left(\frac{T_0}{T}\right)^2 \tag{10}$$

$$\frac{Ti}{T} = 1.597 + 0.1957\left(\frac{\theta}{T}\right) + 0.2983\left(\frac{T_0}{T}\right) + 0.02309\left(\frac{\theta}{T}\right)^2 + 0.05089\left(\frac{\theta}{T}\right)\left(\frac{T_0}{T}\right) \tag{11}$$

4.2 Analytical PID Controller Tuning Rules

In this section tuning formulae for a PID controller will be provided by following the procedure given above. The plant transfer is again assumed to be given by (6). The following

$$G_c(s) = K_c(1 + \frac{1}{T_i s} + T_d s) \tag{12}$$

ideal PID controller transfer function is assumed. Similar to derivation of PI controller tuning formulae, the following tuning rules were determined based on ISTE criterion for calculating tuning parameters of a PID controller:

$$KKc = 3.421 - 3.818\left(\frac{\theta}{T}\right) - 2{,}849\left(\frac{T_0}{T}\right) + 2.227\left(\frac{\theta}{T}\right)^2 + 3.015\left(\frac{\theta}{T}\right)\left(\frac{T_0}{T}\right) + 0.8179\left(\frac{T_0}{T}\right)^2 - 0.6089\left(\frac{\theta}{T}\right)^3 - 1.084\left(\frac{\theta}{T}\right)^2\left(\frac{T_0}{T}\right) - 0.7628\left(\frac{\theta}{T}\right)\left(\frac{T_0}{T}\right)^2 + 0.06428\left(\frac{\theta}{T}\right)^4 \; 0.1215\left(\frac{\theta}{T}\right)^3\left(\frac{T_0}{T}\right) + 0.1728\left(\frac{\theta}{T}\right)^2\left(\frac{T_0}{T}\right)^2 \tag{13}$$

$$\frac{Ti}{T} = 1.924 + 0.166\left(\frac{\theta}{T}\right) - 0.003891\left(\frac{T_0}{T}\right) + 0.04266\left(\frac{\theta}{T}\right)^2 + 0.1025\left(\frac{\theta}{T}\right)\left(\frac{T_0}{T}\right) \tag{14}$$

and

$$\frac{T_d}{T} = 0.5616 + 0.1503\left(\frac{\theta}{T}\right) - 0.09421\left(\frac{T_0}{T}\right) + 0.05702\left(\frac{\theta}{T}\right)^2 + 0.3218\left(\frac{\theta}{T}\right)\left(\frac{T_0}{T}\right) + 0.03904\left(\frac{T_0}{T}\right)^2 - 0.01702\left(\frac{\theta}{T}\right)^3 - 0.08411\left(\frac{\theta}{T}\right)^2\left(\frac{T_0}{T}\right) - 0.09746\left(\frac{\theta}{T}\right)\left(\frac{T_0}{T}\right)^2 + 0.00162\left(\frac{\theta}{T}\right)^4 \; 0.0124\left(\frac{\theta}{T}\right)^3\left(\frac{T_0}{T}\right) - 0.01076\left(\frac{\theta}{T}\right)^2\left(\frac{T_0}{T}\right)^2 \tag{15}$$

For the IST^{2}E criterion, the same procedure was followed to obtain tuning formulae for KK_c, T_i / T and T_d / T which are given below:

$$KKc = 3.328 - 3.956\left(\frac{\theta}{T}\right) - 2,646\left(\frac{T_0}{T}\right) + 2.394\left(\frac{\theta}{T}\right)^2$$
$$+2.925\left(\frac{\theta}{T}\right)\left(\frac{T_0}{T}\right) + 0.7362\left(\frac{T_0}{T}\right)^2 - 0.6714\left(\frac{\theta}{T}\right)^3$$
$$-1.088\left(\frac{\theta}{T}\right)^2\left(\frac{T_0}{T}\right) - 0.698\left(\frac{\theta}{T}\right)\left(\frac{T_0}{T}\right)^2 + 0.07191\left(\frac{\theta}{T}\right)^4 \quad (16)$$
$$0.1274\left(\frac{\theta}{T}\right)^3\left(\frac{T_0}{T}\right) + 0.159\left(\frac{\theta}{T}\right)^2\left(\frac{T_0}{T}\right)^2$$

$$\frac{Ti}{T} = 1.951 + 0.2232\left(\frac{\theta}{T}\right) + 0.007484\left(\frac{T_0}{T}\right) + 0.03265\left(\frac{\theta}{T}\right)^2 \quad (17)$$
$$+0.06075\left(\frac{\theta}{T}\right)\left(\frac{T_0}{T}\right)$$

and

$$\frac{T_d}{T} = 0.4767 + 0.18\left(\frac{\theta}{T}\right) + 0.048\left(\frac{T_0}{T}\right) + 0.01159\left(\frac{\theta}{T}\right)^2$$
$$-0.04473\left(\frac{\theta}{T}\right)\left(\frac{T_0}{T}\right) - 0.02375\left(\frac{T_0}{T}\right)^2 - 0.01302\left(\frac{\theta}{T}\right)^3$$
$$+0.06679\left(\frac{\theta}{T}\right)^2\left(\frac{T_0}{T}\right) + 0.0482\left(\frac{\theta}{T}\right)\left(\frac{T_0}{T}\right)^2 + 0.00298\left(\frac{\theta}{T}\right)^4 \quad (18)$$
$$-0.05388\left(\frac{\theta}{T}\right)^3\left(\frac{T_0}{T}\right) - 0.03439\left(\frac{\theta}{T}\right)^2\left(\frac{T_0}{T}\right)^2$$

5. Simulation Example

Simulation example illustrating the use of the proposed design method is given in this section. Comparisons have been performed with some design methods existing in the literature to show that very good closed loop performances have been obtained by using the proposed optimum tuning rules. Let us consider, a plant transfer function of

$$G(s) = \frac{(-0.2s+1)e^{-1.6s}}{(s+1)^2}.$$

This plant transfer function was first studied by Luyben [3]. Luyben [3] suggested a PI controller with tuning parameters of $K_c = 0.31$, $T_i = 1.14$. Comparisons will be given for the design method suggested by Chien et al. [5] who proposed a PI controller having tuning parameters of $K_c = 0.29$, $T_i = 1.00$ and $T_d = 1.00$.

For the proposed design method, the normalized dead time ratio is given by $\theta / T = 1.6/1 = 1.6$. T_0 / T ratio is 0.2. Table 1 summarizes calculated optimum tuning parameters based on the ISTE and IST^{2}E criteria for a PI and PID controller. Fig. 4 illustrates closed loop performances for a unit step input. Responses to a disturbance with magnitude of -0.3, entering the system at time $t = 30$ s are also depicted in the figure. It is observed that PI controllers designed based on ISTE and IST^{2}E criteria results in similar closed loop performances. This comment holds for DID controllers as well. Also, it is seen that PID controllers gives a faster response and hence results in slightly higher overshoot when compared to PI controllers. Finally, it is noticed that PID controllers yields faster disturbance rejections than PI controllers. Fig. 5 illustrates comparison of performances for the proposed and other design methods. For the proposed design method, the results only for IST^{2}E PI and PID controllers have been provided in order to avoid complexity in the figure. Control signal for all design methods used in comparison are shown in Fig. 6.

Table 1. Optimum PI and PID controller parameters

Optimization	Controller	K_c	T_i	T_d
ISTE	PI	0.5997	2.3786	-
	PID	0.8816	2.3308	0.9344
IST^{2}E	PI	0.4597	1.9914	-
	PID	0.8034	2.4126	0.7798

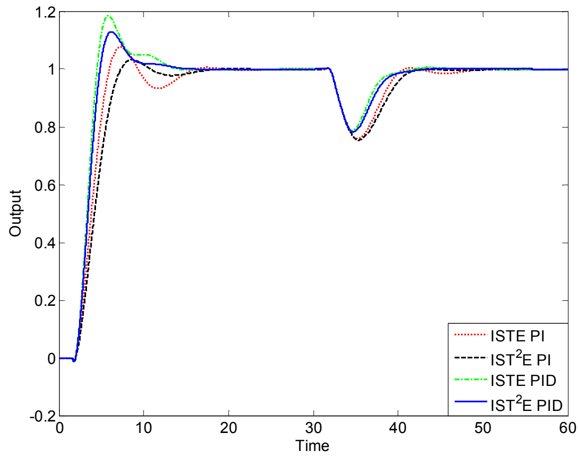

Fig. 4. Set-point tracking and disturbance rejection performances

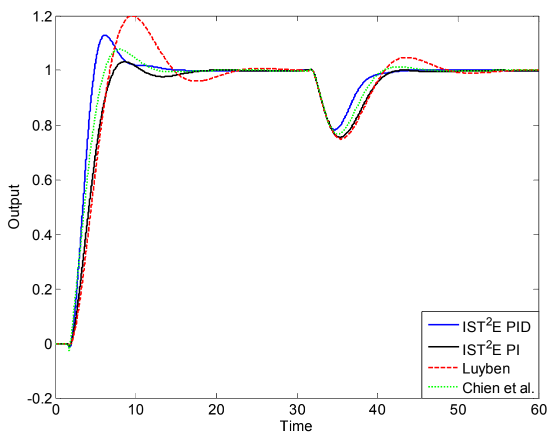

Fig. 5. Comparison of closed loop performances for the proposed and other design methods

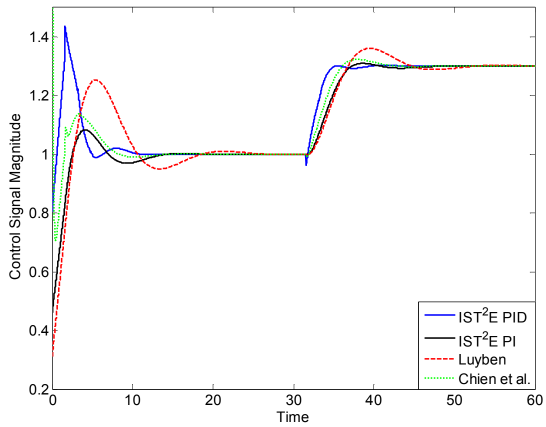

Fig. 6. Control Signal Magnitudes

Performance specifications for all design methods used in comparison are given in Table 2. In the table, ISE (Integral of squared error) (ISE) and IAE (Integral of absolute error) are two widely used criteria for evaluating closed loop performances. TV (total variation) of the input (*u(t)*), which is given by $\sum_{i=1}^{\infty}|u_{i+1}-u_i|$, is a good measure of the smoothness of a signal [16].

It is seen that in the sense of ISE and IAE values, proposed IST^{2}E PID gives the best result. In the sense of TV value, proposed IST^{2}E PI results in the best solution.

Table 2. Performance Specifications

Controller Design	ISE	IAE	TV
IST^{2}E PID	3.1675	4.759	0.0033
IST^{2}E PI	3.783	5.778	0.0021
Luyben	4.163	7.223	0.0030
Chien et al.	3.277	5.149	0.0077

6. Conclusions

Processes exhibiting inverse response characteristics are possible to encounter in industrial applications. However, so far, little attention has been given to control of such processes. Therefore, the paper has provided optimal tuning rules based on integral performance criteria for controlling stable processes with inverse response. Optimal tuning rules have been obtained, particularly, for the ISTE and IST^{2}E criteria as these criteria results in pretty good closed loop performances in response to step input changes. Simulation results have been presented to compare solutions with some design methods existing in the literature and suggested for the control of stable processes with inverse response.

7. References

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