Modelling, Simulation and Control of Quadruple Tank Process

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Abstract

Simple processes with only one output that may controlled by one input (variable) are known as single input single output process. But many processes are not such simple. They have more than one input (variable) and one output, which are called Multi Input-Multi Output (MIMO) processes. Common MIMO systems have some difficulties, such that they are large and complex. In addition, they have nonlinearities and also loop interactions which are between inputs and outputs. On the purpose of studying multivariable systems and designing controllers, the quadruple tank process (QTP) is chosen as a benchmark. This system is suitable for studying linear and nonlinear controllers and exhibits minimum and non-minimum system behaviors by simply changing configuration valve positions. We linearize nonlinear process model, than apply various control methods and compare the controlled system performance results.

1. Introduction

Multivariable system involves more than one control loop, These loops interact with each other, in such a manner that single input not only affects its own output but also affects other process outputs.[1,2] QTP has four tanks and two pumps, this benchmark is simply a water level control problem. The aim of the process is to keep the liquid level in the lower tanks at the desired values. Related laboratory process introduced by Johansson in Figure 2.1. [3], the process is used to show multivariable interactions which are also known as coupling and it limits performance in multivariable control systems. Multivariable interactions in a QTP are each output (water levels) of the system has affected by two pumps. Due to these reasons, it can be regarded as a prototype for many MIMO control applications in industry such as paper production processes, chemical processes, metallurgy and biotechnological areas, medical industries.

The structure of paper is as follows. The mathematical modeling of QTP benchmark system is described in Section 2, Linearization detail is given in Section 3. Centralized and Decentralized control design steps are given in Section 4, Simulation results are depicted in Section 5 and the Conclusions of the article are in Section 6.

2. Mathematical Model

The process inputs are pumps voltages and the process outputs are lower tanks water levels in Figure 2.1. For each tank the mathematical model is obtained by using Bernoulli's law yields and mass balance law. Tank numbers are represented by 'i', which may be 1,2,3,4.

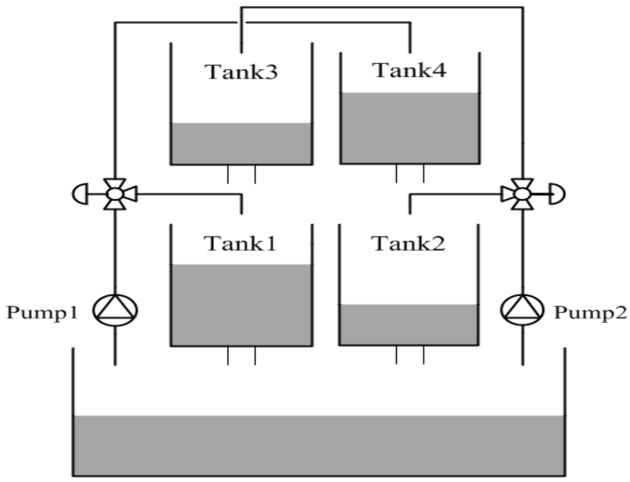


Figure 2.1. Quadruple-Tank Process

2.1. The Nonlinear Model

The mathematical equivalent of the process is given by Bernoulli's law and mass balance equation as follows:

Rate of accumulation = (Rate of in-flow)-(Rate of out-flow)

$$\frac{d(\rho V)}{dt} = \rho q_{in} - \rho q_{out} \quad (\text{since } \rho = \rho_1 = \rho_2 \text{ as same liquid})$$
$$A_i \frac{dh_i}{dt} = q_{in} - q_{out} \tag{1}$$

A_i=cross sectional area of the tank
h_i= the tank water level
q_{in_i}= in-flow of the tank
q_{out_i}= out-flow of the tank
The inflow of the tank (q_{in_i}) only depends on the input pump voltage and out-flow of the tank (q_{out_i}) depends on the gravity and acceleration due to head of the water in the tank.
Based on Bernoulli's equation q_{out_i} can be determined as follows

$$q_{in_i} = k_1 V_1 \qquad q_{in_3} = k_2 V_2 (1-\gamma_2)$$
$$q_{in_2} = k_2 V_2 \qquad q_{in_4} = k_1 V_1 (1-\gamma_1) \tag{2}$$

where k_1, k_2 are the pump constants; γ_1, γ_2 value ratio of valve positions

$$q_{out_i} = a_i \sqrt{2gh_i} \tag{3}$$

a_i, cross sectional area of the outlet pipes;
g, acceleration due to gravity

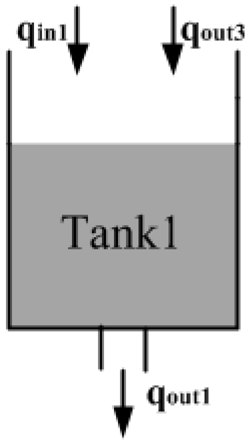


Figure 2.2. Single Tank Diagram

Using the law of conservation of mass

$$A_i \frac{dh_i}{dt} = q_{in_1} + q_{out_3} - q_{out_1} = \gamma_1 k_1 V_1 + a_3\sqrt{2gh_3} - a_1\sqrt{2gh_1} \quad (4)$$

The non-linear equations of the QTP are given as follow:

$$A_1 \frac{dh_1}{dt} = -a_1\sqrt{2gh_1} + a_3\sqrt{2gh_3} + \gamma_1 k_1 v_1$$

$$A_2 \frac{dh_2}{dt} = -a_2\sqrt{2gh_2} + a_4\sqrt{2gh_4} + \gamma_2 k_2 v_2$$

$$A_3 \frac{dh_3}{dt} = -a_3\sqrt{2gh_3} + (1-\gamma_2) k_2 v_2$$

$$A_4 \frac{dh_4}{dt} = -a_4\sqrt{2gh_4} + (1-\gamma_1) k_1 v_1 \quad (5)$$

2.2. Relative Gain Array

If decentralized control structure is chosen as a multi input multi output controller, an appropriate pairing of input and outputs is needed. In this case of an *m x m* plant transfer function, there is *m!* different pairings. By the way, physical interpretation of system gives idea about which pairing is useful or which one is not. Relative Gain Array (RGA) is a method that can be used to suggest pairings through a known quantity. RGA is defined as a matrix Λ [4]:

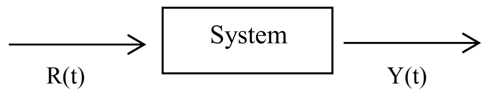
$$\Lambda = G(0) * G^{-T}(0) \quad (6)$$

If diagonal entries of Λ is negative than controlling the system is particularly difficult. A pairing with $0.67 < \lambda < 1.50$ in main diagonal elements usually gives good performance [4]. The RGA of the quadruple system is given as:

$$\Lambda = \begin{bmatrix} \lambda & 1-\lambda \\ 1-\lambda & \lambda \end{bmatrix}, \quad \lambda = \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2 - 1} \quad (7)$$

2.3. Linearized Model

The equations in [4] have square root terms which cause to nonlinearity. Because of that to design a controller becomes more difficult. The equation [5] is solved using by Taylor series expansion at the operating points and Jacobian matrix transformation to get a state space form of QTP.



R(t): input as voltage
Y(t): output as water level

$$\frac{dx_1}{dt} = f_1(h_1, h_2, h_3 \ldots h_n, u_1, u_2, u_3 \ldots u_n)$$

$$\frac{dx_n}{dt} = f_n(h_1, h_2, h_3 \ldots h_n, u_1, u_2, u_3 \ldots u_n) \quad (8)$$

The general vector form $\dot{x} = f(x, u)$ (x represents states)

Let

$$H_e = h_e + \Delta h \qquad U_e = u_e + \Delta u$$

Using Taylor series to yield the linear approximation

$$\dot{x} = \frac{dx}{dt} f(H_e, U_e) = f(h_e + \Delta h, u_e + \Delta u)$$

$$f(x,u) = f(h_e, u_e) + \frac{df}{dh}(h_e, u_e) + \frac{df}{du}(h_e, u_e) + \underbrace{\text{higher order terms}}_{0}$$

For simplification the higher order terms are neglected.

$$\frac{dh_1}{dt} = -\frac{a_1\sqrt{2gh_1}}{A_1} + \frac{a_3\sqrt{2gh_3}}{A_3} + \frac{\gamma_1 k_1 V_1}{A_1}, (u_1 = V_1;\ u_2 = V_2)$$

$$\frac{dh_1}{dt} - \frac{dh_{10}}{dt} = -\frac{a_1}{A_1}\sqrt{\frac{g}{2h_{10}}}(h_1 - h_{10}) + \frac{a_3}{A_1}\sqrt{\frac{g}{2h_{30}}}(h_3 - h_{30}) + \frac{\gamma_1 k_1}{A_1}(V_1 - V_0)$$

Let $x_i := h_i - h_{io}$ and $u_i := v_i - v_{io}$; so the system can be represented in state space form as follows:

$$\dot{x}_1 = -\frac{a_1}{A_1}\sqrt{\frac{g}{2h_{1o}}} x_1 + \frac{a_3}{A_1}\sqrt{\frac{g}{2h_{3o}}} x_3 + \frac{\gamma_1 k_1}{A_1} u_1$$

$$\dot{x}_2 = -\frac{a_2}{A_2}\sqrt{\frac{g}{2h_{2o}}} x_2 + \frac{a_4}{A_2}\sqrt{\frac{g}{2h_{4o}}} x_4 + \frac{\gamma_2 k_2}{A_2} u_2$$

$$\dot{x}_3 = -\frac{a_3}{A_3}\sqrt{\frac{g}{2h_{3o}}} x_3 + \frac{(1-\gamma_2)}{A_3} k_2 u_2$$

$$\dot{x}_4 = -\frac{a_4}{A_4}\sqrt{\frac{g}{2h_{4o}}} x_4 + \frac{(1-\gamma_1)}{A_4} k_1 u_1 \quad (9)$$

and

$$\frac{dx}{dt}=\begin{bmatrix} -\frac{1}{T_1} & 0 & \frac{A_3}{A_1T_3} & 0 \\ 0 & -\frac{1}{T_2} & 0 & \frac{A_4}{A_2T_4} \\ 0 & 0 & -\frac{1}{T_3} & 0 \\ 0 & 0 & 0 & -\frac{1}{T_4} \end{bmatrix} x + \begin{bmatrix} \frac{\gamma_1 k_1}{A_1} & 0 \\ 0 & \frac{\gamma_2 k_1}{A_1} \\ 0 & \frac{(1-\gamma_2)k_2}{A_3} \\ \frac{(1-\gamma_1)k_1}{A_4} & 0 \end{bmatrix} u$$

$$y=\begin{bmatrix} k_c & 0 & 0 & 0 \\ 0 & k_c & 0 & 0 \end{bmatrix} x$$

where the time constants are

$$T_i=\frac{A_i}{a_i}\sqrt{\frac{2h_{io}}{g}}\ ,\ i=1,..,4\ ,$$

$$u=\begin{bmatrix} \Delta u_1 \\ \Delta u_2 \end{bmatrix} \quad x=\begin{bmatrix} \Delta h_1 \\ \Delta h_2 \\ \Delta h_3 \\ \Delta h_4 \end{bmatrix} \quad y=\begin{bmatrix} \Delta y_1 \\ \Delta y_2 \end{bmatrix} \tag{10}$$

Transfer function matrix of the linearized system can be written as:

$$G(s)=\begin{bmatrix} \frac{\gamma_1 c_1}{1+T_1} & \frac{(1-\gamma_2)c_1}{(1+T_3 s)(1+sT_1)} \\ \frac{(1-\gamma_1)c_2}{(1+sT_4)(1+sT_2)} & \frac{\gamma_2 c_2}{1+sT_2} \end{bmatrix} = \begin{bmatrix} \frac{\gamma_1 c_1}{1+T_1} & \frac{(1-\gamma_2)c_1}{(1+T_3 s)(1+sT_1)} \\ \frac{(1-\gamma_1)c_2}{(1+sT_4)(1+sT_2)} & \frac{\gamma_2 c_2}{1+sT_2} \end{bmatrix} \tag{11}$$

where $c_1 = T_1 k_1 k_c / A_1$ and $c_2 = T_2 k_2 k_c / A_2$. Here the ratio k_1/k_2 is approximately equal to 1. The parameters $\gamma_1, \gamma_2 \in (0,1)$ are determined from how the valves are set prior to an experiment [3]. Due to values process act minimum or nonminimum phase which is shown on Table 1.

Table 1. Valve Setting.

Valve values	Process	Zero Location
$1 < \gamma_1 + \gamma_2 < 2$	minimum phase	Zero is in left half plan
$0 < \gamma_1 + \gamma_2 < 1$	nonminimum phase	Zero is in right half plan
$\gamma_1 + \gamma_2 = 1$		Zero is located at the origin

Table 2.Process parameters

Parameter	Value
Height of tanks, h_{max}	20 cm
Bottom area, Tank1, Tank3, A_1, A_3	28 cm^2
Bottom area, Tank1, Tank3, A_2, A_4	32 cm^2
Out pipe cross-sections, a_1, a_3	0.071 cm^2
Out pipe cross-sections, a_2, a_4	0.057
Level measurement device constant, k_c	0.500 V/cm
Gravity g	981 cm/s^2

Table 3. Operating points

Parameters	Minimum-Phase	Nonminimum-Phase
(h_{1o}, h_{2o}) cm	(12.26, 12.78)	(12.44, 13.17)
(h_{3o}, h_{4o}) cm	(1.63, 1.41)	(4.73, 4.99)
(v_1, v_2) V	(3.00, 3.00)	(3.15, 3.15)
(k_1, k_2) cm^3/Vs	(3.33, 3.35)	(3.14, 3.29)
(γ_1, γ_2)	(0.70, 0.60)	(0.43, 0.34)

By using the operating point parameters, physical modeling gives minimum-phase and non-minimum-phase transfer matrices in Equation 12 and Equation 13 respectively:

$$G(s)_{-}=\begin{bmatrix} \frac{2.6}{1+62.3s} & \frac{1.48}{(1+22.8s)(1+62.3)} \\ \frac{1.4}{(1+30s)(1+90.6s)} & \frac{2.84}{1+90.6s} \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \tag{12}$$

$$G(s)_{+}=\begin{bmatrix} \frac{1.5}{1+62.8s} & \frac{2.7}{(1+38.7s)(1+62.8)} \\ \frac{1.4}{(1+56.6s)(1+92s)} & \frac{1.61}{1+92s} \end{bmatrix} \tag{13}$$

4. Centralized and Decentralized control

In this part, various control methods are applied to the nonlinear system simulation to validate modeling and observe the system performance. Firstly, a state feedback controller is designed for minimum-phase system. The system is controllable and observable. The goal is to obtain a level tracking controller for lower two tanks. By Equation 13, it's clear to see the plant is a Type 0 system which has no integrator. The basic idea to design Type 1 servo system is adding an integrator in the feedforward path between the error comparator and the plant as shown in Figure 4.1.

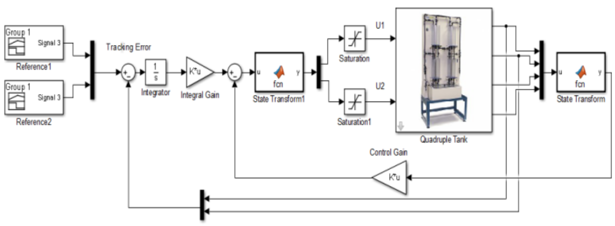


Figure 4.1. Augmented error state feedback controlled system block structure

After mathematical calculations and derivations, linearized system state space model is obtained at minimum phase operating point as given in Equation 14.

$$\frac{dx}{dt}=\begin{bmatrix} -0.016 & 0 & 0.044 & 0 \\ 0 & -0.011 & 0 & 0.033 \\ 0 & 0 & -0.044 & 0 \\ 0 & 0 & 0 & -0.033 \end{bmatrix} x + \begin{bmatrix} 0.0833 & 0 \\ 0 & 0.0628 \\ 0 & 0.0479 \\ 0.0312 & 0 \end{bmatrix} u$$

$$y=\begin{bmatrix} 0.5 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \end{bmatrix} x \tag{14}$$

The system and controller equations are as follows [5]:

$$\dot{x} = Ax + Bu\,; \qquad y = Cx\,; \quad u = -Kx + k_I e$$

$$\dot{\xi} = r - y = r - Cx \tag{15}$$

where ξ is the output of the integrator, r is the reference control signal. By augmenting the states ξ with states x, we can get the integral action in the controller for better tracking. The augmented system equations are as follows:

$$\begin{bmatrix} \dot{\tilde{x}}_e(t) \\ \dot{\tilde{\xi}}_e(t) \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \begin{bmatrix} x_e(t) \\ \xi_e(t) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u_e(t) \tag{16}$$

where

$$\tilde{x}_e(t) = x(t) - x(\infty)\,,$$

$$\xi_e(t) = \xi(t) - \xi(\infty)\,,$$

$$u_e(t) = -Kx_e(t) + k_I \xi_e(t)$$

Here new states become $x(t) = [x_e(t) \quad \xi_e(t)] \in R^{6x1}$.

Closed loop poles of system are placed:

$$P = [-0.0678 \pm 0.0683i \quad -0.0617 \pm 0.0591i \quad -0.0172 \quad -0.0562]$$

The controlled system performance is observed via simulations. System simulation time is 250s; system initially starts with operating point parameters given in Table 3.2. After 50s a step change of 2 cm in the reference signal is applied for Tank1 level. For Tank 2 a constant reference is chosen to track. The response of the system can be seen from Figure 4.2. The level of Tank1 is tracking the reference signal with zero steady state error and a settling time of approximately 60s. Tank2 level deviates from its reference a little between 60s and 110s as a result of interaction of tanks.

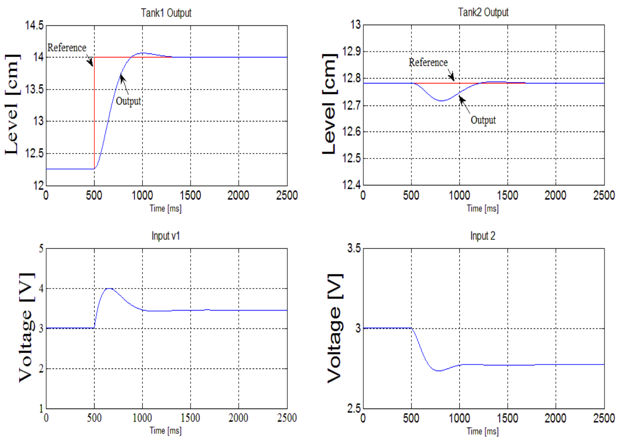


Figure 4.2. State feedback controlled system responses.

Secondly, a decentralized PI controller is designed by using system transfer matrix. Tank1 and Tank2 levels are the outputs to be controlled. Since we use minimum phase system configuration, the interaction between tanks are small in contrast to nonminmum phase model. We can also see that by looking at diagonal elements of RGA matrix. From Table 3, valve positions γ_1 and γ_2 are 0.7 and 0.6 respectively. RGA matrix in this configuration is:

$$\Lambda = \begin{bmatrix} 1.4 & -0.4 \\ -0.4 & 1.4 \end{bmatrix} \tag{17}$$

RGA, in this case, suggests (y_1, u_1) and (y_2, u_2) pairings.

PI controllers transfer function:

$$G_{ci}(s) = K_i\left(1 + \frac{1}{T_i s}\right), \quad i = 1, 2 \tag{18}$$

Controller parameters are tuned so that they give acceptable performance such as less than %10 and 50s settling time. The controller settings (K_1, T_1) = (3.0, 30) and (K_2, T_2) = (2.7, 40) give the response shown in Figure 4.4.

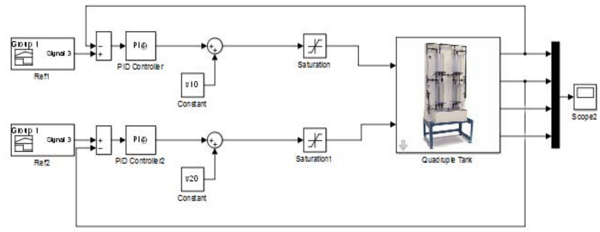


Figure 4.3. PI-controlled system block structure

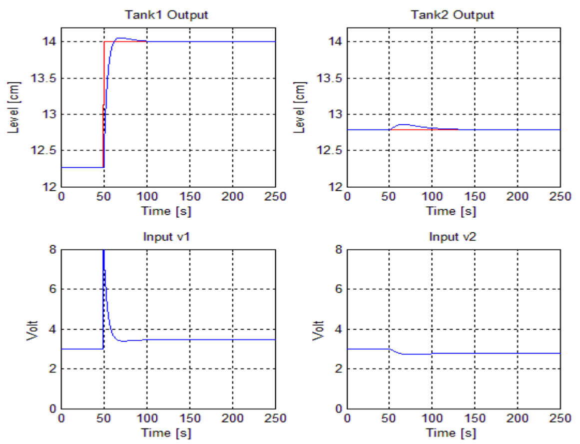


Figure 4.4. PI-controlled system simulation (__, reference signal)

4.1. Decoupler

MIMO problems can be converted to SISO problems by several methods. One of these methods is non–interacting or decoupling control schemes. This kind of control avoids the effects of loop interactions totally. The decoupler divides a MIMO process into a few independent single-loop subsystems. [6]. Figure 4.5 shows the decoupling control plot.

According to ideal decoupling procedure in [7] as given in Equation (4.6).

$$T(s) = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \tag{19}$$

where the diagonal elements, $T_{11} = T_{22} = 1$ (ideal decoupler case) and off diagonal elements, $T_{12}=-\frac{G_{12}}{G_{22}}$ and $T_{12}=-\frac{G_{21}}{G_{22}}$.

$$T_{12}=-\frac{0.57}{(1+22.7s)},\ T_{21}=-\frac{0.5}{(1+30.07s)} \tag{20}$$

Comparison of PI controlled system and Decentralized system is shown on Figure 4.61

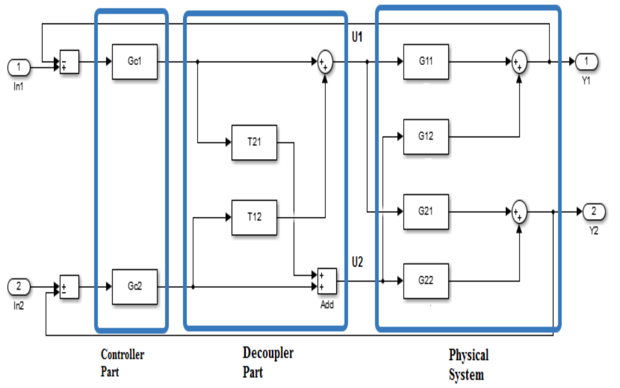


Figure 4.5. Block diagram of system with decoupler

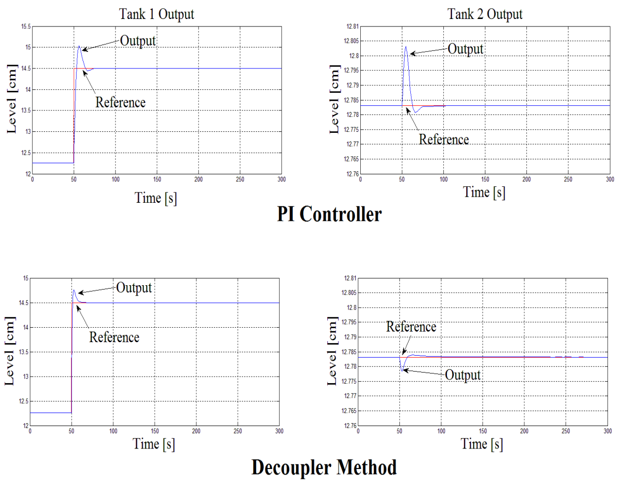


Figure 4.6. Comparison of controller

6. Conclusions

In this paper, a multiple interacting coupled tanks system is chosen as a case study. The linear system model is obtained and the conditions which the system behaves as minimum or non-minimum phase are given. Minimum phase system structure is chosen for this study than a centralized augmented state feedback controller is designed. For this controller, any further performance improvement is not considered. We only show that such a controller design is possible. Since interactions between levels of tanks are in acceptable range a decentralized control system design can be another option. In this case, interactions between tanks may be considered as disturbance. However, for cases such as high changes in the reference signal for one tank, the disturbance will have more effect on the other tank level. For such a case, a dynamical decoupler is designed and integrated to decentralized PI controller. Performance of controllers is compared via simulations.

Acknowledgement

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7. References

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