Scaled Consensus of Descriptor Multi-agent Systems

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Abstract

Multi-agent systems have been studied from different perspectives. In this paper, the problem of scaled consensus is considered for descriptor multi-agent systems. In fact, each agent is considered as a descriptor system. This is significant due to the fact that descriptor systems describe a larger group of systems. Furthermore, an algorithm is proposed to drive the agents to different consensus points, which is denoted as scaled consensus. This can also have very important applications in areas such as multi-robot systems where each robot needs to perform a different task. Moreover, in contrast with the works in the literature on scaled consensus, higher order systems are allowed in this paper. Also, as practical systems may suffer from time delay, these systems under a constant delay are considered. Finally, numerical examples are provided to further demonstrate the effectiveness of the proposed algorithms.

Keywords— multi-agent; scaled consensus; descriptor systems

1. Introduction

Various aspects of multi-agent systems have been studied in the literature throughout the past decades. A well-known issue in this area is the consensus problem. Consensus means agreement of agents on a common value. Different schemes have been considered to tackle the consensus problem of multi-agent systems [1-3]. As descriptor systems describe a larger class of systems compared with nondescriptor ones [4], researchers have tried to study these systems in a multi-agent framework.

Consensus conditions for descriptor multi-agent systems were introduced in [5]. The work of [5] was further studied in [6], in which necessary and sufficient conditions for the consensus problem of descriptor multi-agent systems were derived. An output feedback control law is employed to ensure the consensusability of the agents. Two algorithms were proposed for the consensus problem of descriptor multi-agent systems in [7]. These algorithms were based on linear matrix inequalities as well as a modified Riccati equation. Moreover, the admissible consensus problem of heterogeneous descriptor multi-agent systems was addressed in [8], and a novel consensus protocol was proposed. Two consensus algorithms based on a dynamic compensator are proposed in [9], and the consensus problem is tackled through dynamic output feedback. The consensus problem of descriptor multi-agent systems was further studied by introducing distributed observer-based protocols in [10]. In addition, multi-step algorithms was proposed to design the gain matrices.

In all of the above mentioned papers, consensus denotes the state in which all agents reach agreement on one common value. In [11], however, the notion of scaled consensus was introduced. Scaled consensus is a new concept in the field of consensus control. Using scaled consensus algorithm, the states of the agents reach assigned proportions instead of a common value.

The main contribution of our paper is to introduce the scaled consensus problem to descriptor multi-agents systems. To the best of authors' knowledge, this has not been addressed in the literature. The importance of this paper is twofold. Firstly, the agents are considered as descriptor systems. These systems include a larger class of dynamical systems. Second, the scaled consensus is considered for these systems. This is important mainly due to the fact that sometimes it is need for the agents to reach different final values in cooperation with each other, rather than a common value. To make the problem even more interesting, we also study these systems under constant delays.

The subsequent parts of the paper are organized as follows: Section II introduces the graph theory basics needed for this paper. In addition, it discusses the basics of descriptor systems. In Section III, the problem formulation is presented, and the stability result is given in Section IV. In section V, scaled consensus of descriptor multi-agent systems with delay is presented, and its stability is considered. In section VI, simulation results are studied and finally, conclusions are drawn in Section VII.

2. Preliminaries

2.1. Graph Theory

Graph theory is a powerful tool which has been successfully employed in the literature to study the connection of multi-agent systems. Let $G(\nu,\varepsilon,q)$ denote a weighted graph, in which $\nu=\{v_1,v_2,\cdots,v_n\}$ is the finite set of vertices, ε is the set of edges, and $Q=[q_{ij}]\in\mathbb{R}^{n\times n}$ represents a weighted adjacency matrix. The adjacency matrix of G is defined such that $q_{ij}>0$ if

$(j,i)\in\varepsilon$ and $q_{ij}=0$ otherwise. In addition, the Laplacian matrix $L=[l_{ij}]\in\mathbb{R}^{n\times n}$ associated with graph G is defined as $l_{ii}=\sum_{j=1,j\neq i}^{n}q_{ij}$ and $l_{ij}=-q_{ij}, i\neq j$. Moreover, a graph is called strongly connected if for every pair of vertices there is a path between them.

2.2 Descriptor Systems

As mentioned earlier, descriptor systems describe a larger class of systems. For example, descriptor systems have been applied in different areas such as economics, electrical and mechanical models. Researchers may find [12-15] useful resources on descriptor systems. Consider a descriptor system of the following form:

$$E\dot{x}=Ax \tag{1}$$

where $x\in\mathbb{R}^{p}$ is the state vector. Moreover $E\in\mathbb{R}^{p\times p}$, $A\in\mathbb{R}^{p\times p}$, and $rank(E)=r\leq p$. Since E is a singular matrix, these systems are also called singular systems.

Definition 1. System (1) is said to be regular if $\det(sE-A)\neq 0$.

Definition 2. System (1) is said to be impulse-free if $\deg(\det(sE-A))=rank(E)$.

3. Problem Formulation

In this section, the problem of scaled consensus for a group of descriptor agents is considered. Motivated by the work of [11], the following protocol is considered for agents to reach scaled consensus.

$$E_i\dot{x}_i=\mathrm{sgn}(\alpha_i)\sum_{j\in U(i)}k_{ij}(\alpha_j x_j(t)-\alpha_i x_i(t)) \tag{2}$$

where α_i is the scale for *i*-th state, $x_i\in\mathbb{R}^{p}$ and $U(i)$ is the set of vertices which are neighbors with i-th agent. Defining the stack vector $X=[x_1^T,\ldots,x_n^T]^T$, (2) can be written as follows:

$$E\dot{X}=(A\otimes I_p)X \tag{3}$$

where $E=diag(E_i)$ and $A=[diag(\mathrm{sgn}(\alpha))]\ K\ [diag(\alpha)]$ with $\alpha=[\alpha_1,\alpha_2,\cdots,\alpha_n]$, $[K]_{ij}=k_{ij}$ for $i=1,\ldots,n$, $j\in U(i)$; and $[K]_{ii}=-\sum_{j\in U(i)}k_{ij}$ for $i=1,\ldots,n$; $[K]_{ij}=0$ for $j\notin U(i)$. In other words, K is the negative of the Laplacian matrix defined in section II. Furthermore, the symbol $\otimes$ represents the Kronecker product, and I_p is the identity matrix with $p\times p$ dimension.

Remark 1. It is worthwhile to mention that the difference between (2) in this paper and equation (2) in [11] is that the proposed approach is a more general case compared to singular systems. Furthermore, in contrast with [11], higher dimensions are also considered.

In section IV the stability of (3) is studied, and proper conditions are provided for the stability of this system, and the final values of the agents are derived.

The final values of the agents can be determined when the system is stable. For stable systems, the following lemma was introduced in [11].

Lemma 1. [11] The final values of the agents can be determined with the following equation

$$x_f=\beta w^T x_0$$

where $\beta=[1/\alpha_1,\ldots,1/\alpha_n]^T$, and w is the left eigenvector of the state matrix. Note that w is appropriately normalized. Moreover, x_0 represents the vector of initial states.

4. Stability

This section provides the proper conditions for the stability of the system defined in (3). Moreover, the scaled final values of the agents are determined by modifying Lemma 1.

Theorem 1. System (3) is said to be stable if the following inequality holds for a symmetric positive semi-definite matrix P.

$$(A\otimes I_p)^T PE+E^T P(A\otimes I_p)<0 \tag{4}$$

Proof. In order to study the stability of (3), consider the following Lyapunov function

$$V(EX)=X^T E^T PEX \tag{5}$$

Calculating the derivative of (5)

$$\dot{V}=\dot{X}^T E^T PEX+X^T E^T PE\dot{X}$$

Noting that $\dot{X}^T E^T=(E\dot{X})^T$ and replacing the right hand side of (3)

$$\dot{V}=X^T(A\otimes I_p)^T PEX+X^T E^T P(A\otimes I_p)X$$

which yields

$$\dot{V}=X^T\left\{(A\otimes I_p)^T PE+E^T P(A\otimes I_p)\right\}X$$

Therefore $(A\otimes I_p)^T PE+E^T P(A\otimes I_p)<0$ is a sufficient condition for $\dot{V}<0$.

Remark 2. In order to determine the final values of the system, Lemma 1 should be modified due to the fact that the agents in this paper are assumed to have more states ($p\geq 1$). The modified result of Lemma 1 is rewritten as follows to obtain the final values of (3),

$$x_f=(\beta\otimes I_p)(w\otimes I_p)^T x_0 \tag{6}$$

where β is defined in Lemma 1.

5. MULTI-AGENT DESCRIPTOR SYSTEMS WITH DELAY

In some real systems, it is undeniable to have delays in the process of sending and receiving information between agents. As a result, a constant delay between each agent is considered and it is assumed that, each agent has access to its own state without any delay.

Rewrite (2) as:

$$E_i\dot{x}_i(t) = \mathrm{sgn}(\alpha_i)\sum_{j\in u(i)} k_{ij}\left(\alpha_j x_j(t-\tau) - \alpha_i x_i(t)\right) \tag{7}$$

Then, the overall system can be found as:

$$E\dot{X}(t) = A_0 X(t) + A_1 X(t-\tau) \tag{8}$$

where $E = diag(E_i)$ and A_0 is a diagonal matrix with diagonal entries of $A \otimes I_p$ in (3), and A_1 is a matrix with zero diagonal elements and off-diagonal elements of $A \otimes I_p$, i.e. $A_0 + A_1 = A \otimes I_p$. Note that, in this section, we assume that the matrix E has the following form.

$$E = \begin{bmatrix} I_n & 0 \\ 0 & 0 \end{bmatrix}$$

where n represents the total number of dynamic states in (8).

Theorem 2: The system (8) is delay independently stable if the following LMI holds

$$\begin{bmatrix} P^T A_0 + A_0^T P + Q & P^T A_1 \\ A_1^T P & -Q \end{bmatrix} \prec 0 \tag{9}$$

where $Q > 0$, and $P = \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix}$ with $P_1 = P_1^T > 0$, $P_1 \in \mathbb{R}^{n\times n}$, and P_2, and P_3 have appropriate dimensions.

Proof: In order to study the stability of (8), consider the following Lyapunov function

$$\mathrm{V} = \mathrm{X}^T EPX + \int_{t-\tau}^{t} \mathrm{X}^T(s)QX(s)ds \tag{10}$$

Considering the first part of V and the structure of P, we have

$$\frac{d}{dt}[X^T(t)EPX(t)] = 2X_1^T(t)P_1\dot{X}_1(t) = 2X^T(t)P^T\begin{bmatrix} \dot{X}_1(t) \\ 0 \end{bmatrix}$$

where X_1 is part of the vector X, also we know :

$$\begin{bmatrix} \dot{X}_1(t) \\ 0 \end{bmatrix} = E\dot{X}(t)$$

From equation (8), the derivative of V is

$$\dot{V} = 2\mathrm{X}^T(t)\mathrm{P}^T A_0 X(t) + 2\mathrm{X}^T(t)\mathrm{P}^T A_1 X(t-\tau) + \mathrm{X}^T(t)QX(t) - \mathrm{X}^T(t-\tau)QX(t-\tau)$$

which can be written as

$$\dot{V}(t) = \xi^T(t)\varphi\xi(t)$$

Where

$$\varphi = \begin{bmatrix} P^T A_0 + A_0^T P + Q & P^T A_1 \\ A_1^T P & -Q \end{bmatrix}$$

and

$$\xi^T = \begin{bmatrix} X^T(t) & X^T(t-\tau) \end{bmatrix}$$

It is clear that $\varphi < 0$ is a sufficient condition for the stability of (8). □

6. SIMULATION RESULTS

Example 1. A network of agents is considered in Figure 1.

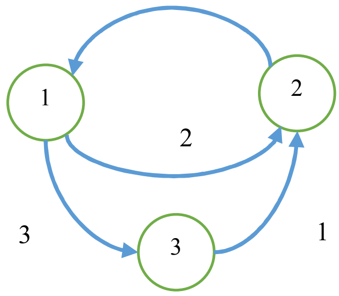

Fig. 1. The topology of the system Example 1

The following parameters are considered for the agents in example 1.

$$E_i = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad K = \begin{bmatrix} -1 & 1 & 0 \\ 2 & -3 & 1 \\ 3 & 0 & -3 \end{bmatrix} \quad \alpha = \begin{bmatrix} 1 \\ 0.5 \\ -2 \end{bmatrix}$$

In contrast with [11], the agents are allowed to have more than one state; therefore, in this example, agents are assumed to have two states; that is $p = 2$. Using the presented information, (3) is solved with the following initial condition.

$$x_0 = \begin{bmatrix} 2.08 & 1.76 & 2.02 & 3.53 & 8.21 & -0.88 \end{bmatrix}^T$$

The results are depicted in Figure 2. It is obvious that the agents have reached scaled consensus. The final values of the agents are obtained according to (6) as follows:

$$x_f = \begin{bmatrix} 1.07 & 1.76 & 2.14 & 3.53 & -0.53 & -0.88 \end{bmatrix}^T$$

which are consistent with the results presented in Figure 2.

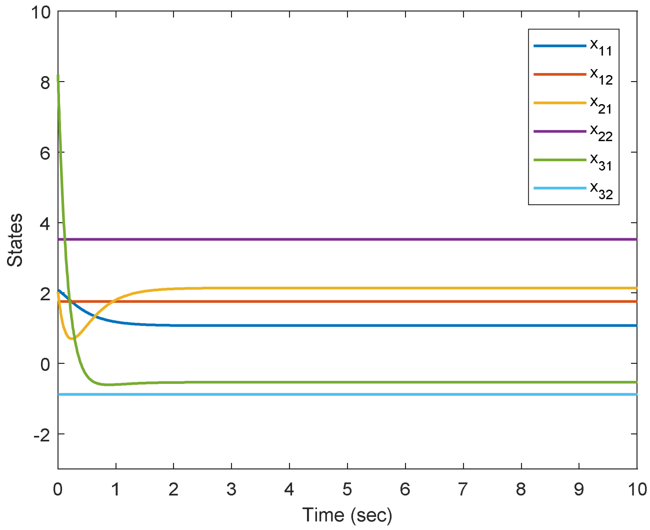

Fig. 2. State trajectories of agents in Example 1

Example 2. In order to validate the findings of this paper, another example is provided with four agents. Consider the topology for the agents which is shown in figure 3.

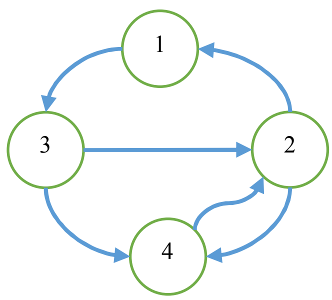

Fig. 3. The network topology of the system in Example 2

The following parameters are considered for the agents.

$$E_i = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad K = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -2 & 1 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & -2 \end{bmatrix} \quad \alpha = \begin{bmatrix} 2 \\ 0.7 \\ 0.3 \\ 1.1 \end{bmatrix}$$

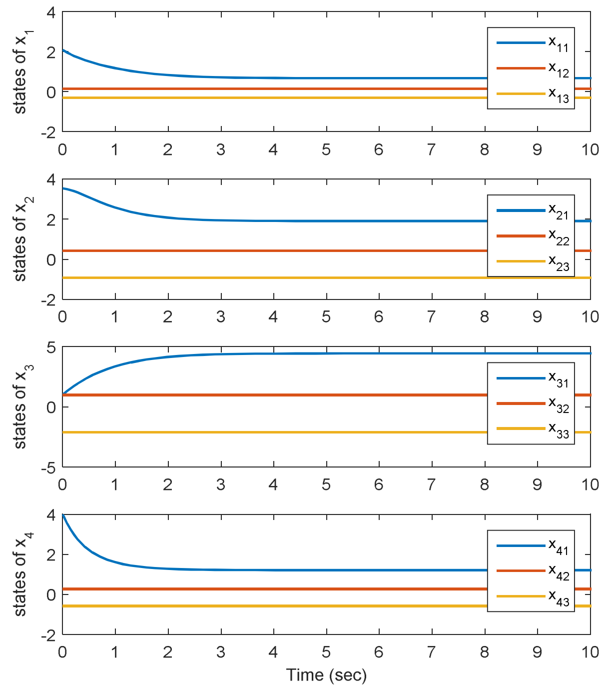

Fig. 4. State trajectories of the agents 1-4 in Example 2

The results shown in Figure 4 indicate that the system defined in (3) can reach a scaled consensus. The initial conditions assumed for this system are:

$$x_{01} = \begin{bmatrix} 2.08 \\ 0.28 \\ -0.63 \end{bmatrix} \quad x_{02} = \begin{bmatrix} 3.53 \\ 0.41 \\ -0.91 \end{bmatrix} \quad x_{03} = \begin{bmatrix} 1.00 \\ 0.96 \\ -2.13 \end{bmatrix} \quad x_{04} = \begin{bmatrix} 4.00 \\ 0.26 \\ -0.58 \end{bmatrix}$$

Note that the final values of agents in example 2 are computed using (6) as

$$x_{f1} = \begin{bmatrix} 0.66 \\ 0.14 \\ -0.31 \end{bmatrix} \quad x_{f2} = \begin{bmatrix} 1.89 \\ 0.41 \\ -0.91 \end{bmatrix}$$

$$x_{03} = \begin{bmatrix} 4.43 \\ 0.96 \\ -2.13 \end{bmatrix} \quad x_{04} = \begin{bmatrix} 1.20 \\ 0.26 \\ -0.58 \end{bmatrix}$$

which confirms the results of Figure 4.

Example 3: Consider the graph of Example 1 with a constant delay τ between agents. The simulation results are given for different delays in Figures 5-7. Moreover, using LMI solvers, P and Q in (10) are determined as follows.

$$P=\begin{bmatrix} 46.61 & 0 & 1.76 & 0 & 0 & 0 \\ 0 & 56.27 & 0 & 0 & 0 & 0 \\ 1.76 & 0 & 7.47 & 0 & 0 & 0 \\ 0 & -3.61 & 0 & 11.10 & 0 & -0.32 \\ -0.32 & 0 & 0.38 & 0 & 5.45 & 0 \\ 0 & 0.38 & 0 & -0.32 & 0 & 11.98 \end{bmatrix}$$

$$Q=\begin{bmatrix} 41.61 & 0 & -0.72 & 0 & -19.99 & 0 \\ 0 & 47.03 & 0 & -2.64 & 0 & -29.07 \\ -0.72 & 0 & 14.10 & 0 & 4.79 & 0 \\ 0 & -2.64 & 0 & 16.26 & 0 & 3.49 \\ -19.99 & 0 & 4.79 & 0 & 46.05 & 0 \\ 0 & 29.07 & 0 & 3.49 & 0 & 30.85 \end{bmatrix}$$

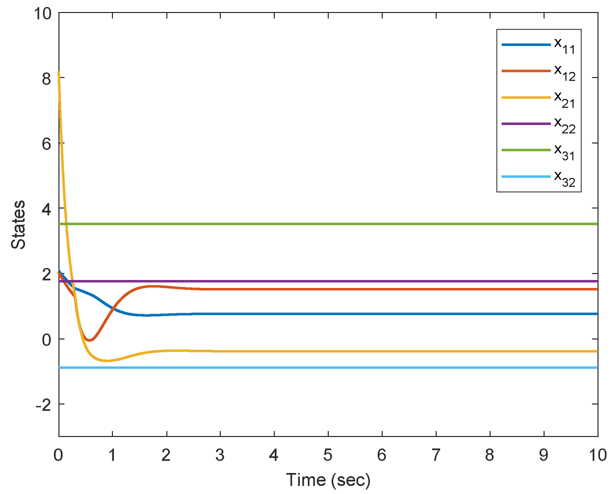

Fig. 5 State trajectories of Example 3 with $\tau = 0.2$

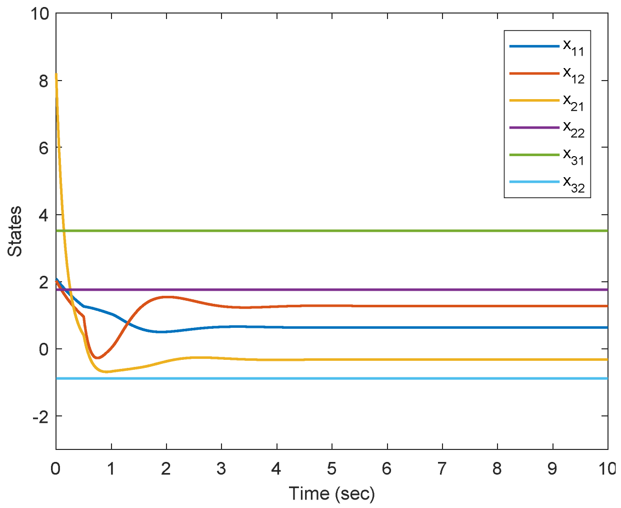

Fig. 6 State trajectories of Example 3 with $\tau = 0.5$

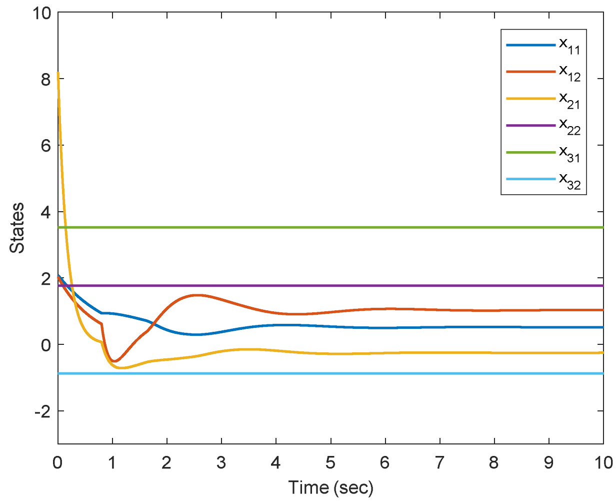

Fig. 7 State trajectories of Example 3 with $\tau = 0.8$

Based on Simulation results of Example 3, it is obvious by increasing the amount of τ , states need more time for scaled consensus.

7. CONCLUSION

In this paper, the scaled consensus of descriptor multi-agent systems was addressed. As a matter of fact, the stability of these systems with and without time delay was studied. Three numerical examples were presented to validate the findings of this paper. Results demonstrated that scaled consensus occurred in all examples and simulation results fit with theories.

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