Investigation of the Effects of Fractional and Integer Order Fuzzy Logic PID Controllers on System Performances

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Abstract

In this study, the effects of fractional and integer order fuzzy logic PID controllers on system performances are investigated. In that respect, linear and nonlinear fuzzy decision rule bases are taken into consideration. To be fair on comparison, the parameters of the controllers are all optimized using BBBC optimization algorithm according ITSE performance index. This investigation is done on second order linear system with time delay since many high order systems may be represented by this model. Moreover, further study is done on a nonlinear benchmark system. It has been observed that linear decision surface in both controllers generally gives better performance than nonlinear decision surface. It is concluded that nonlinearity of fractional operators is more effective than the nonlinearity produced by fuzzy rule base.

1. Introduction

Fuzzy logic PID (FL-PID) controllers are broadly used and proven to be practical amongst fuzzy logic controllers (FLC) [1-5]. FL-PID controllers and their performances have been analyzed with respect to the variations of their internal structure such as fuzzy rule base, input/output parameters, fuzzy linguistic sets, membership functions (MF), inference and defuzzification methods. Some other studies make observations on the change of performance with adjustments of input/output scaling factors and membership functions [6-8].

Integer order PID (IO-PID) controllers are known to be ineffective while controlling non-minimum phase processes or highly nonlinear systems with dead time [9]. Consequently, applications of fractional order calculus gradually increased over the years in various areas of science and engineering. Deployment of fractional order calculus to dynamic systems was introduced by Manabe [10]. Oustaloup showed that performance of fractional order controller surpasses the performance of IO-PID [11]. Later, Podlubny proposed a generalization of the PID controller called fractional order PID [12].

Fractional order PID (FO-PID) inherently owns two extra parameters, one from the integrator and the other one from the derivative operator [13-17]. Classical FL-PID controllers generally use error and its integer order derivative as input and integer order integrating operator at the output. Usage of fractional order derivation and integration instead of their integer order counterparts may provide design flexibility and additional nonlinearity while improving the FL-PID performance [18].

In fractional order fuzzy PIDs (FO-FL-PID), nonlinearities from fractional order operations and the nonlinearity from fuzzy mapping are added together and this provides more complex design options. In this study, effects of the nonlinearity caused by fractional operators and the nonlinearity caused by fuzzy rule bases of FO-FL-PID and FL-PID are compared with each other. In order to investigate the effects and dominancy of two types of nonlinearity, two distinct rule bases are formed, one with nonlinear control surface and the other one with linear control surface. The controllers are tested on nonlinear and linear systems with different dead times and the parameters of the controllers are optimized using Big-Bang-Big-Crunch (BBBC) optimization algorithm according to Integral of Time multiplied Squared Error (ITSE) performance index [19].

Rest of the paper is organized as follows: Section 2 shows the FL-PID controller structure, Section 3 discusses the fractional calculus and approximations used on control structure, Section 4 presents the FO-FL-PID structure and the rule bases, Section 5 provides the simulation results for the benchmark systems and Section 6 is the discussion and conclusion of the study.

2. Fuzzy Logic PID Controller

FL-PID controller structure considered in this study is shown in the Fig. 1.

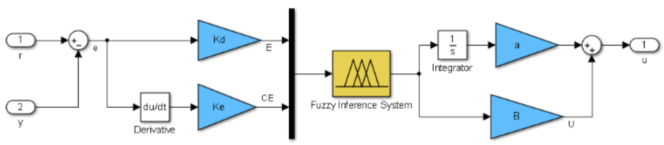

Fig. 1. Fuzzy Logic PID Controller block diagram

In this figure, K_e, K_d, α and β are input and output scaling factors, respectively. Output u of the FL-PID is given by

$$u = \alpha U + \beta \int U \, dt \qquad (1)$$

where U is the output of FLC. Mamdani-type inference mechanism is preferred. Product-sum inference method with triangular membership functions for both input and output are used. Fuzzy output is transformed to a crisp value using centroid defuzzification method.

3. Fractional Calculus

In Fractional Calculus, d^n/dt^n can have fractional positive or negative n values. Generalized fractional order operator can be defined as

$${}_a D_t^a = \begin{cases} \frac{d^a}{dt^a} \\ I \\ \int_a^t (dT)^a \end{cases} \qquad (2)$$

Riemann-Liouville, Caputo and Grünwald-Letnikov descriptions of fractional operator are the most commonly used ones [20].

Linear time invariant (LTI) fractional order differential equations are in the form of

$$\alpha_n D^{\alpha_n} y(t) + \alpha_{n-1} D^{\alpha_{n-1}} y(t) + \cdots + \alpha_0 D^{\alpha_0} y(t) = \beta_m D^{\beta_m} u(t) + \beta_{m-1} D^{\beta_{m-1}} u(t) + \cdots + \beta_0 D^{\beta_0} u(t) \quad (3)$$

By taking Laplace transformation continuous fractional order system can be shown as

$$G(s) = \frac{b_m s^{\beta_m} + \ldots + b_1 s^{\beta_1} + b_0 s^{\beta_0}}{a_n s^{\alpha_n} + \ldots + a_1 s^{\alpha_1} + a_0 s^{\alpha_0}} \quad (4)$$

where αk, βk (k=0,1,2,…) are real numbers, βk< βk-1< …< β0, αk< αk-1 <…< α0 and ak, bk (k=0,1,2…) are arbitrary constants.

In order to define fractional order differentiation, approximations using integer order systems must be carried. Oustaloup filter is one of the most commonly used approximation method [11]. Oustaloup filter is defined as

$$s^a \cong G_f(s) = K' \prod_{k=1}^{N} \frac{s+w_k'}{s+w_k} \quad (5)$$

where $a \in (0,1)$, gain zeros and poles are

$$K' = w_h^a \quad (6)$$

$$w_k = w_l \left(\frac{w_h}{w_l}\right)^{\frac{2k-1+a}{2N}}; \; w_k' = w_l \left(\frac{w_h}{w_l}\right)^{\frac{2k-1-a}{2N}} \quad (7)$$

Low and high frequencies for the Oustaloup filter approximation are w_l and w_h , respectively. N represents the order of integer transfer function used in Oustaloup approximation. In order to define exact fractional order transfer function, N must be infinite. Thus, larger N value gives a better approximation [11, 16].

4. Fractional Order Fuzzy Logic PID Controller

Fuzzy PID structure used in this paper has fuzzy PD and fuzzy PI controllers. In the case of FO-FL-PID, derivative of error at the input and integral operator at the output are replaced by their fractional counterparts [20]. The basic internal block diagram of FO-FL-PID is shown in Fig. 2. In this figure μ is the fractional derivative and λ is the fractional integrator orders.

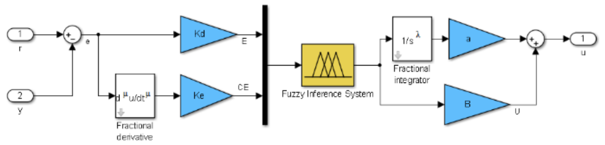

Fig. 2. Fractional order fuzzy logic control block diagram

Two distinct rule bases are used in order to analyze the nonlinearity effects of fractional operators and fuzzy mapping. For both of the rule bases, Mamdani type inference and standard triangular input and output membership functions are used. Centroid method is used for the defuzzification.

Fig. 3 provides the non-linear rule base [21]. The meta-rule of the rule base is that "when the system error and its derivative is large and moderately far from the reference the control input is large". Membership functions are triangular and equally spaced ranging in [-1,1]. The linguistic variables are defined as NL, NM, NS, Z, PS, PM, PL where N, P, Z, S, M, L represent Negative, Positive, Zero, Small, Positive, Medium, Large, respectively. Fig. 4 shows the nonlinear decision surface of the given rule base.

The linear rule base is given in Fig.5. In order to stretch the control surface and make it as much as linear, additional output membership functions are added. Fuzzy linguistic variables at Fig. 5 starting from the left, are NLL, NML, NSL, NL, NM, NS, Z, PS, PM, PL, PSL, PML, PLL where the same nomenclature is used as above. Linear decision surface is given in Fig. 5.

de \ e	NL	NM	NS	Z	PS	PM	PL
PL	Z	PS	PM	PL	PL	PL	PL
PM	NS	Z	PS	PM	PL	PL	PL
PS	NM	NS	Z	PS	PM	PL	PL
Z	NL	NM	NS	Z	PS	PM	PL
NS	NL	NL	NM	NS	Z	PS	PM
NM	NL	NL	NL	NM	NS	Z	PS
NL	NL	NL	NL	NL	NM	NS	Z

Fig. 3. Rule base for nonlinear control surface

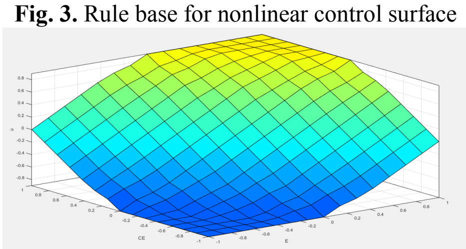

Fig. 4. Nonlinear decision surface

de \ e	NL	NM	NS	Z	PS	PM	PL
PL	Z	PS	PM	PL	PSL	PML	PLL
PM	NS	Z	PS	PM	PL	PSL	PML
PS	NM	NS	Z	PS	PM	PL	PSL
Z	NL	NM	NS	Z	PS	PM	PL
NS	NSL	NL	NM	NS	Z	PS	PM
NM	NML	NSL	NL	NM	NS	Z	PS
NL	NLL	NML	NSL	NL	NM	NS	Z

Fig. 5. Rule base for linear control surface

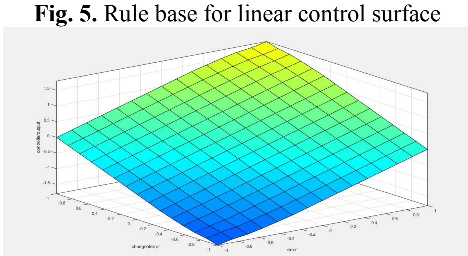

Fig. 6. Linear decision surface

5. Investigation of the Effects of FO-FL-PID and FL-PID on Nonlinearity via Simulations

Integer order classical controllers are known to be ineffective while controlling nonlinear system and linear systems with large dead-time. In this respect, fuzzy logic and/or fractional order controllers are more effective. Our aim is to compare performances induced by the nonlinearity caused by fractional operators and fuzzy rule bases of FO-FL-PID and FL-PID controllers. For this reason, second order linear system with time delay is chosen as a benchmark system since many high order systems may be represented by this model [22]. Moreover, a nonlinear benchmark system model is also selected to make the same above stated performance comparisons. The parameters of the controllers are optimized using Big-Bang-Big-Crunch (BBBC) optimization algorithm according to Integral of Time multiplied Squared Error (ITSE) performance index. The values of different performance indices (%OS, Tr, Ts) are also provided for comparison purposes in different tables for each process separately.

5.1. Nonlinear Control Surface Case

Firstly, linear second order systems with time delay is chosen as follows:

$$G_1(s) = \frac{1}{s^2+as+b} e^{-Ls} \quad (8)$$

FL-PID and FO-FL-PID control structures are applied to above system for various a, b and L values. Fig. 7 and Fig. 8 shows the output and control signals for only a=1, b=1, L=2 s, L=3s.

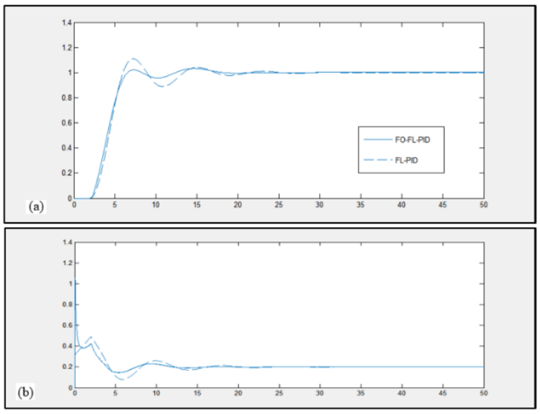

Fig. 7. a) Output signal for LSOS-TD with L=2 b) Control signal for LSOS with L=2

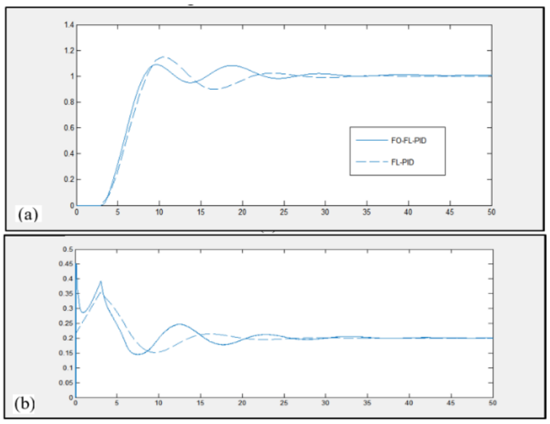

Fig. 8. a) Output signal for LSOS with L=3 b) Control signal for LSOS with L=3

Table 1: Performance analysis for nonlinear control surface with LSOS system

Dead time	FLC	ITSE	%OS	Ts (%5)	Tr
2s	FO-FL-PID	6.5452	%2.2	6.01s	5.671s
	FL-PID	7.6255	%11.1	12.57s	5.54s
3s	FO-FL-PID	13.6769	%8.9	20.74s	7.48s
	FL-PID	15.4436	%14.8	19.35s	8.11s

Table 1 compares the controller performances for different delay times.

As a second benchmark system the following nonlinear system [23] is taken into consideration to show whether the same effects on the performance will be seen or not.

$$\frac{d^2y}{dt^2} + \frac{dy}{dt} + 0.25y^2 = u(t-L) \quad (9)$$

Table 2 compares all the controller performances for different delay times and Fig. 9 and Fig. 10 shows the output and control signals for L=0.8 and L=1.2 delay times.

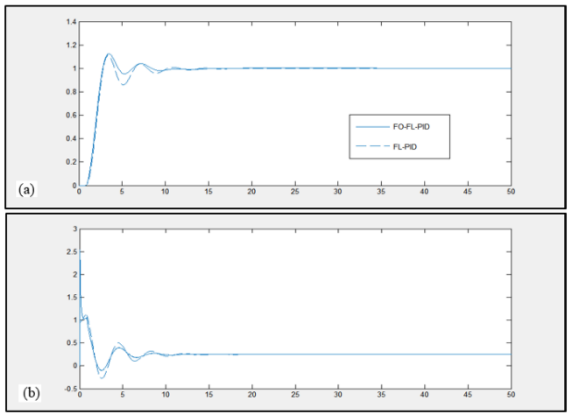

Fig. 9. a) Output signal for nonlinear system with L=0.8 b) Control signal for nonlinear system with L=0.8

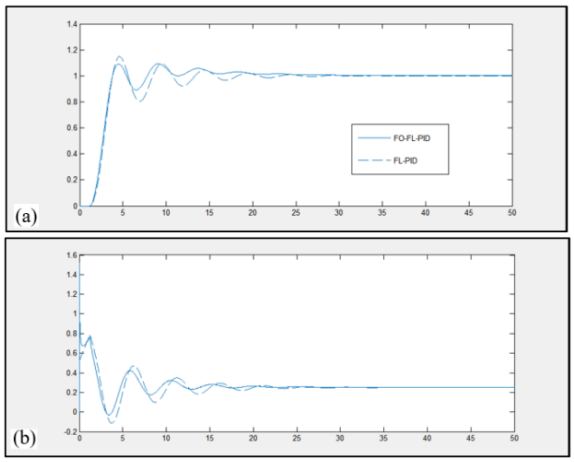

Fig. 10. a) Output signal for nonlinear system with L=1.2 b) Control signal for nonlinear system with L=1.2

Table 2: Performance analysis for nonlinear control surface with nonlinear system

Dead time	FLC	ITSE	%OS	Ts (%5)	Tr
0.8s	FO-FL-PID	1.5269	%12.2	4.214s	2.554s
	FL-PID	1.6097	%11.1	6.08s	2.61s
1.2s	FO-FL-PID	3.2921	%9.2	14.3s	3.51s
	FL-PID	3.7835	%14.8	12.69s	3.57s

5.2. Linear Control Surface Case

In this case, linear control surface is implemented to FL-PID and FO-FL-PID structures. Table 3 compares all the controller performances for different delay times and Fig.11 and Fig.12 shows the output and control signals for only a=1, b=1, L=2 and L=3 in (8).

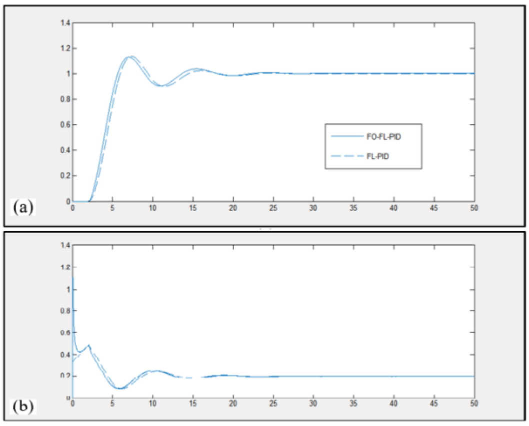

Fig. 11. a) Output signal for LSOS-TD with L=2 b) Control signal for LSOS-TD with L=2

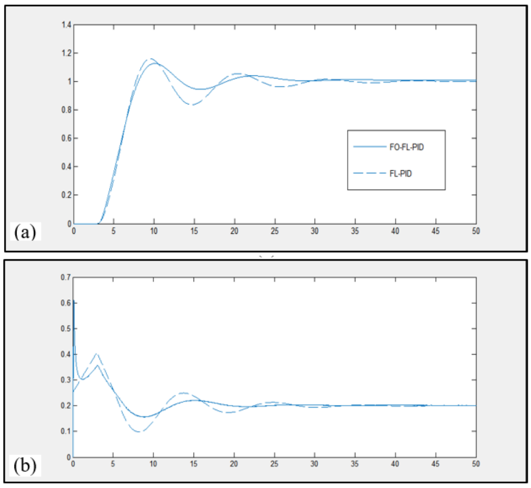

Fig. 12. a) Output signal for LSOS with L=3 b) Control signal for LSOS with L=3

Table 3: Performance analysis for linear control surface with LSOS-TD

Dead time	FLC	ITSE	%OS	Ts (%5)	Tr
2s	FO-FL-PID	6.6337	%12.8	12.66s	5.21s
	FL-PID	7.6023	%13.6	13.24s	5.53s
3s	FO-FL-PID	13.1927	%12.6	16.76s	7.423s
	FL-PID	15.0373	%15.8	21.14s	7.26s

Table 4 compares all the controller performances for different delay times and Fig 13 and Fig. 14 shows the output and control signals for L=0.8 and L=1.2 delay times in (9).

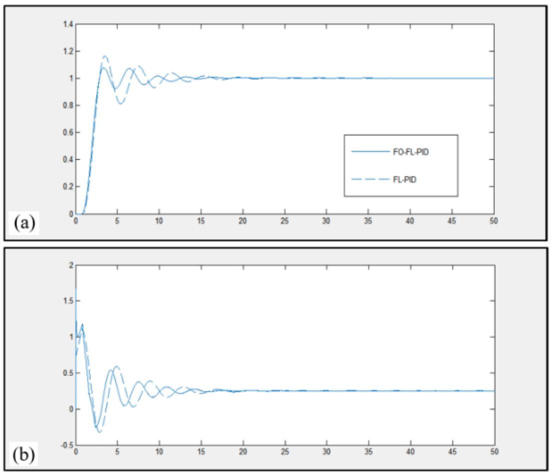

Fig. 13 a) Output signal for nonlinear system with L=0.8 b) Control signal for nonlinear system with L=0.8

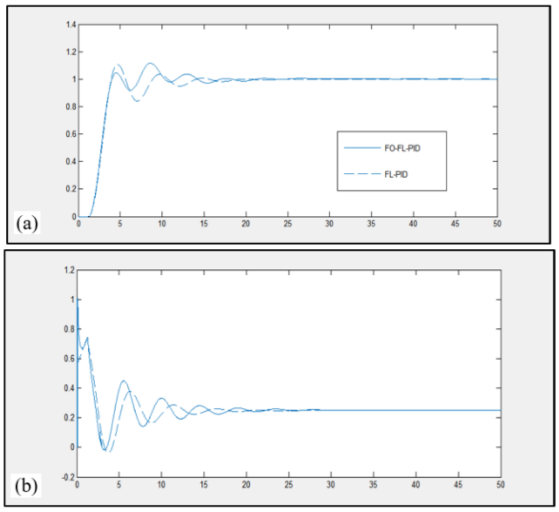

Fig. 14. a) Output signal for nonlinear system with L=1.2 b) Control signal for nonlinear system with L=1.2

Table 4: Performance analysis for linear control surface with nonlinear system

Dead time	FLC	ITSE	%OS	Ts (%5)	Tr
0.8s	FO-FL-PID	1.5674	%7.8	6.93s	2.627s
	FL-PID	2.0484	%16.3	9.98s	2.67s
1.2s	FO-FL-PID	2.7907	%11.7	9.703s	3.636s
	FL-PID	3.3962	%10.9	8.4s	3.62s

5.3. Evaluation of the performance results of nonlinearity effects

When we investigate the results given in the Tables 1- 4, the first conclusion that could be made is that FO-FL-PID controller performs better on both nonlinear and linear surfaces in all benchmark models. This not a surprising result since FO-FL-PID owns two more design parameters which produces a flexibility. We can also investigate the effects of the linear and nonlinear decision surfaces on FO-FL-PID and FL-PID controllers. For this reason, we provide the ITSE values of the decision surfaces for the linear (a=1, b=1) and nonlinear benchmark systems with delay times in Table 5.

Table 5: ITSE values for nonlinear and linear control surfaces

	Nonlinear Control Surface		**Linear Control Surface**	
Systems	FO-FL-PID	FL-PID	FO-FL-PID	FL-PID
LSOS with L=2	6.5452	7.6255	6.6337	7.6023
LSOS with L=3	13.6769	15.4436	13.1927	15.0373
Nonlinear L=0.8	1.5269	1.6097	1.5674	2.0484
Nonlinear L=1.2	3.2921	3.7835	2.7907	3.3962

From all these analysis, we conclude that FO-FL-PID and FL-PID controllers with linear decision surface generally gives better performance when compared to the controllers with nonlinear decision surface.

6. Conclusion

In this study, we investigate the performances induced by the nonlinearities caused by fractional operators and fuzzy rule bases of FO-FL-PID and FL-PID controllers. In that respect, one linear and the other one nonlinear rule bases are taken into consideration. To be fair on comparison, the parameters of the controllers are all optimized using BBBC optimization algorithm according ITSE performance index. This investigation is done on second order linear system with time delay and a nonlinear benchmark system. FO-FL-PID controller performs much better than FL-PID controllers both on nonlinear and linear surfaces since it owns two more design parameters. Linear decision surface in FO-FL-PID and FL-PID controllers generally gives better performance than nonlinear decision surface. Therefore, it can be concluded that nonlinearity of fractional operators are more effective than the nonlinearity produced by fuzzy rule base.

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