Pattern Synthesis of Linear Antenna Array Using Hybrid Particle Swarm-Taguchi Optimization

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Abstract

In this paper, Hybrid Particle Swarm-Taguchi Optimization (HPSTO) is developed and pattern synthesis of a linear antenna array based on HPSTO is presented. HPSTO uses Taguchi Algorithm (TA) and Particle Swarm Optimization (PSO) in order to obtain desired side lobe level and half power beamwidth. The results obtained by TA, PSO and HPSTO are compared. The results show that HPSTO is much better than TA and PSO. Results of HPSTO are also compared with the results obtained by High Frequency Structural Simulator (HFSS).

1. Introduction

Global optimization methods like Taguchi Algorithm and Particle Swarm optimization have been used for pattern synthesis of antenna arrays [1]-[4].

In this work, Hybrid Particle Swarm-Taguchi Optimization(HPSTO) algorithm is developed to synthesize the pattern of linear antenna array with desired side lobe level (*SLL*) and half power beam width (*HPBW*). The array excitation amplitudes are optimized to minimize *SLL* and adjust *HPBW*. HPSTO is realized by applying Taguchi Algorithm(TA) and Particle Swarm Optimization(PSO) on certain members of population.

PSO is a well-known evolutionary optimization technique introduced by Kennedy and Eberhart [5] inspired by social behavior of bird flocking or fish schooling. PSO is used in many research fields. PSO is easy to implement and has few parameters to adjust. Besides being used in many applications of electronics, PSO has been successfully applied to synthesis of the antenna arrays.

Taguchi method is proposed by Genichi Taguchi [6]. Although Taguchi method is not extensively used in electromagnetics, it has been used in various disciplines like finance, chemistry, mechanics, and electronics. To obtain optimum results in only a few step, the orthogonal arrays and signal-to-noise ratio are employed in TA.

In this study, a linear antenna array has been synthesized. Array antenna elements are identical rectangular microstrip antennas because of low cost, low profile and easy to fabricate. The desired side lobe level and half power beam width are selected as aim of optimization. The amplitudes of excitation of antenna elements have been optimized. Linear antenna array has also been simulated by HFSS [7] using magnitudes of excitation currents of array elements which are obtained from HPSTO that giving the best results. The results of HFSS simulation and results of HPSTO have been compared. Optimization results of TA, PSO and HPSTO are also compared with respect to convergence rate and the error.

2. Hybrid Particle Swarm-Taguchi Optimization Method

A hybrid optimization algorithm based on Genetic Algorithm (GA) and PSO which is called HIGAPSO is proposed by Wen Tao Li and others [8]. In HIGAPSO, a different optimization scheme is used. Wen Tao Li and others are adapted the grafting process of plants to the pattern synthesis of conformal antenna arrays. HIGAPSO makes difference by dividing the population to three parts and using the offspring of one method as the parents of the other one.

In this work, HPSTO is inspired by HIGAPSO. Instead of Genetic algorithm of HIGAPSO, Taguchi method is used in HPSTO with PSO. In this paper, HPSTO based on PSO and TA is developed for the pattern synthesis of the linear antenna array.

PSO algorithm starts with creation of the initial *swarm* (population). After the generation of initial swarm, each particle searches the optimum point for the problem. The members of population is called *particle* in PSO. The particles move for searching with a *velocity*. Each particle stores its position and best result of it. The particles share the results which are obtained by moving [9]. So the *global best* result is determined from within these *local best*s.

Each particle has a *cost* value which is obtained by *objective function*. In the every iteration, the particles move a new position. The movement is determined by *velocity* of the particle. The position of fittest particle is used to update velocity of particles. The particle velocities are adjusted by using [5]

$$\begin{aligned} v(n+1) = wv(n) + c_1 r_1[\hat{x}(n) - x(n)] \\ + c_2 r_2[g(n) - x(n)] \end{aligned} \quad (1)$$

where v is the velocity of particle, w, c_1 and c_2 are *learning factors*, r_1 and r_2 are random numbers, $\hat{x}(n)$ is best position for particle and g(n) is best solution for swarm.

Taguchi method is a statistical procedure which is developed for improvement of quality in industry. The purpose of the TA is to find relation between the factors (or variables) wanted to search with least number of experiments or trials. In order to achieve this, Taguchi method uses *orthogonal arrays* (*OA*) and *signal-to-noise ratio* (*SNR*). The optimum results are obtained by Taguchi method with a few trial.

*Orthogonal array*s are substantial tool for TA. The control parameters are selected by orthogonal array. An orthogonal array can be shown as *OA(N,k,s,t)* [1], where N is number of experimental runs (rows), k is number of variables (columns), s is number of levels for each factor and t is strength.

Number of variables (columns) is selected according to the number of parameters to be optimized. Two level orthogonal arrays are used in this study. An example of OA(12,11,2,2) is presented in Table 1 [10].

In Taguchi method signal-to-noise ratio is used as a control factor. Taguchi method maximizes *SNR*s by running experiments using orthogonal arrays[1]. The *SNR* values of the experiments are calculated by using;

$$\eta = -20 \log(\text{Fitness}) \qquad (2)$$

After generation of first population according to number of particles and variables, two members (levels) are selected randomly from population. A new population is created by using the selected two members and the OA. After the creation of new population, *SNR* values of all members are computed. Then the total effects of the variables are calculated by using orthogonal array and *SNR* values of members [11]. Total effects are found by sum of *SNR* values(η) for variable v at level l.

Table 1. OA(12,11,2,2)

Var. / Exp.	1	2	3	4	5	6	7	8	9	10	11
1	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	1	1	1	1	1	1
3	0	0	1	1	1	0	0	0	1	1	1
4	0	1	0	1	1	0	1	1	0	0	1
5	0	1	1	0	1	1	0	1	0	1	0
6	0	1	1	1	0	1	1	0	1	0	0
7	1	0	1	1	0	0	1	1	0	1	0
8	1	0	1	0	1	1	1	0	0	0	1
9	1	0	0	1	1	1	0	1	1	0	0
10	1	1	1	0	0	0	0	1	1	0	1
11	1	1	0	1	0	1	0	0	0	1	1
12	1	1	0	0	1	0	1	0	1	1	0

This process is used for all variables and the optimum offspring particle is generated by comparing the effects of levels [12]. The variable which has the largest *SNR* gives the optimal level [11]. The optimum particles are generated as expected number.

HPSTO combines TA and PSO. The initial population is separated into three parts.

The fittest members of the population (the superiors) are transferred to the next generation directly.

The members that have average fitness value (the averages), are applied to the TA and generate their offspring. TA stops when the expected number of optimal members are generated. The expected number is $\frac{1}{2}Qp_c$ [11] where Q is the number of population members and p_c is the probability of cross-over (mating). The offspring of TA is also used in PSO.

The worst fitness valued members (the worst) and TA improved members are together form the population of PSO. The population worst members and TA improved members are applied to PSO and produce the new members.

Then, offspring of the superiors, the averages and the worst members are reunited. The reunited population is initial population of the next iteration. The algorithm is continued until the termination criteria is met. In this work the termination criteria is selected as desired *SLL* and *HPBW*. Flow chart of HPSTO is given in Fig. 1.

In HPSTO, average members are improved by TA because of the convergence speed and robustness of TA.

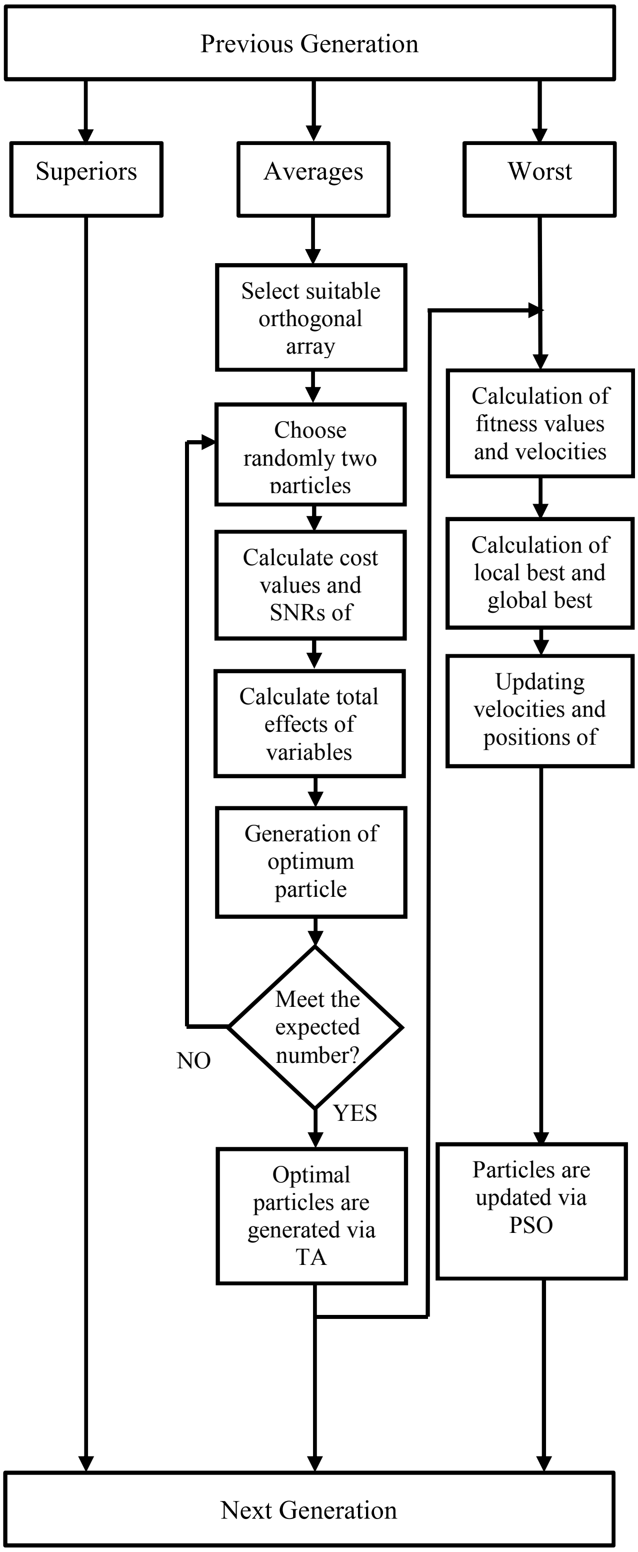

Fig. 1. Flow chart of HPSTO

3. Linear Antenna Array

The shape of the linear antenna array with N elements located at $\{x_1, x_2, \ldots x_N\}$ is shown in Figure 2.

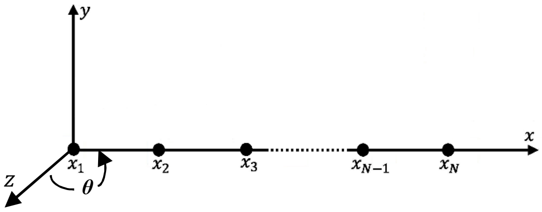

Fig. 2. N-element linear antenna array

The radiation pattern of N-element linear antenna array can be written as $F(\theta)$ [2,13];

$$F(\theta) = \sum_{n=1}^{N} I_n f_n(\theta) exp[jkx_n cos\theta + \psi_n] \quad (3)$$

$$\psi_n = -kx_n cos\theta_0 \quad (4)$$

where k is the free space wave number ($k=2\pi/\lambda$), λ is the wavelength, θ is the angle measured with respect to the x-axis, θ_0 is steering angle, I_n and ψ_n are the excitation amplitude and phase of the n^{th} antenna respectively, $f_n(\theta)$ is the radiation pattern of individual antenna array element, x_n is the distance between the first and the n^{th} antennas measured in wavelength λ. ($x_1 = 0$). The cost function is calculated as in [14];

$$cost = w_1 \times (SLL_{max} - SLL_{comp}) + w_2 \times (HPBW_{max} - HPBW_{comp}) \quad (5)$$

where SLL_{max} is the desired maximum side lobe level and SLL_{comp} is the computed side lobe level. $HPBW_{max}$ is the desired maximum half power beam width and $HPBW_{comp}$ is the computed desired maximum half power beam width. w_1 and w_2 are the weight coefficient.

The algorithms are applied to N=11 elements linear antenna array. Identical rectangular microstrip patch antenna elements are used in the array [15]. The element spacing is half-wavelength ($\lambda/2$) on x-axis. HFSS drawing of the linear antenna array is presented in Fig. 3.

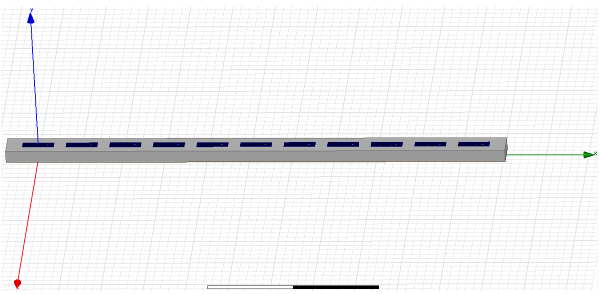

Fig. 3. HFSS drawing of the linear array

4. Examples

The linear antenna array have 11 antenna elements (Fig. 3). OA(12,11,2,2) is used to optimize amplitudes of these 11 excitation of antenna elements. N=11 linear antenna array is given in Fig 2.

Linear antenna array factor pattern is synthesized using different global optimization algorithms. The aim of the optimization is to obtain the maximum *HPBW* 10 degrees and the maximum *SLL* -20 dB.

Chosen value of number of particles(Q) is 100, number of variables(N) is 11, probability of cross-over(p_c) is 0.75, maximum number of generations is 100. The weights, w_1 and w_2 in (5) are selected as 1. The excitation amplitudes of antennas are optimized in the range {0, 0.5}. The cognitive parameters of PSO, c_1 and c_2 are chosen as 1 and 2 respectively. Number of the fittest members of the population(superiors) which will be transferred to the next generation is selected as 4. Half of the remaining particles are applied to TA and the other half to PSO.

Radiation pattern of the array is obtained by (3) from the optimization of (5) and shown in Fig. 4. The desired *SLL* and *HPBW* have been obtained by TA and HPSTO. The phase of each element is equal to zero for (θ_0=90˚).

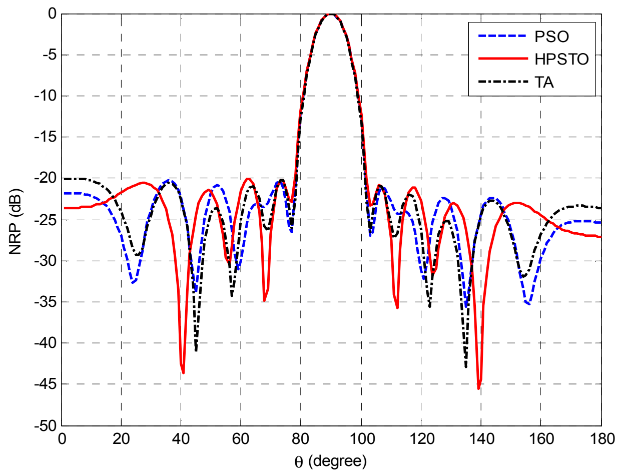

Fig. 4. Normalized radiation pattern of the linear antenna array scan direction (θ_0=90˚)

Cost function values for scan direction (θ_0=90˚) for TA, PSO and HPSTO are shown in Fig. 5.

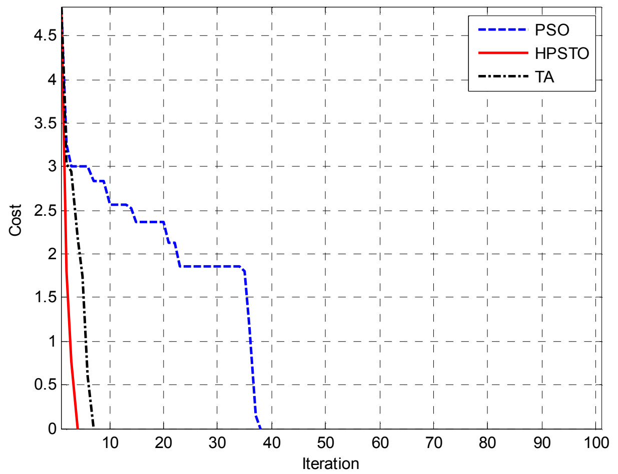

Fig. 5. Convergence of TA, PSO and HPSTO for scan direction (θ_0=90˚)

TA is converged at 7th iteration, HPSTO is converged at 4th iteration and PSO is converged at 38th iteration. The optimization goals are achieved by all of four algorithms successfully for scan direction (θ_0=90˚)

The antenna element excitation amplitudes obtained by TA, PSO and HPSTO scan direction (θ_0=90˚) are given in the Table 2.

Table 2. Optimized antenna excitation amplitudes (I_n) of 1x11 linear antenna array for (θ_0=90˚)

n	TA	PSO	HPSTO
1	0.1997	0.3088	0.2937
2	0.2515	0.3348	0.2677
3	0.3129	0.5000	0.3787
4	0.3407	0.5000	0.4074
5	0.3224	0.5000	0.5000
6	0.3553	0.5000	0.4032
7	0.3599	0.5000	0.3615
8	0.2712	0.5000	0.3096
9	0.2383	0.2900	0.2249
10	0.1290	0.1737	0.2581
11	0.1828	0.2284	0.3793

The result of HFSS simulation is compared with the result obtained by HPSTO for scan direction (θ_0=90˚) in Fig. 6. HPSTO and HFSS give similar results.

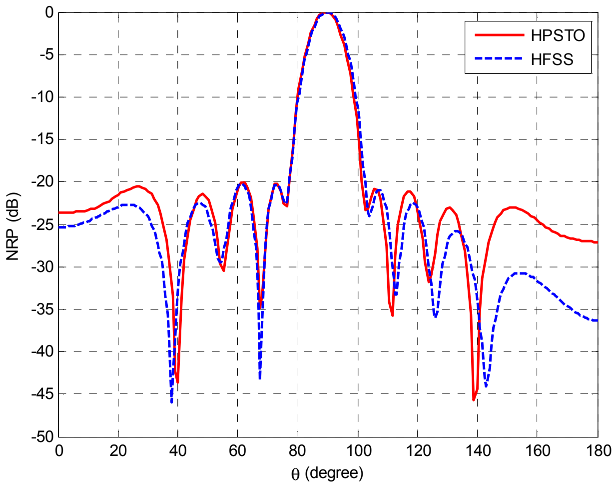

Fig. 6. Comparison of HFSS simulation and result of HPSTO for (θ_0=90˚)

Optimized radiation pattern for scan direction (θ_0=60˚) is given in Fig. 7. The desired side lobe level and *HPBW* has been obtained by TA and HPSTO. The phase of each element is calculated by (4) for steering the main lobe to the (θ_0=60˚).

Cost values for scan direction (θ_0=60˚) obtained by TA, PSO and HPSTO are presented in Fig. 8. TA is converged at 6th iteration, HPSTO is converged at 4th iteration But, PSO could not have converged and has reached the maximum number of iterations with maximum value of error. HPSTO is converged faster than other algorithms.

The antenna array element excitation amplitudes obtained by TA, PSO and HPSTO for scan direction (θ_0=60˚) are shown in the Table 3.

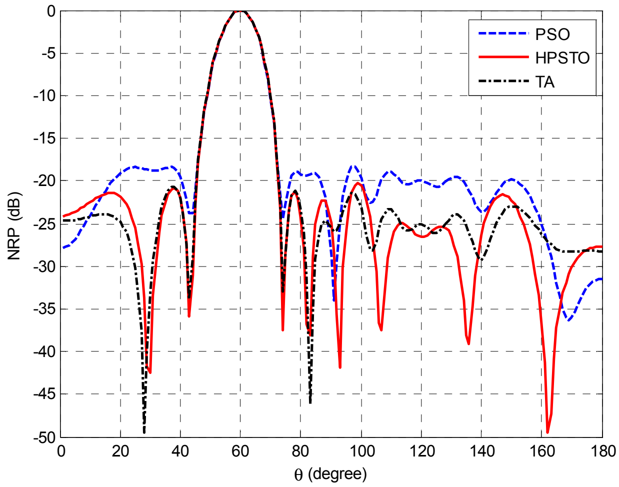

Fig. 7. Normalized radiation pattern of the linear antenna array for scan direction (θ_0=60˚)

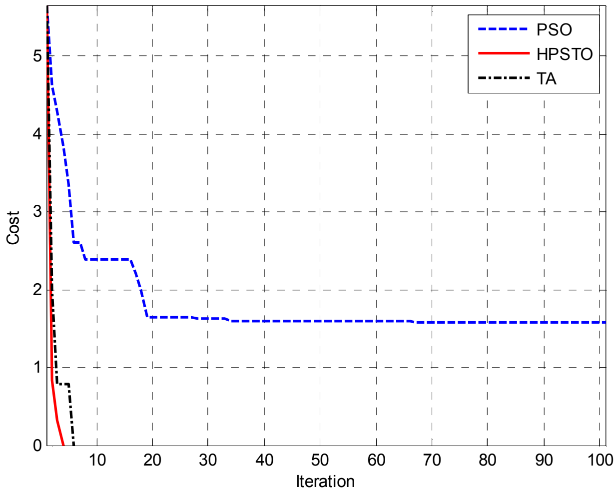

Fig. 8. Convergence of TA, PSO and HPSTO for scan direction (θ_0=60˚)

Table 3. Optimized antenna excitation amplitudes (I_n) of 1x11 linear antenna array for (θ_0=60˚)

n	TA	PSO	HPSTO
1	0.1410	0.2547	0.2132
2	0.2636	0.5000	0.3400
3	0.3517	0.5000	0.3101
4	0.4687	0.5000	0.4781
5	0.4735	0.5000	0.4825
6	0.4720	0.5000	0.5000
7	0.4993	0.5000	0.4614
8	0.4702	0.5000	0.4743
9	0.3580	0.4179	0.2667
10	0.2351	0.0000	0.3552
11	0.3473	0.3468	0.1945

The result of HFSS simulation is compared with the result obtained by HPSTO for scan direction (θ_0=60˚) in Fig. 9. HPSTO and HFSS give similar results.

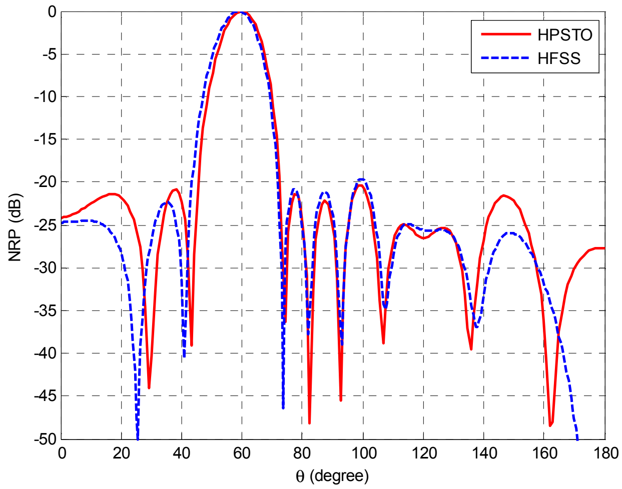

Fig 9. Comparison of HFSS simulation and result of HPSTO for (θ_0=60˚)

HPSTO has been converged faster than TA and PSO for both examples. PSO has reached maximum number of generations with maximum value of error.

5. Conclusion

In this work, HPSTO is developed and applied to the pattern synthesis of a linear antenna array. Linear antenna array is optimized by using TA, PSO and HPSTO. The optimization goals are selected as -20 dB side lobe level and 10 degrees half power beamwidth for scan directions of (θ_0=90˚) and (θ_0=60˚)

Results show that desired side lobe level and half power beam widths are successfully obtained. The results obtained by TA, PSO and HPSTO are compared. The best results are obtained by HPSTO.

The radiation pattern of the linear antenna array is simulated by HFSS using the best excitation amplitude values obtained by HPSTO. The similar results are obtained.

6. References

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