Extending SCAN and SHAPE Algorithms for Designing MIMO Radar Sequence Sets

Ozan Kozakçıoğlu[1] and Olcay Akay[2]

Dokuz Eylül University, Dept. of Electrical and Electronics Eng.,
Tınaztepe Campus, 35390, Buca / İZMİR TURKEY
[1]ozan.kozakcioglu@ogr.deu.edu.tr [2]olcay.akay@deu.edu.tr

Abstract

In radar systems, interference between the transmit signal and communication signals possibly existing in the same spectrum band is a serious concern. As a possible solution to this problem, cognitive radars having transmit signals whose spectra contain notches are proposed. Two algorithms named SCAN and SHAPE can be used for designing such radar transmit signals with desirable spectral characteristics. The SCAN algorithm can also reduce sidelobes of the temporal autocorrelations of the designed sequences via some additional constraints. In this manuscript, we introduce generalizations of both SCAN and SHAPE algorithms for multiple-input multiple-output (MIMO) radar systems. SCAN and SHAPE are both iterative algorithms employing fast Fourier transform (FFT). Hence, they allow design of long sequences in an efficient manner. We also provide numerical simulation examples of MIMO SCAN and MIMO SHAPE algorithms comparing their performances against each other.

1. Introduction

Recently, cognitive radar has drawn the attention of many researchers [1]. Put simply, cognitive radar can adapt itself to the changing dynamics of the environment. It can be employed when there is a possibility for the radar transmit signal to interfere with signals emenating from other sources such as local communication systems, navigation systems, or military communication systems whose operating frequency band might overlap with the spectral band of the radar transmit signal. This overlap could be prevented to a certain degree by designing radar transmit signals with spectral notches (stopbands) over the interfering frequency bands. There exist various works proposing design of such radar transmit signals with desirable spectral characteristics [2], [3], [4].

In this manuscript, we first review two existing techniques that are named as SCAN (stopband cyclic algorithm-new) [5] and SHAPE [6] algorithms which have been proposed for designing radar transmit signals with some spectral domain constraints. Differing from the SHAPE algorithm, the SCAN algorithm also aims at reducing sidelobe levels of the temporal autocorrelations of the designed sequences via some additional constraints. Then, we utilize SCAN and SHAPE for designing transmit signals of multiple-input multiple-output (MIMO) radar systems and present some simulation examples.

Notation: Bold lowercase letters denote vectors while bold uppercase letters denote matrices. $[\cdot]^H$ and $[\cdot]^T$ represent Hermitian and transpose operations, respectively, and $||\cdot||$ denotes the Euclidean norm for vectors and matrices. $(.)^*$ is reserved for denoting conjugate of complex numbers and the phase of a complex number is represented by $\arg\{\cdot\}$. $\mathbf{X}[n,m]$ shows the $(n,m)^{th}$ element of the matrix $\mathbf{X}$. The normalized cyclic frequency values ranging from 0 to 1 Hz. have been used throughout the manuscript.

2. SCAN and SHAPE Algorithms

In this section, we briefly review the SCAN and SHAPE algorithms for single-input single-output (SISO) systems.

2.1. SCAN Algorithm

The SCAN algorithm was proposed for designing unimodular (i.e. having a constant modulus of unity) sequences by applying constraints both in temporal and spectral domains [5]. SCAN can be computed by utilizing the fast Fourier transform (FFT), and hence, is computationally quite efficient. Another advantage of the SCAN algorithm is that it can be initialized by random phased unimodular sequences of large lengths. Every realization of the algorithm with random initialization produces a different new sequence with similar good properties.

Let us assume that the spectrum of the complex valued length-N radar transmit sequence, $x[n]$, for $n = 1, 2, \ldots, N$, to be designed has notches in the following set of normalized frequency bands

$$\Omega = \bigcup_{s=1}^{N_s} (f_{s1}, f_{s2}) \quad (1)$$

where (f_{s1}, f_{s2}) corresponds to the s^{th} stopband and N_s represents the total number of stopbands. The number of samples, $\tilde{N}$, for calculating the discrete Fourier transform (DFT) is chosen large enough for densely covering Ω. Here, the $(n,m)^{\text{th}}$ element of $\tilde{N} \times \tilde{N}$ DFT matrix $\mathbf{F}_{\tilde{N}}$ is given as

$$\mathbf{F}_{\tilde{N}}[n,m] = \frac{1}{\sqrt{\tilde{N}}} \mathrm{e}^{j2\pi \frac{nm}{\tilde{N}}}; \quad n,m = 0, \ldots, \tilde{N}-1. \quad (2)$$

A matrix $\mathbf{S}$ is created by including the columns of $\mathbf{F}_{\tilde{N}}$ corresponding only to the normalized frequencies within Ω. Another matrix $\mathbf{G}$ is formed by the remaining columns of $\mathbf{F}_{\tilde{N}}$. Then, suppression of spectral power of $x[n]$ within Ω can be realized by solving the following minimization problem [5]

$$\begin{aligned} &\min_{\mathbf{x},\boldsymbol{\alpha}} J_1(\mathbf{x},\boldsymbol{\alpha}) = \left\|\tilde{\mathbf{x}} - \mathbf{G}\boldsymbol{\alpha}\right\|^2 \\ &\text{subject to} \quad |x[n]| = 1 \quad n = 1, \ldots, N \end{aligned} \quad (3)$$

where $\tilde{\mathbf{x}} = [x[1] \ \ldots \ x[N] \ 0 \ \ldots \ 0]^T_{\tilde{N}\times 1}$ and $\boldsymbol{\alpha}$ is a vector of auxiliary variables. In addition to spectral suppression, SCAN

can also manage to reduce autocorrelation sidelobes of $x[n]$ by utilizing the CAN (cyclic algorithm-new) algorithm [7]. CAN aims to minimize the performance metric of integrated sidelobe level (ISL) which is defined [7] as

$$\text{ISL} = 2\sum_{k=1}^{N-1} |r_x[k]|^2. \tag{4}$$

In Eqn. (4), $r_x[k]$ denotes the aperiodic autocorrelation of $x[n]$. It is defined [7] as

$$r_x[k] = \sum_{n=k+1}^{N} x[n]x^*[n-k] = r_x^*[-k]; \;\; k = 0, \ldots, N-1. \tag{5}$$

Utilizing $2N \times 2N$ DFT matrix $\mathbf{F}_{2N}$, suppression of autocorrelation sidelobes can be accomplished by solving the following minimization problem [5]

$$\begin{aligned} \min_{\mathbf{x},\mathbf{v}} \; & J_2(\mathbf{x}, \mathbf{v}) = \left\| \mathbf{F}_{2N}^H \begin{bmatrix} \mathbf{x} \\ \mathbf{0}_{N\times 1} \end{bmatrix} - \mathbf{v} \right\|^2 \\ \text{subject to} \;\; & |x[n]| = 1, \quad n = 1, \ldots, N \\ & |v[n]| = \frac{1}{\sqrt{2}}, \quad n = 1, \ldots, 2N \end{aligned} \tag{6}$$

where $\mathbf{x} = [x[1]\; x[2]\; \ldots\; x[N]\,]^T$ is the designed sequence and $\mathbf{v} = [v[1]\; v[2]\; \ldots\; v[2N]\,]^T$ is a constant-valued vector.

The two minimization problems in Eqns. (3) and (6) can be combined so that both spectral stopband and temporal autocorrelation sidelobe constraints are combined in a single minimization problem which can be formulated [5] as

$$\begin{aligned} \min_{\mathbf{x},\boldsymbol{\alpha},\mathbf{v}} \; & J(\mathbf{x}, \boldsymbol{\alpha}, \mathbf{v}) = \lambda \left\| \tilde{\mathbf{x}} - \mathbf{G}\boldsymbol{\alpha} \right\|^2 \\ & + (1-\lambda) \left\| \mathbf{F}_{2N}^H \begin{bmatrix} \mathbf{x} \\ \mathbf{0}_{N\times 1} \end{bmatrix} - \mathbf{v} \right\|^2 \\ \text{subject to} \;\; & |x[n]| = 1, \quad n = 1, \ldots, N \\ & |v[n]| = \frac{1}{\sqrt{2}}, \quad n = 1, \ldots, 2N \end{aligned} \tag{7}$$

where $0 \le \lambda \le 1$ is a weighting factor controlling the relative weight of the two cost functions J_1 and J_2.

2.2. SHAPE Algorithm

Especially for wideband radar applications, designing sequences by shaping their spectrum becomes important. Unlike the SCAN algorithm, the SHAPE algorithm is purely based on spectral constraints [6]. For a wideband radar, a waveform might be required to contain notches in certain predefined spectral bands. The SHAPE algorithm can manage shaping of the spectrum in a computationally efficient manner by employing FFT and with the aid of some predefined upper and lower spectral bounds.

The cost function to be minimized can be expressed as

$$\begin{aligned} \min_{\mathbf{x},\boldsymbol{\theta}} \quad & \left\| \mathbf{F}_N^H \mathbf{x} - \mathbf{y} \odot e^{j\boldsymbol{\theta}} \right\|^2 \\ \text{subject to} \; & |x[n]|^2 = h[n], \text{ for } \; n = 1, \ldots, N \end{aligned} \tag{8}$$

where $\odot$ represents element-wise product operation, $\mathbf{x} \in \mathbb{C}^{N\times 1}$ is the designed sequence, $\mathbf{y} \in \mathbb{R}^{N\times 1}$ is the nonnegative valued desired spectrum magnitude, and $\mathbf{F}_N \in \mathbb{C}^{N\times N}$ represents the unitary DFT matrix. $\boldsymbol{\theta} \in \mathbb{R}^{N\times 1}$ is an auxiliary phase vector. The time-domain envelope constraint is signified by the vector $\mathbf{h} = [h[1]\; h[2]\; \ldots\; h[N]\,]^T$ which could be formed by utilizing common window functions (rectangular, raised cosine, triangular, etc.) [6].

Instead of fitting to an exact spectrum as $\mathbf{y}$ in Eqn. (8), one can allow the amplitude of the spectrum to stay between an upper spectral bound, $u(f)$, and a lower spectral bound, $l(f)$. These bound functions can be approximated as vectors $\mathbf{u} = [u[1]\; u[2]\; \ldots\; u[N]]^T$ and $\mathbf{l} = [l[1]\; l[2]\; \ldots\; l[N]]^T$ sampled on the used frequency grid points. Relaxing on the exact spectrum shape by employing bounds makes the problem easier and more manageable. Thus, one can search for a spectrum, $\mathbf{z}$, with its modulus contained within the upper and lower spectral bounds. The accordingly modified minimization problem becomes [6]

$$\begin{aligned} \min_{\mathbf{x},\beta,\mathbf{z}} \quad & \left\| \mathbf{F}_N^H \mathbf{x} - \beta \mathbf{z} \right\|^2 \\ \text{subject to} \; & |x[n]|^2 = h[n], \text{ for } n = 1, \ldots, N \\ & |z[n]| \le u[n], \text{ for } n = 1, \ldots, N \\ & |z[n]| \le l[n], \text{ for } n = 1, \ldots, N. \end{aligned} \tag{9}$$

In Eqn. (9), β is an auxiliary scale factor introduced to compensate for any likely energy mismatch and phase offset between the designed time domain sequence, $\mathbf{x}$, and the spectrum, $\mathbf{z}$.

3. MIMO SCAN and MIMO SHAPE

In this section, we extend the SCAN and SHAPE algorithms for MIMO systems. In generalizing SISO versions of SCAN and SHAPE algorithms, we have been inspired by [8] where the CAN algorithm was generalized into MIMO systems. By transmitting orthogonal waveforms, MIMO radar systems provide better detection performance, improved parameter estimation, and better resolution [8].

Both MIMO SCAN and MIMO SHAPE algorithms are initialized with a set of sequences which can be represented as columns of the following matrix

$$\mathbf{X}^{(0)} = [\,\mathbf{x}_1 \mid \mathbf{x}_2 \mid \ldots \; \mid \mathbf{x}_M]_{N\times M} \tag{10}$$

where the m^{th} column represents the m^{th} initial sequence $\mathbf{x}_m = [x_m[1]\; x_m[2]\; \ldots\; x_m[N]\,]^T$ and the superscript of the matrix denotes the iteration number. Note that there are M initial sequences of length N. The aperiodic cross-correlation of two sequences $\mathbf{x}_{m_1}$ and $\mathbf{x}_{m_2}$ can defined [8] as

$$r_{m_1,m_2}[k] = \sum_{n=k+1}^{N} x_{m_1}[n] x_{m_2}^*[n-k] = r_{m_1 m_2}^*[-k] \tag{11}$$

where $m_1, m_2 = 1, 2, \ldots, M$ and $n = 1, 2, \ldots, N$. It is desired to have low level cross-correlations between designed transmitted sequences. Good cross-correlation means that any transmitted waveform is approximately uncorrelated with other time-shifted transmitted waveforms.

The cross-energy spectral density (CESD), $P_{m_1,m_2}(f)$, can be defined [9] via the discrete-time Fourier transform (DTFT) of $r_{m_1,m_2}[k]$ as

$$P_{m_1,m_2}(f) = \sum_{k=-\infty}^{\infty} r_{m_1,m_2}[k] e^{-j2\pi f k}. \tag{12}$$

3.1. MIMO SCAN Algorithm

Before outlining the steps of the MIMO SCAN algorithm, the following input parameters are to be determined. The input matrix $\mathbf{X}^{(0)}_{N\times M}$ in Eqn. (10) involving the initial set of sequences, the weighting parameter λ in Eqn. (7), the DFT size of the algorithm $\tilde{N}$, and the set of frequency bands, Ω, to be suppressed are assigned first. Then, the matrix $\mathbf{G}$ is formed as explained prior to Eqn. (3). After those required initializations, the steps of the MIMO SCAN algorithm can be executed as follows:

Step #1: Form the zero-padded matrix, $\tilde{\mathbf{X}}_{\tilde{N}\times M}$, in an analogous manner to $\tilde{\mathbf{x}}$ in Eqn. (3). Then, calculate $\boldsymbol{A} = \mathbf{G}^H\tilde{\mathbf{X}}_{\tilde{N}\times M}$.

Step #2: Form the zero-padded matrix, $\mathbf{X}_{2N\times M}$, and compute $2N \times M$ matrix, $\mathbf{V} = \frac{1}{\sqrt{2}}e^{j\arg\left\{\mathbf{F}^H_{2N}\mathbf{X}_{2N\times M}\right\}}$.

Step #3: Rename the first N rows of $\mathbf{G}\boldsymbol{A}$ and $\mathbf{F}_{2N}\mathbf{V}$ as $\mathbf{C_1}$ and $\mathbf{C_2}$, respectively.

Step #4: Find the resultant matrix at iteration i as $\mathbf{X}^{(i)} = e^{j\arg\{\lambda\mathbf{C_1}+(1-\lambda)\mathbf{C_2}\}}$.

Iteration: Perform Step 1 through Step 4 for a predetermined number of iterations.

3.2. MIMO SHAPE Algorithm

MIMO SHAPE algorithm also requires some initial input parameters. The matrix $\mathbf{X}^{(0)}_{N\times M}$ contains M initial sequences of length N as its columns. The $M \times 1$ vector of auxiliary scale factors is initilized as $\boldsymbol{\beta}^{(0)} = [1\ 1 \ldots\ 1]^T$. The upper, $\mathbf{u} = [u[1]\ u[2]\ \ldots\ u[N]\ ]^T$, and lower, $\mathbf{l} = [l[1]\ l[2]\ \ldots\ l[N]\ ]^T$, bound vectors for spectral suppression and the window vector, $\mathbf{h} = [h[1]\ h[2]\ \ldots\ h[N]\ ]^T$, for forming time-domain envelope are assigned. After determining these initial parameters, the steps of the MIMO SHAPE algorithm are performed as follows.

Step #1: Initialize the temporary matrix $\mathbf{Q}_{N\times M} = \mathbf{F}^H_N\mathbf{X}^{(0)}_{N\times M}$ and divide the m^{th} column of $\mathbf{Q}_{N\times M}$ by the corresponding scalar value, β_m, which is the m^{th} element of the vector $\boldsymbol{\beta}^{(0)}$.

Step #2: In parallel to the pseudocode of the SISO SHAPE algorithm given in [6], perform comparisons of the elements in each and every column of the temporary matrix $\mathbf{Q}_{N\times M}$ with the corresponding elements of the upper, $\mathbf{u}$, and lower, $\mathbf{l}$, spectral bound vectors. Via execution of these comparisons, determine the elements of the auxiliary matrix $\mathbf{Z}^{(i)}$ at the i^{th} iteration. In these comparisons; if $\mathbf{Q}[n,m] > u[n]$, then assign $\mathbf{Z}^{(i)}[n,m] = u[n]\frac{\mathbf{Q}[n,m]}{|\mathbf{Q}[n,m]|}$; if $\mathbf{Q}[n,m] < l[n]$, then assign $\mathbf{Z}^{(i)}[n,m] = l[n]\frac{\mathbf{Q}[n,m]}{|\mathbf{Q}[n,m]|}$. Otherwise, assign $\mathbf{Z}^{(i)}[n,m] = \mathbf{Q}[n,m]$.

Step #3: Calculate the m^{th} element of the vector $\boldsymbol{\beta}^{(i)}$ at the i^{th} iteration using the m^{th} columns of both $\mathbf{X}^{(i-1)}$ and $\mathbf{Z}^{(i)}$, as $\beta_m = \frac{\mathbf{Z}^H[n,m]\mathbf{F}^H_N\mathbf{X}_{N\times M}[n,m]}{\|\mathbf{Z}[n,m]\|^2}$

Step #4: Compute the m^{th} column of the matrix $\mathbf{V}^{(i)}$ by multiplying the m^{th} column of $\mathbf{F}_N\mathbf{Z}^{(i)}$ by the corresponding m^{th} element of the vector $\boldsymbol{\beta}^{(i)}$.

Step #5: In parallel to the pseudocode of the SISO SHAPE algorithm given in [6], calculate the elements of the m^{th} column of $\mathbf{X}^{(i)}$ using preassigned window vector $\mathbf{h}$ and the corresponding m^{th} column of $\mathbf{V}^{(i)}$. If $\mathbf{V}[n,m] \neq h[n]$, then assign $\mathbf{X}^{(i)}[n,m] = \sqrt{h[n]}\frac{\mathbf{V}[n,m]}{|\mathbf{V}[n,m]|}$. Otherwise, assign $\mathbf{X}^{(i)}[n,m] = \mathbf{V}[n,m]$.

Step #6: Calculate $\mathbf{Q}_{N\times M} = \mathbf{F}^H_N\mathbf{X}^{(i)}_{N\times M}$ and divide the m^{th} column of $\mathbf{F}^H_N\mathbf{X}^{(i)}_{N\times M}$ by the corresponding scalar value, β_m, where β_m is the m^{th} element of the vector $\boldsymbol{\beta}^{(i)}$.

Iteration: Perform Step 2 through Step 6 for a predetermined number of iterations.

4. Simulation Examples

In the following, we present simulation examples of the MIMO SCAN and MIMO SHAPE algorithms developed above. Through the examples, we compare performances of the two algorithms against each other. In all of the examples, both algorithms are initialized with uniformly distributed random phased unimodular sequences of length $N = 100$.

4.1. Simulation Example for MIMO SCAN

The length of the sequences to be designed and the number of designed sequences are taken as $N = 100$ and $M = 2$, respectively. Thus, when finished, the algorithm produces two unimodular sequences as columns of a 100×2 matrix. The weighting factor λ introduced in Eqn. (7) determines the preference between the temporal and spectral constraints and is chosen as $\lambda = 0.8$ favouring spectral shaping more than lowering correlation sidelobes. Placement of a spectral notch in the normalized frequency band, $\Omega = [0.65, 0.8)$, is required for both designed sequences. The FFT size is taken as $\tilde{N} = 1000$ and the number of iterations is fixed as 2×10^5. The resultant designed sequences are shown in Fig. 1. It is interesting to observe that similar to the individual spectra in Figs. 1a and 1b, the CESD of the designed sequences in Fig. 1c also contains a spectral notch in the required stopband. Moreover, cross-correlation of the designed sequences displayed in Fig. 1d remains low obeying almost zero cross-correlation requirement for nearly orthogonal sequences. Finally, sidelobes of the autocorrelations of both designed sequences are greatly suppressed as indicated in Figs. 1e and 1f.

4.2. Simulation Example for MIMO SHAPE

Again, we take the sequence length as $N = 100$ and the number of designed sequences as $M = 2$. Except for the notch, the spectral upper bound, $\mathbf{u}$, is merely applied to force the spectrum below 0 dB across the whole frequency range. Over the stopband, $\Omega = [0.65, 0.8)$, the designed spectrum is forced to be under -40 dB. No spectral lower bound, $\mathbf{l}$, is applied in the design. The FFT size is taken to be $\tilde{N} = 1000$ and the number of iterations is fixed as 2×10^5. The resultant designed sequences can be seen in Fig. 2 where the employed spectral upper bound is shown using a green line. As can be seen in Figs. 2a and 2b, the spectra of both designed sequences contain the desired notch of -40 dB as specified by the employed spectral upper bound. It is interesting to see from Fig. 2c that the CESD of the designed sequences contains a deeper notch of -60 dB although it does not obey the spectral upper bound at other frequency values. Finally, cross-correlation sidelobes of the designed sequences remain quite low as seen in Fig. 2d providing near orthogonality of the designed sequences. Furthermore, sidelobes of the autocorrelations of both designed sequences are greatly suppressed as indicated in Figs. 2e and 2f.

4.3. Performance Investigation

We can mesaure the performance of MIMO SCAN and MIMO SHAPE algarithms by comparing certain performance metrics of the initial and final designed sequences. Integrated

Sidelobe Level (ISL) defined in Eqn. (4) and Merit Factor (MF) which is defined below are two of these metrics. We can also measure the stopband level of the designed spectra. Table 1 displays the ISL values of the initial and final designed sequences in the above subsection by using MIMO SCAN and MIMO SHAPE algorithms. One should keep in mind that it is desirable to obtain ISL values as low as possible. Notice that ISL values of the sequences designed by MIMO SHAPE algorithm is worse than the initial sequences. This is because the SHAPE algorithm constrains the spectrum of the designed sequences but not their autocorrelations. On the other hand, looking at Eqn. (7) one can see that when $\lambda < 0.5$ the MIMO SCAN algorithm constrains the correlations of the designed sequences more than their spectra. This is apparent in the third row of Table 1 ($\lambda = 0.2$) where the ISL values are lower than that of initial sequences. When $\lambda = 0.8$, on the contrary, spectra of the designed sequences are constrained more. Hence, the ISL values in the fourth row are worse, as expected.

Table 1. ISL performance of MIMO algorithms

	1^{st} Sequence	2^{nd} Sequence
Initial	8.538e+03	8.834e+03
MIMO SCAN ($\lambda = 0.2$)	1.0186e+03	759.7848
MIMO SCAN ($\lambda = 0.8$)	2.6824e+03	2.5194e+03
MIMO SHAPE	1.2554e+04	1.3386e+04

The MF can be defined [7], [8] as

$$\mathrm{MF} = \frac{|r(0)|^2}{\sum_{\substack{k=-(N-1)\\ k\neq 0}}^{N-1} |r(k)|^2}. \tag{13}$$

Similar to ISL, the metric of MF also measures suppression of autocorrelation sidelobes, although, contrary to ISL, MF is desired to be as large as possible. Table 2 represents MF values of the initial and final designed sequences via the MIMO SCAN and MIMO SHAPE algorithms. Similar to the results in Table 1, MF values of the designed sequences via MIMO SHAPE are worse than that of initial sequences. Again, the MF values of MIMO SCAN with $\lambda = 0.2$ are better compared to the values obtained when $\lambda = 0.8$.

Table 2. MF performance of MIMO algorithms

	1^{st} Sequence	2^{nd} Sequence
Initial	1.1712	1.1319
MIMO SCAN ($\lambda = 0.2$)	9.8172	13.1616
MIMO SCAN ($\lambda = 0.8$)	3.7280	3.9692
MIMO SHAPE	0.7966	0.7470

By virtue of their definitions, the ISL and MF performance metrics are concerned with the temporal autocorrelation sidelobes, but not spectral properties. Hence, to evaluate the spectral performances of MIMO SCAN and SHAPE algorithms, we measure the stopband levels in (dB). Table 3 exhibits the spectral suppression levels of the initial and the final designed sequences via the MIMO SCAN and MIMO SHAPE algorithms. It can be seen that, in contrast to ISL and MF, when the value of λ is larger, performance of MIMO SCAN is better. This is because, when $\lambda > 0.5$ the spectrum is constrained more than autocorrelation. Moreover, performance of MIMO SHAPE is

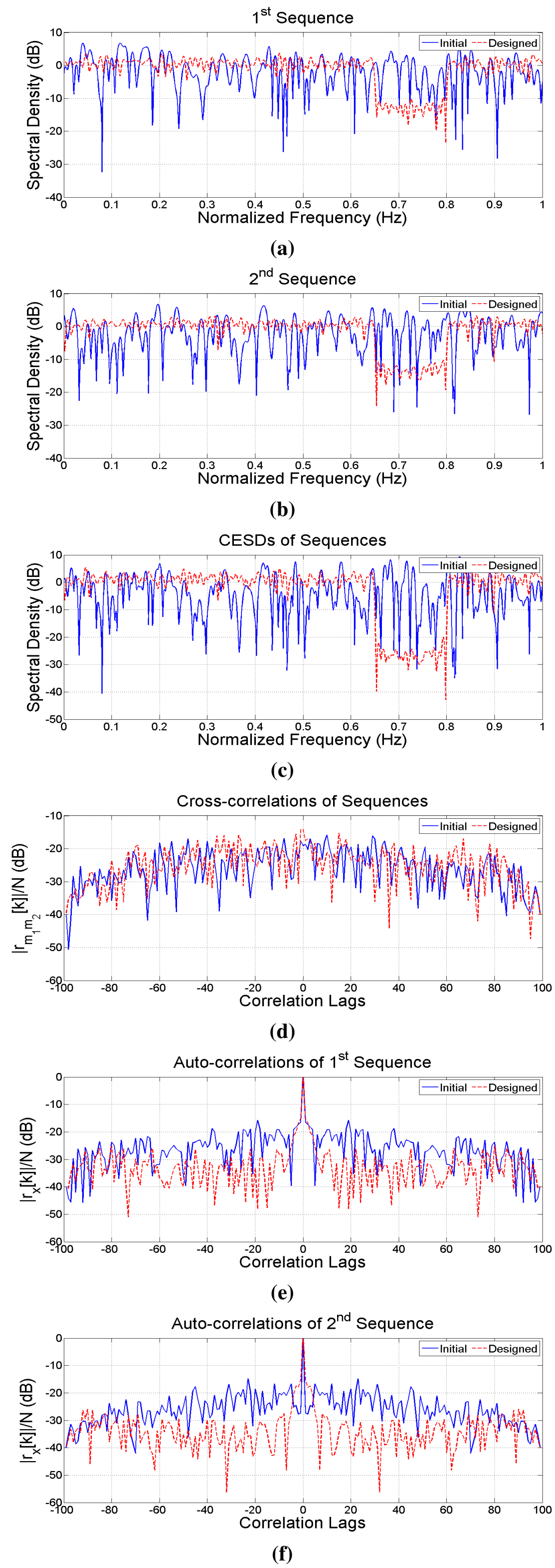

Figure 1. MIMO SCAN example. a) Initial and final spectra of the first designed sequence. b) Initial and final spectra of the second designed sequence. c) CESDs of initial and final designed sequences. d) Aperiodic cross-correlations of the initial and final designed sequences. e) Initial and final autocorrelations of the first designed sequence. f) Intial and final autocorrelations of the second designed sequence

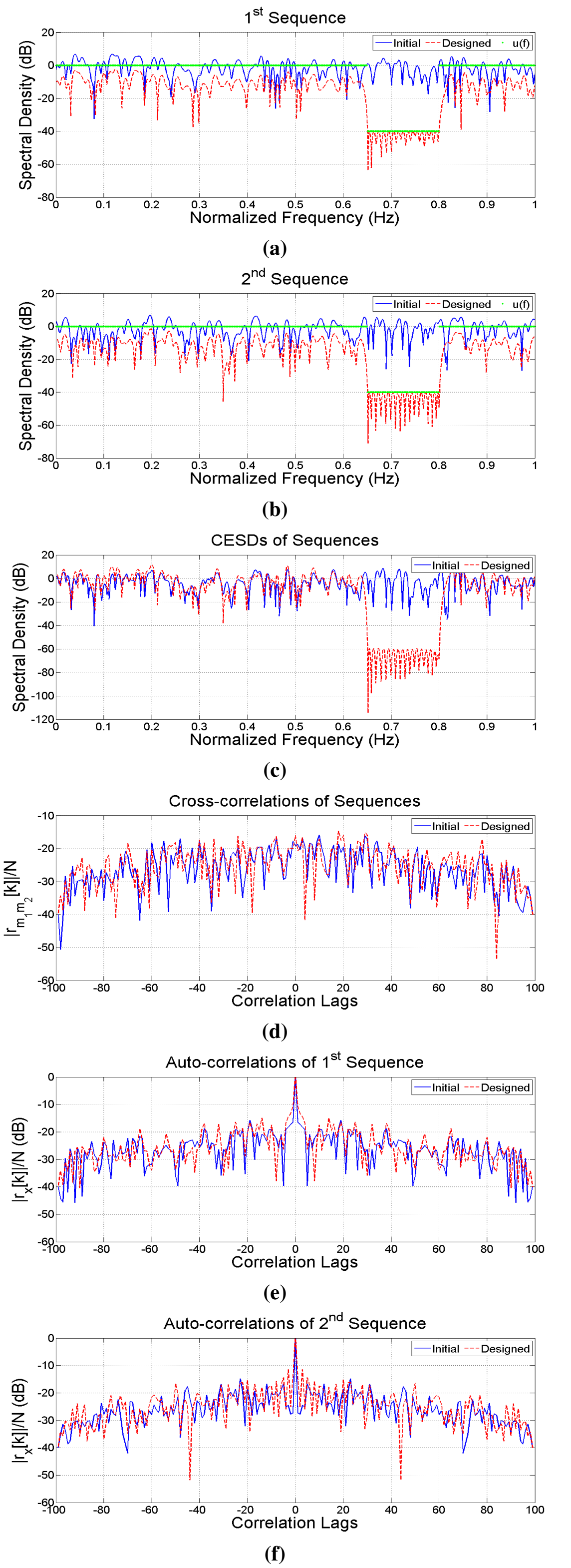

Figure 2. MIMO SHAPE example. a) Initial and final spectra of the first designed sequence. b) Initial and final spectra of the second designed sequence. c) CESDs of the initial and final designed sequences. d) Aperiodic cross-correlations of the initial and final designed sequences. e) Initial and final autocorrelations of the first designed sequence. f) Intial and final autocorrelations of the second designed sequence

much better than that of MIMO SCAN. This is no surprise because, by design, the SHAPE algorithm solely constrains the spectrum and is not concerned with autocorrelation sidelobes.

Table 3. Stopband levels of MIMO algorithms (dB)

	1^{st} Sequence	2^{nd} Sequence
Initial	-17.3108	-25.9472
MIMO SCAN ($\lambda = 0.2$)	-9.0008	-8.0448
MIMO SCAN ($\lambda = 0.8$)	-23.5015	-24.0894
MIMO SHAPE	-63.9190	-70.8986

5. Conclusion

In this manuscript, we have proposed generalizations of the radar waveform design methods of SCAN and SHAPE for MIMO systems. We presented the implementation steps of MIMO SCAN and MIMO SHAPE algorithms. We performed simulation examples of both algorithms for designing two nearly orthogonal unimodular sequences. We calculated some temporal and spectral performance metrics to compare performances of MIMO SCAN and MIMO SHAPE algorithms against each other. The SCAN algorithm can weight both temporal and spectral constraints whereas the SHAPE algorithm performs waveform design based solely on spectral constraints. Finally, although in our simulation examples we designed two unimodular sequences, we would like to note that the number of designed sequences can be increased straightforwardly.

6. References

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