RSS Based Direction Finding via Array of Directional Antennas with Normal Density Distribution in Magnitude

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Abstract

In this study, the theoretical framework of received signal strength (RSS) based direction finding with array of directional antennas with normal density distribution in magnitude are established. Directional antenna systems offer simple and effective solutions, when the antennas must be placed within a limited area, such as on a car. While some important studies exist, the theoretical framework to create such systems where the radiation patterns of directional antennas are modeled as von Mises distributions (i.e. circular version of normal distribution) in magnitude are lacking in the literature. Therefore, in this study first, Cramer Rao Lower Bound (CRLB) is explicitly calculated for joint estimation of angle of arrival and incident signal power. It is found that CRLB is not a function of angle of arrival, and it decreases with the increasing directivity of the antennas. Finally, maximum likelihood estimation (MLE) is discussed by analyzing its ability of convergence.

1. Introduction

Direction of arrival estimation of incoming signals is a quite old research area [1], therefore today, direction finding (DF) technology are primarily divided into classical and modern (i.e. sensor array processing) methods. In classical methods, the systems were including only electrical and analog circuits, therefore, DF algorithms were mathematically established in the simplest form. Classical methods include directional antenna systems, Watson-Watt method, Doppler-based direction finding, monopulse and interferometer etc. Sensor array processing has been developed especially with the developing computer and digital technology. Beamforming, subspace methods, MUSIC and ESPRIT and other complex algorithms are included within sensor array processing. Most of these systems are phase based solutions, while signal strength or amplitude based systems are also preferred for simplicity when no special hardware is required.

Among all these methods, this study will focus on RSS based DF by directional antennas. RSS based directional antenna systems can offer simple and inexpensive solutions. These systems are especially suitable when the area that the antennas can be placed is limited (such as placing the antennas on a car rather than a wide landscape) as shown in Fig.1. The simplest version of these systems consists of a single directional antenna rotating around itself by recording the received signal strength (RSS) for each angle. This system does not function in situations such as the short duration of the incoming signal [2], therefore instead of a single rotating directional antenna, direction finding systems were proposed with multiple stationary directional antennas that are facing different directions [3].

Historically, the first attempts at direction finding involve use of the directional characteristics of antennas [1], so DF systems exploiting directional antennas are regarded as classical methods. Therefore, today they remain in the shadow of other methods developed within the field of sensor array processing. Except a few articles, these systems have not been thoroughly investigated from the point of view of sensor array processing. In [3], RSS based angle of arrival (AOA) estimation by directional antennas are discussed by providing a measurement model for array of directional antennas. Even though some comments on Cramer-Rao Lower Bounds (CRLB) are provided, the CRLB calculations and the proposed solution as well as the gain function of directional antennas are not explicitly discussed. In [4], it was mentioned that AOA estimation can be accomplished via the difference in the signal strengths between the directional antennas facing different directions. In [5], the problem AOA detection with directional antennas is defined by matrix equations, and the nonlinear least squares (NLS) solution is proposed.

Positioning and direction finding studies with directional antennas have been sporadic [6-11] after these initial studies. Most of these studies [6-9] are aiming at range-free positioning. Recently, sectorized antennas are employed to create DOA and localization systems [15]. In [15], the radiation patterns of antennas are regarded as Von Mises distributions in decibel. Consequently, RSS based direction finding with array of directional antennas has not been studied where the radiation patterns of antennas are regarded as Von Mises distributions in magnitude. Therefore, the aim of this study is to enrich the literature about RSS based DF with directional antennas.

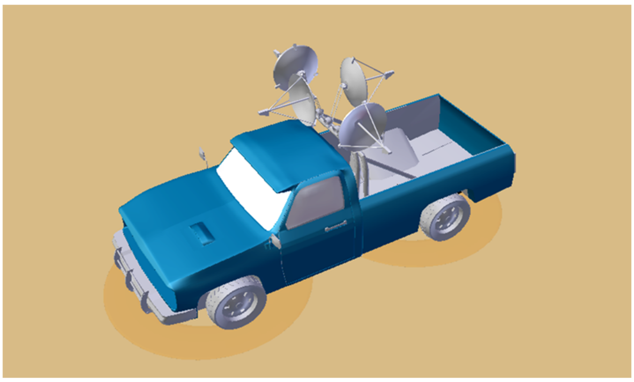

Fig. 1. Directional Antenna Array mounted on a car as a DF system.

In this study, Cramer Rao Lower Bound (CRLB) is explicitly calculated for joint estimation of angle of arrival and incident signal power. The major component of the CLRB calculations is to model the radiation patterns of directional antennas as von Mises distributions (i.e. circular version of normal distribution) in magnitude. It is found that CRLB is not a function of angle of arrival. After analyzing maximum likelihood estimation (MLE) and its convergence, it is shown that MLE can attain CRLB. It is found that error level of these systems are quite acceptable, so this declares that these systems have a high potential for affordable yet reliable real applications.

2. Measurement Model and Radiation Patterns

In this section, the RSS measurement model for directional antennas and radiation patterns will be discussed. RSS values measured by directional antennas can be modeled as the following [11]:

$$R(\theta, d) = 10\log_{10}\left(\frac{P_t\, G_t\, \lambda^2}{(4\pi)^2\, d^2\, L}\right) + 10\log_{10} G_r(\theta) \qquad (1)$$

where $R(\theta, d)$ refers to the received signal strength as a function of both the angle of arrival θ and the distance to the emitter d. P_t is the transmitted power, G_t is the transmitter antenna gain, λ is the wavelength, L is the system loss factor and finally $G_r(\theta)$ is the receiver antenna gain as function of the angle of arrival θ. To separate the effects of d and θ, the function $R(\theta, d)$ can be expressed as:

$$R(\theta, d) = P(d) + g(\theta) + W \sim N(0, \sigma) \qquad (2)$$

where $P(d)$ is RSS value as a function of distance and $g(\theta)$ is the antenna gain determined by the radiation of pattern of receiver antenna. In this study, $P(d)$ is regarded as the highest power received by the directional antenna, therefore, $g(\theta)$ will vary within $(-\infty, 0]$ as dB or within $[0, 1]$ as magnitude. The error term can be expressed as log-normal shadowing W as zero-mean normal distribution with standard deviation σ [3]. After defining measurement model, we need to find a closed from expression for the radiation pattern $g(\theta)$.

In the literature, there exists a couple of closed form expression for the radiation patterns of unidirectional antennas [12]. Among those, the following functions are the most basic ones without any side lobes. Below, $g_1(\theta)$ is the unidirectional cosine function and $g_2(\theta)$ is the ellipse function to express the radiation pattern of unidirectional antennas. The parameters n and w respectively for $g_1(\theta)$ and $g_2(\theta)$ can be adjusted to model the desired directivity for the directional antennas. There are also expressions which include sinc function to model the side lobes of the radiation patterns.

$$\begin{aligned} g_1(\theta) &= 20\log_{10}|\cos^n(\theta)| \quad for\ \theta\ \epsilon\ [0, \pi/2)\ and\ [0, -\pi/2\,) \\ &= -60\ dB \qquad\qquad otherwise \end{aligned} \qquad (3)$$

$$\begin{aligned} g_2(\theta) &= 20\log_{10}|\cos(\theta)\ /\ (\cos^2(\theta) + \sin^2(\theta)\ /\mathrm{w}^2)\ |\ \ldots \\ &\qquad \ldots\ for\ \theta\ \epsilon\ [0, \pi/2)\ and\ [0, -\pi/2\,) \\ &= -60\ dB \qquad\qquad otherwise \end{aligned} \qquad (4)$$

The major problem with these models is that they are not differentiable for the domain [0, 2π] because they are piecewise functions and they include absolute value operators. This make it difficult to build AOA estimators and to calculate CRLB based on these radiation patterns. Therefore, at least a second order differentiable radiation pattern is required. When the unidirectional radiation patterns in magnitude are plotted in cartesian plane (as shown in Fig.2.b), it can be observed that all these patterns resemble normal distributions. Therefore, it can be a suitable idea to model the radiation patterns as von Mises distribution [13] which is the circular version of Gaussian distribution:

$$f(x|\,\mu, K\,) = \frac{1}{2\pi\, I_0(K)} e^{K\,\cos(x-\mu)} \qquad (5)$$

where μ and $1/K$ is the analogy of mean and standard deviation respectively in the Gaussian distribution, and $I_0(K)$ is the zero order modified Bessel function. As mentioned previously, we would like to normalize this function to make it take values it within [0,1], so our final measurement model is as the following:

$$\begin{aligned} R_\beta(\theta, p_d) = p_d + 20\ \log_{10}\!\left(e^{K\,(\cos(\theta-\beta)-1)}\right) \\ + W \end{aligned} \qquad (6)$$

where β is the counter clockwise angle between x-axis and the antenna direction. For homogenously oriented N antennas as shown in Fig.2.a, the β values are written as the following:

$$\beta_i = \frac{2\pi}{N} i \qquad for\ i = 1, 2, \ldots, N \qquad (7)$$

As the parameter K increases, the beam width of the directional antenna decreases, in other words directional concentration (directivity) of the antenna increases.

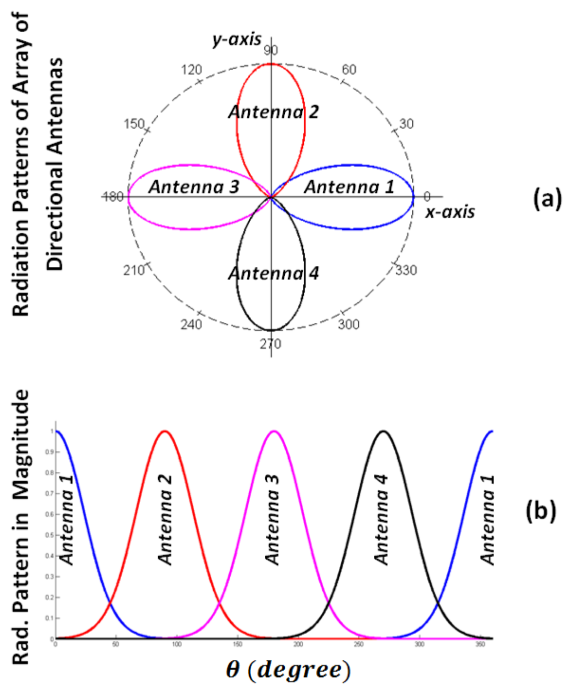

Fig. 2. Radiation Patterns of Array of Directional Antennas

3. Cramer Rao Lower Bound

In this section, we will explicitly calculate the Cramer Rao Lower Bound for jointly estimating θ and p_d with an intention to find the angle of arrival of an incident signal. First, we need to the obtain following differentiations:

$$\frac{\partial\, R_{\beta_i}(\theta, p_d)}{\partial\, \theta} = -\frac{20\, K}{ln(10)}\, sin(\theta - \beta_i) \tag{8}$$

$$\frac{\partial\, R_{\beta}(\theta, p_d)}{\partial\, p_d} = 1 \tag{9}$$

After these calculations, Fisher information matrix (FIM) can be written as the following:

$$I(\theta, p_d) = \frac{1}{\sigma^2} \cdots$$

$$\begin{bmatrix} \sum_{i=0}^{N-1}\left(\frac{20\, K}{ln(10)}\, sin(\theta - \beta_i)\right)^2 & -\sum_{i=0}^{N-1}\left(\frac{20\, K}{ln(10)}\, sin(\theta - \beta_i)\right) \\ -\sum_{i=0}^{N-1}\left(\frac{20\, K}{ln(10)}\, sin(\theta - \beta_i)\right) & N \end{bmatrix} \tag{10}$$

Now, we have to calculate each element of the FIM. To be able to continue with the calculations, the following identities must be presented [14]:

$$\sum_{i=0}^{N-1} sin(x + i\, y) = \frac{sin\left(\frac{Ny}{2}\right)}{sin\left(\frac{y}{2}\right)}\, sin\left(x + \frac{N-1}{2}\, y\right) \tag{11}$$

$$\sum_{i=0}^{N-1} cos(x + i\, y) = \frac{sin\left(\frac{Ny}{2}\right)}{sin\left(\frac{y}{2}\right)}\, cos\left(x + \frac{N-1}{2}\, y\right) \tag{12}$$

Now, let us rewrite the first element of the FIM as:

$$\left(\frac{20\, K}{ln(10)}\right)^2 \sum_{i=1}^{N} \frac{1}{2}\, (1 - cos(2\theta - 2\beta_i)) \tag{13}$$

$$\left(\frac{20\, K}{ln(10)}\right)^2 \left(\frac{N}{2} - \frac{1}{2}\sum_{i=0}^{N-1} cos(2\theta - i\,.\frac{4\pi}{N})\right) \tag{14}$$

Based on (12), let us rearrange (14) as the following:

$$\left(\frac{20\, K}{ln(10)}\right)^2 \left(\frac{N}{2} - \frac{1}{2}\frac{sin(2\pi)}{sin(\frac{2\pi}{N})}\, cos\left(2\theta + \frac{N-1}{2}\,\frac{4\pi}{N}\right)\right) \tag{15}$$

Because $sin(2\pi)$ equals to 0, the first element of FIM finally can be written as the following:

$$\left(\frac{20\, K}{ln(10)}\right)^2 \left(\frac{N}{2}\right) \tag{16}$$

Let us continue with the second element of FIM:

$$-\frac{20\, K}{ln(10)} \sum_{i=0}^{N-1} sin\left(\theta - i\,\frac{2\pi}{N}\right) \tag{17}$$

Based on (11), the second element of FIM can be written as the following:

$$-\frac{20\, K}{ln(10)}\frac{sin(\pi)}{sin(\pi/N)}\, sin\left(\theta + \frac{N-1}{2}\,\frac{2\pi}{N}\right) \tag{18}$$

Because of the existence of $sin(\pi)$, the second element of FIM is found to be 0. Consequently, FIM in (10) can be rewritten as the following:

$$I(\theta, p_d) = \frac{1}{\sigma^2}\begin{bmatrix} \left(\frac{20\, K}{ln(10)}\right)^2 \left(\frac{N}{2}\right) & 0 \\ 0 & N \end{bmatrix} \tag{19}$$

So, we obtained a diagonal FIM. Now, we are ready to establish the CRLB for estimation of the parameter θ based on N measurements from N directional antennas as the following:

$$\sigma_\theta \geq \frac{\sigma\, ln(10)}{20\, K}\sqrt{\frac{2}{N}} \tag{20}$$

Base on this equation, the following useful comments can be made about this CRLB:

- CRLB is independent from θ: All angles of arrival have the same level of estimation error as opposed to common sense that estimating the angles around the orientations of antennas would be easier.
- CRLB is inversely proportional to K: This means that as the directional concentration (i.e. directivity) of the antennas increases, estimation capability also increases.
- CRLB is inversely proportional to $\sqrt{N}$: This means that as the number of antenna increases, estimation capability increases with a decreasing rate of change.
- CRLB is proportional to σ: Estimation capability decreases with the increasing level of RSS measurement error.
- Uncertain transmitted power does not increase CRLB: Usually, as the number of unknown parameters increases, CRLBs for the parameters increase. However, here we obtained a diagonal FIM, so the uncertainty in the transmitted power does not decrease the capability of estimating θ.

After these comments, in the next section, the CRLB that has been found will be verified by the means of simulations.

4. Simulations

In this section, θ and p_0 will be jointly estimated by the means of maximum likelihood estimation (MLE) as the following equation:

$$\underset{(\theta, p_d)}{\text{argmin}}\left(\sum_{i=0}^{N-1}\left(p_d + 20\, \log_{10}\left(e^{K\,(\cos(\theta - \beta_i) - 1)}\right) - R_i\right)^2\right) \tag{21}$$

where R_i is RSS measurement at the ith antenna within the array. Before comparing the MLE solution with CRLB, the surface of the cost function that is minimized will be investigated in term of any convergence issue. When the cost function of $(\theta,\ p_d)$ is plotted in the form of polar coordinates such that $x = p_d\cos(\theta)$ and $y = p_d\sin(\theta)$, the surface shown in the oblique view in Fig.3.a is obtained. For this example, the parameters of the incoming signal i.e. p_d and θ are set to 15 dBm and 60 degrees respectively. Therefore, the global minimum of the surface is supposed to be 15 dB distant from the origin and at 60 degree with respect to x-axis.

The surface shown in Fig.3.b belongs to a cost function where 2 dB noise are added to R_i's. As seen, the global minimum occurs around the desired location without any local minima. Next, it must be investigated if any convergence issue arise with increasing level of measurement error. For the cost functions shown in Fig.3.c and Fig.3.d, the standard deviations of measurements are 5 dB and 10 dB respectively. As the standard deviation increases, the curvature around the minimum point decreases. In other words, the global minimum lies at the bottom of a smoother hole. However, no local minima or saddle point emerges, therefore no convergence issue arises. As a result, we can anticipate that MLE solution can attain CRLB.

The root mean squared (RMS) estimation errors for θ are obtained by the means of Monte Carlo Simulations. For each iteration, the θ value is randomly selected within the interval [0, 360) degrees, and 1000 iterations are conducted to obtain the mean squared error:

$$MSE_\theta = \frac{1}{1000}\sum_{m=1}^{1000}(\theta - \hat{\theta})^2$$

where $\hat{\theta}$ is the estimation of angle of arrival which is estimated together with the $\hat{p}_d$ as illustrated in Equation (21). Please note that for the angle of arrival in degrees (rather than in radians), CRLB must be rearranged as:

$$\sigma_{\hat{\theta}} \geq \frac{9\,\sigma\, ln(10)}{\pi\, K}\sqrt{\frac{2}{N}}$$

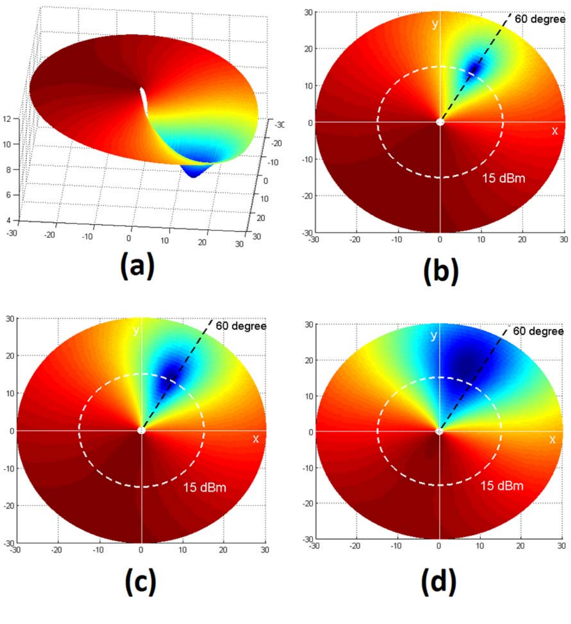

Fig. 3. The surface and global minimum of cost function of MLE: (a) an illustrative 3D oblique view, and top view of the surfaces for standard deviation of (b) 2 dB, (c) 5 dB and finally (d) 10 dB. (N = 4 and K = 5)

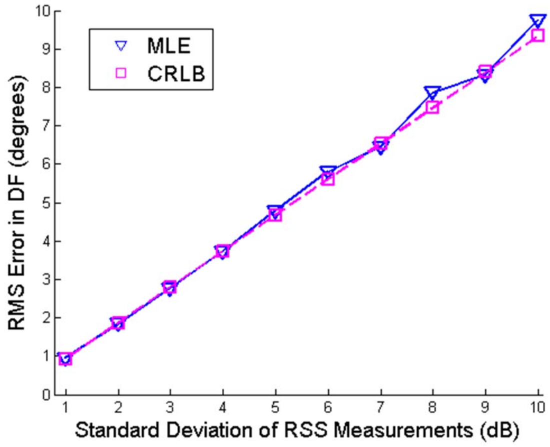

Fig. 4. CRLB and RMSE with respect to σ (standard deviation of RSS measurement error). (N = 4 and K = 5)

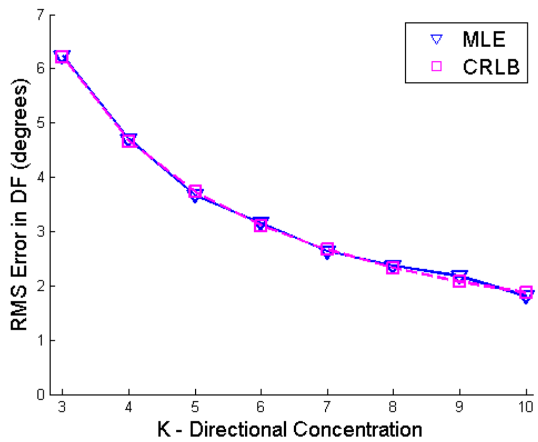

Fig. 5. CRLB and RMSE with respect to K i.e. the directional concentration (directivity) of the antennas. (N = 4 and σ = 4)

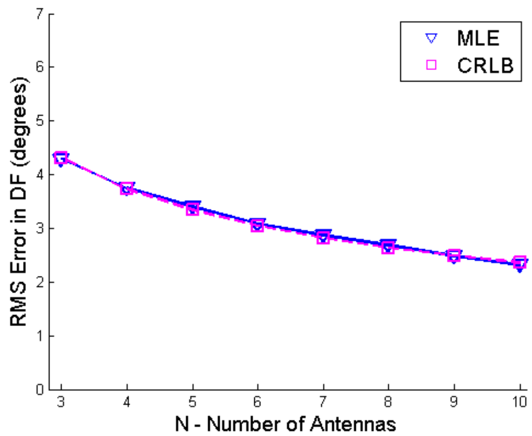

Fig. 6. CRLB and RMSE with respect to N i.e. the number of antennas within the array. (σ = 4 and K = 5)

In Fig.4, both CRLB and RMSE of the MLE solution are plotted for different σ (standard deviation of RSS measurement error) values. In this experiment, $N = 4$ and $K = 5$. As seen, the plots are matching each other. Therefore, CRLB that is calculated in Section III is verified. Moreover, as anticipated, MLE solution is attaining CRLB. As σ increases, RMSE also linearly increases. In Fig.5, CRLB and RMSE are plotted for different K values i.e. the directional concentration of the antennas. In this experiment, $N = 4$ and $\sigma = 4$. Again, CRLB and RMSE plots are overlapping each other. As the directivity of antennas increases, RMSE decreases. Therefore, using antennas with high directivity can significantly increase the capability of DF system. Finally, in Fig.6, CRLB and RMSE are plotted for different N values i.e. the number of antennas within the array. Again, two plots are matching each other. While RMSE decreases, it can be observed that increasing the number of antennas are not as effective as replacing antennas with high directivity ones.

6. Conclusions

In this study, the theoretical framework of RSS based direction finding by the means of array of directional antennas are established where the radiation patterns of directional antennas are modeled as von Mises distributions (i.e. circular version of normal distribution) in magnitude. These systems are suitable when there is a limited area for the antennas such as placing them on a car. In the literature, some important studies exist which are interested in localization and direction finding by the means of directional antennas. However, the theoretical framework for these systems are not thoroughly discussed. Therefore, in this study first, Cramer Rao Lower Bound (CRLB) is explicitly calculated for joint estimation of angle of arrival and incident signal power. The key component of these calculations is to model radiation patterns of directional antennas as von Mises distributions (i.e. circular version of normal distribution) in magnitude. This distribution is preferred because it is at least second order differentiable.

After careful calculations, it is found that CRLB is not a function of angle of arrival θ. Therefore, all angles have the same level of error as opposed to common sense that estimating the angles around the orientations of antennas would be easier. CRLB is also found to be inversely proportional to the directional concentration (i.e. directivity) of the antennas increases. Moreover, a diagonal FIM is obtained, so the uncertainty in the transmitted power does not decrease the capability of estimating θ.

After CRLB calculations, Monte Carlo Simulations are conducted to compare MLE solution with CRLB. It is proposed that MLE does not have a convergence issue, so it can attain CRLB. Finally, it is shown that the plots of MLE and CRLB are overlapping each other. The value of parameters within the model are chosen to be close to the real values to simulate the real cases, and estimation error of these systems are found to be quite acceptable. Consequently, it is proposed that RSS based directional antenna systems can provide very effective solutions, when these systems are regarded as a component of Sensor Array Processing rather than as a classical method.

6. Acknowledgement

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7. References

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