Implementation and Comparison of LQR-MPC on Active Suspension System

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Abstract

Suspensions are systems produced to minimize the effect of road surface defects on the vehicle, and are divided into three as passive, semi-active and active suspensions. In this study, quarter car suspension model is examined with linear quadratic regulator (LQR) and model predictive controller (MPC), respectively. Quanser active suspension experiment set is used to obtain experimental results. First, LQR which minimizes the performance criteria related to state and input signals is used to control the system. Then, MPC which is very popular in the industry is used as a second control method. After obtaining nominal system responses for both control methods, comparison between LQR and MPC under different load characteristics and parameter variations is done. By changing the road characteristics and the plant, and by applying disturbance to the system, system responses with LQR and MPC are examined and compared.

1. Introduction

The suspension system, generally contains spring, damper and axle mechanism, and the main purpose of it is to reduce the effect of road disturbances. Suspension systems can be divided into three subcategories as passive, semi-active and active suspension. The active suspension system has an actuator which distinguishes it from other suspension types, and the active suspension system can be used in transportation, especially road and railways to minimize the disturbances caused by road.

Several control methods have been proposed in the literature for the control of the active suspension system. Some of these control methods include Skyhook, LQG (Linear Quadratic Gaussian control), LQR (Linear Quadratic Regulator), Delayed Resonator and H_∞ [1-2].

Within the scope of this study, it is aimed to examine the quarter vehicle suspension model, to control the active suspension system with two different methods which are LQR and MPC, to obtain both simulation and experimental results of system, and finally to compare the controllers under different load characteristics and parameter variations.

Firstly, the system is controlled by LQR, which is an optimal control method and has numerous detailed researches in the literature. Then, the system is controlled by using the MPC method, which is very popular in the industry. Having a dynamic structure is the most important feature of MPC that separates the method from the LQR which has a static structure. After that, simulation and experimental results for nominal system are obtained for both controllers. Finally, system with LQR and system with MPC are compared by changing the road characteristics, changing the plant, and applying disturbance. System responses are examined and interpreted.

The main motivation of this study is that there are a few studies in the literature where more than one method is applied experimentally and the results are compared.

2. Active Suspension System

The suspension system generally consists of a spring, a damper, and an axle mechanism and reduces the effect of the road-related disturbances by the help of the spring and damper elements it contains. The aim of the suspension systems is to improve driving comfort, road holding, and stability.

Suspensions are divided into three groups which are passive, semi-active and active suspensions. While passive suspension has fixed characteristics, characteristics of semi-active suspension can be adjusted by changing the spring and damper coefficients. Active suspension system, which is examined in this paper, differs from other types due to its actuator it has that suppresses the road disturbances by implementing the energy directly to the system.

For this study, Quanser active suspension experiment set is used since it has a stable system architecture, its interface which is easily adaptable with MATLAB in real-time experience, it enables different control methods to be applied, and it is a widely used set in researches.

As it can be seen in Figure 1, the active suspension set consists of three plates. The upper mass (blue plate) represents the vehicle body, the middle mass (red plate) corresponds to one of the tires of the vehicle, and the bottom mass (silver plate) simulates the vertically moving road surface.

The Quanser active suspension system model consists of two masses and one road disturbance, each supported by a spring and a damper. The system has fundamentally two inputs which are actuator force (F_c) and road disturbance (z_r). The actuator force is the manipulated variable and the road disturbance is the unmeasured disturbance. The outputs are selected as tire displacement (x_1) and body displacement (x_2). This disturbance profiles are applied to the system by using MATLAB/Simulink. Since system model is derived in earlier studies, in this study it is not examined in detail. State, input, output matrices and state space representation of the system are given below.

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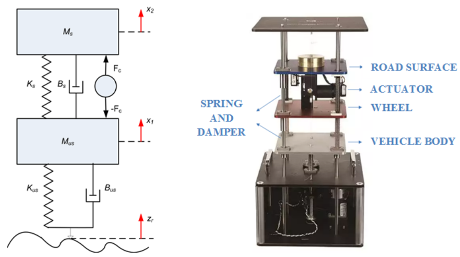


Figure 1. The representation of the plates of active suspension system.

$$x = \begin{bmatrix} z_s - z_{us} \\ \dot{z_s} \\ z_{us} - z_r \\ \dot{z_{us}} \end{bmatrix}, \quad u = \begin{bmatrix} \dot{z_r} \\ F_c \end{bmatrix}, \quad y = \begin{bmatrix} z_s - z_{us} \\ \ddot{z_s} \end{bmatrix} \tag{1}$$

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -\frac{K_s}{M_s} & -\frac{B_s}{M_s} & 0 & \frac{B_s}{M_s} \\ 0 & 0 & 0 & 1 \\ \frac{K_s}{M_{us}} & \frac{B_s}{M_{us}} & -\frac{K_{us}}{M_{us}} & -\frac{(B_s+B_{us})}{M_{us}} \end{bmatrix} \begin{bmatrix} z_s - z_{us} \\ \dot{z_s} \\ z_{us} - z_r \\ \dot{z_{us}} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{M_s} \\ -1 & 0 \\ \frac{B_{us}}{M_{us}} & -\frac{1}{M_{us}} \end{bmatrix} \begin{bmatrix} \dot{z_r} \\ F_c \end{bmatrix} \tag{2}$$

3. Control Methods

In this study, two methods are used to control the active suspension system. Firstly, LQR is used for active suspension system since it provides optimal control solutions and there are numerous detailed researches about it. Then, the system is controlled by using MPC which is a dynamic optimal control approach and becomes more widespread in industry.

3.1. Linear Quadratic Regulator

Designing efficient controllers for non-linear systems which are able to provide desired system performance and also have a basic design process with minimum cost is the fundamental problem in modern control theory. Most dynamical systems in nature are non-linear and the equations of these systems are difficult to solve, thus these systems are commonly linearized by equations which are proper within a small region around the operation point which provides a controllable structure because most of the control methods are designed for linear systems.

The primary purpose of optimal control is to determine control signals which result in the system to satisfy some physical constraints and extremize the desired performance criteria (performance index or cost function). The Linear Quadratic Regulator (LQR) is one of the most studied control problems in the literature, and it has many applications. LQR is a specified form of state feedback control method, and LQR technique makes optimal control decisions considering the states and control input of the dynamical system [16].

The structure of the LQR control technique is same with the structure of the state feedback control; however, the design method of LQR differs from state feedback. The main difference between them is the calculation of the gain matrix K. LQR algorithm solves the equation 3 to find an optimum gain matrix that minimizes the quadratic cost function below:

$$J(u) = \int_0^\infty (x^T Q x + u^T R u) dt \tag{3}$$

where Q is the cost structure to determine the importance of the states and R is the regulator for control signal [17]. The weighting matrix Q is a symmetric positive semi-definite matrix, while R is a symmetric positive definite symmetric matrix [16].

The LQR problem can be explained as minimization with weighting for the linear combinations of the states x and the control input u. The matrix Q which is also called weighting matrix represents the difference of the importance of the states are to be controlled. Moreover, R defines the allowable aggressiveness of the control signal. For instance, large R values result in a smaller control signal [16].

The LQR gain vector K can be calculated as in equation 4.

$$K = R^{-1} B^T P \tag{4}$$

where P is a positive definite symmetric constant matrix which can be obtained from the solution of the Algebraic Riccati Equation as shown in equation 5 [16] [14].

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \tag{5}$$

In the design of LQR controller, the key point is to determine Q and R structures. Basically, there are numerous methods to determine the Q and R matrices. In the project, the diagonal weights are used for calculations as recommended by Quanser (6) [3]. The method's principle is to choose the weights as their relative importance.

For instance, if $1\ cm$ error is acceptable for x_1 state, then $q_1 = \frac{1}{100^2}$ and if $\frac{1}{60}$ radians error is acceptable for the x_3 state, then $q_3 = 60^2$. Moreover, $q_1 {x_1}^2 = 1$ when $x_1 = 1\ cm$ and $q_3 {x_3}^2 = 1$ when $x_3 = \frac{1}{60}\ rad$ [18].

In the project, since Q is chosen as diagonal, the states have been considered that they do not have any effect on each other in terms of error punishment. Similar to Q, R represents punishment rate of the control signal. For instance, larger R values cause smaller control signals.

$$Q = \begin{bmatrix} q_1 & & & \\ & \cdot & & \\ & & \cdot & \\ & & & \cdot \\ & & & & q_n \end{bmatrix} \quad R = \rho \begin{bmatrix} r_1 & & & \\ & \cdot & & \\ & & \cdot & \\ & & & \cdot \\ & & & & r_n \end{bmatrix} \tag{6}$$

LQR weighting diagonal matrices Q and R are selected as in equation 7 which are suggested by Quanser [3]. R is chosen as two different values to observe the differences due to R. Then, the optimal state feedback gain matrix is calculated by using MATLAB (8) and implemented.

$$Q = \begin{bmatrix} 450 & 0 & 0 & 0 \\ 0 & 30 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0.1 \end{bmatrix}; R_1 = 0.01;\ R_2 = 0.001 \tag{7}$$

$$K = [\,24.6621 \quad 49.2526 \quad 68.6037 \quad 5.0940] \tag{8}$$

After calculations, the Simulink diagram which includes both real-time and simulation blocks is designed as in Figure 2.

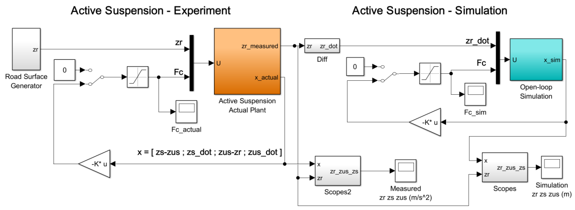


Figure 2. Experiment system block diagram.

The orange colored block in the Simulink diagram is the actual plant and the cyan colored block is the simulation block. After running the experiment, real-time and simulation results of the system can be compared. In this model, the surface disturbance is given as a square wave.

In the real-time experiment, the surface disturbance is generated by a DC motor which is also controlled in a process to generate square wave disturbance. The desired disturbance is created in MATLAB/Simulink, then inserted to system via Quanser hardware in the loop (HIL) analog write block. The states of the system are acquired by using the accelerometer sensor from Quanser HIL read analog blocks and calibrated.

Since the system is a real-time application, there are constraints such as actuator force. Also, the system has internal safety constraints to protect itself from fatal errors which may cause serious damage to the structure.

Two different R values are selected and results are observed. When R is chosen as 0.01 and 0.001, the poles of the system located at different points. Closed loop system poles for R=0.01 and R=0.001 are given below in 9 and 10.

$$\overrightarrow{\lambda}_{R=0.01} = \begin{bmatrix} -8.1159 + 14.5964i \\ -8.1159 + 14.5964i \\ -6.9916 + 57.8225i \\ -6.9916 + 57.8225i \end{bmatrix} \tag{9}$$

$$\overrightarrow{\lambda}_{R=0.001} = \begin{bmatrix} -54.183 \\ -7.6387 \\ -8.2385 - 52.4759i \\ -8.2385 + 52.4759i \end{bmatrix} \tag{10}$$

3.2. Model Predictive Control

Model predictive control (MPC) has a long history in the field of control engineering. MPC and receding horizon control are dated back to 1960s with Propoi (1963) who proposed the moving horizon approach, and Lee and Markus (1967) who have to foresee today's MPC practice in their optimal control textbook [19].

However, MPC became popular and started to use in numerous control engineering areas in the 1980s after the first successful application of model predictive heuristic control (MPHC) algorithm by Richalet et al. on a Fluid Catalytic Cracking Unit in 1978 [20].

MPC emerged in the industry due to its effectiveness for solving multivariable constrained control problems. Since MPC needs high computational power and has slower system response, it is mostly used in process control areas such as power plants, petrochemical, and refinery industries until the last two decades [19].

After the 1990s, usage areas of MPC has been grown greatly in the industry. When higher computational powered machines which work better in real-time experiments becomes affordable and widespread, MPC is started to use in the areas which need faster system responses such as automotive, food processing, metallurgy, aerospace, and defense industries [20].

In past 40 years, numbers of MPC algorithms have been established; for instance, dynamic matrix control (DMC) by Cutler and Ramaker (1979), internal model control (IMC) by Garcia and Morari (1982) and generalized predictive control (GPC) by Clarke et al (1987) [21]. The types of the plant models which represent the system dynamics and the cost functions are the main differences of all MPC algorithms mentioned above.

The main idea of the MPC control scheme is to use a model of the plant to predict the future evolution of the system by optimizing the controller signal. Figure 3 shows the basic structure of MPC procedure.

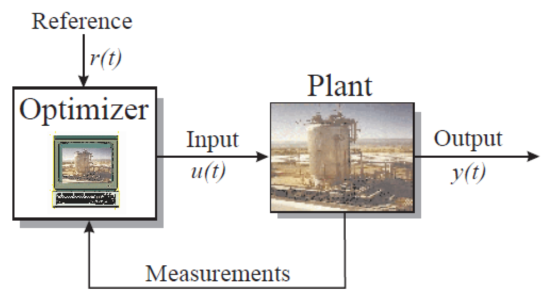


Figure 3. Basic structure of MPC procedure [21].

MPC uses receding horizon principle. At each sample time of the system, the first input of the optimized control signal is applied to the system. Then, at the time $t + 1$, optimal control problem is solved to find new control signal. This procedure is called as receding horizon principle and this principle is common for all types of MPC algorithms which are shown in Figure 4. While the feedback information is collected from the plant at each sample time, the receding horizon enables the plant to perform at desired characteristics [21].

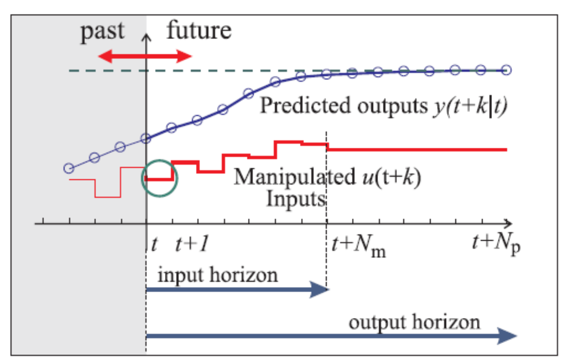


Figure 4. Receding horizon strategy [21].

MPC has three key points which are the model of the plant, constraints of the real system, and the weight tuning. MPC algorithm requires all of them for the state estimations, generating the control signal for real-time experiment, and providing desired characteristics, respectively.

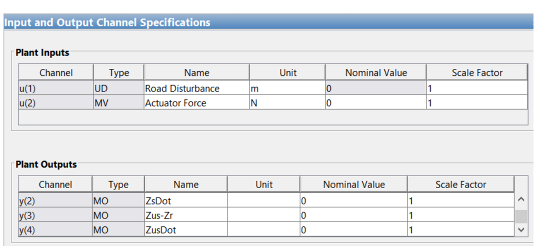
Working with a long range prediction is the most important property which distinguishes the MPC from other control methods. MPC achieves good performance against the limitation from the process dead-time, nonminimum phase, and slow process dynamic due to the predictions over the future horizon [20].

For the calculation of each controller output, the system response is predicted for a finite time horizon. Thus, there is an iterative open loop optimization method which turns to closed loop by the update of the feedback from system output [22].

Although LQR and MPC algorithm is similar in terms of obtaining results by solving quadratic problems, LQR and predictive control differ in various ways. However, the essential difference between them is calculation horizon. While predictive control solves the optimization problem using in moving time horizon, LQR solves the quadratic problem within a fixed time window which is zero to infinity. Using a moving time horizon window enables the controller to perform with hard constraints on for real-time experiments [23].

In this experiment, MATLAB MPC Toolbox is used. While using the MPC Toolbox, there are three steps for generating and implementing the MPC into the plant. The first step is defining the plant model, the second step is defining the MPC inputs and outputs (IOs), and the third step is assigning the MPC parameters [24] [25]. Firstly, the system is defined in terms of state space model which is the key point of the MPC design because the whole process of prediction depends on the accuracy of the plant model. Simulink MPC controller block requires a linear plant model. As a first step of the design, plant model is identified in the previous section and imported to the MPC controller. This defines the number of inputs and outputs of the system.

Then, the plant inputs and outputs should be defined correctly considering the behavior of the system. In active suspension system, the actuator force is defined as the manipulated variable and the road disturbance is defined as the unmeasured disturbance. After that, the feedbacks (measured outputs) of the MPC is chosen as all the states of the plant to make a better comparison between LQR control structure as it can be seen as in Figure 5 and Figure 6. After that, as it is shown in Figure 7, the constraints for MPC are defined by considering the real system limitations.



Input and Output Channel Specifications

Plant Inputs

Channel	Type	Name	Unit	Nominal Value	Scale Factor
u(1)	UD	Road Disturbance	m	0	1
u(2)	MV	Actuator Force	N	0	1

Plant Outputs

Channel	Type	Name	Unit	Nominal Value	Scale Factor
y(2)	MO	ZsDot		0	1
y(3)	MO	Zus-Zr		0	1
y(4)	MO	ZusDot		0	1

Figure 5. MPC IO specifications.

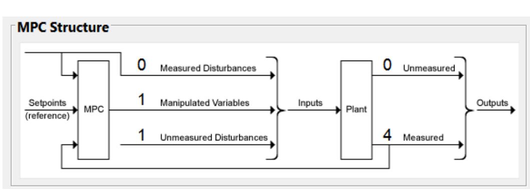
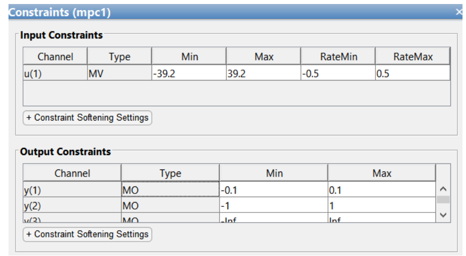


Figure 6. MPC structure.



Constraints (mpc1)

Input Constraints

Channel	Type	Min	Max	RateMin	RateMax
u(1)	MV	-39.2	39.2	-0.5	0.5

+ Constraint Softening Settings

Output Constraints

Channel	Type	Min	Max
y(1)	MO	-0.1	0.1
y(2)	MO	-1	1
y(3)	MO	-Inf	Inf

+ Constraint Softening Settings

Figure 7. Constraints of MPC structure.

The weights of inputs and outputs are selected and the weights determine the importance of the states error rate. If a larger weight is selected, MPC becomes more sensitive to the state. After the selections and definitions of the MPC parameters, the system can be tuned by using performance tuning criteria which are Robustness-Aggressiveness and Slower-Faster State Estimation is shown in Figure 8. Moreover, the sample time, the prediction horizon which determines the number of future steps to be predicted by the controller, and the control horizon which defines the number of parameters is used to capture the future trajectory [23].

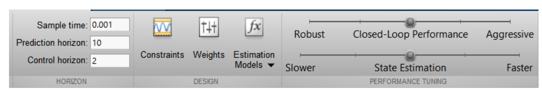


Figure 8. Performance tuning of MPC.

After the parameter definitions, the controller is simulated in MPC Toolbox using the scenario simulation. The scenario setting is defined by considering the system structure and behavior against road disturbance. The references to the states are given as zero because the positions of the plates remain steady in a perfect control. In other words, the desired behavior of the MPC is making the upper plate (zs) to be steady against the road disturbance. Furthermore, the unmeasured disturbance is selected as a pulse signal with 0.1 second period and 0.2 m/sec magnitude which is generated by Quanser Quarc library of road disturbance. After that, the simulation is observed against the tuning of the MPC Toolbox and the controller is created for the system. Then, the Simulink diagrams of simulation and real system are designed and the plant results are acquired for both simulation and real-time system.

4. Simulation and Implementation Results for Nominal System

Simulation and implementation results for the nominal system with LQR for R=0.001, LQR for R=0.01 and MPC are obtained and given below, respectively. It is observed that, R parameter selection directly affects the control's agresiveness by allowing to use larger control signals. Also, MPC's dynamic behavior is observed in experimental results. Also, it is observed that LQR(R=0.001) controller has similar response with MPC by using smaller control signal.

4.1. Simulation

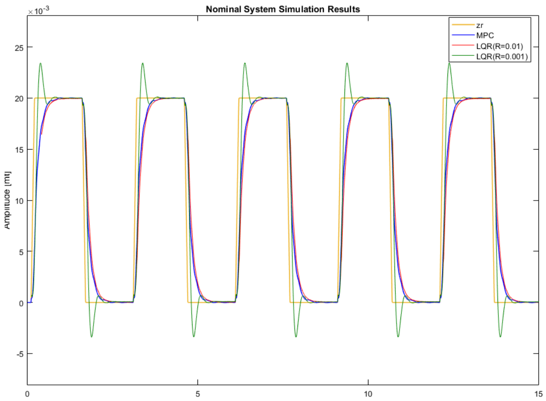


Figure 9. Simulation responses for nominal system.

4.2. Implementation

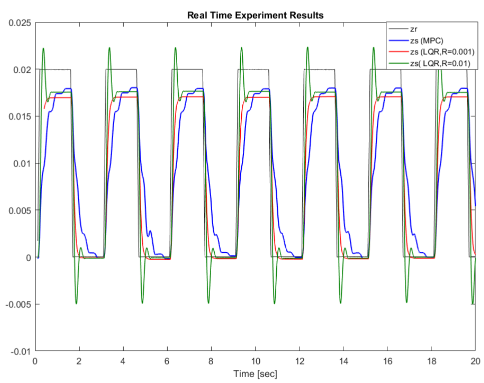


Figure 10. Implementation responses for nominal system.

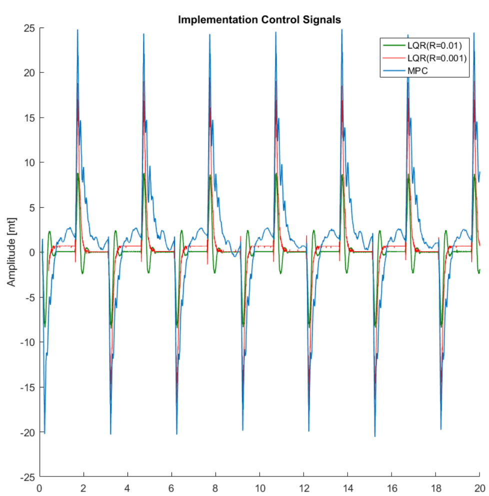


Figure 11. Implementation control signals.

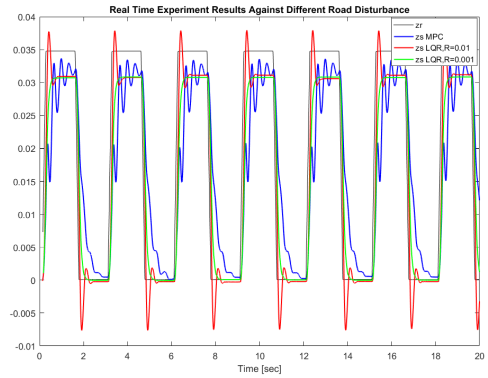


Figure 12. Real-time response with different road disturbance for three controllers.

5. Comparison Under Different Load Characteristic and Parameter Variations

System responses of LQR and MPC methods are examined and compared for three different conditions. First of all, the road disturbance is changed, secondly, the plant is changed by adding an extra mass to the upper plate, and finally disturbances are applied to the upper plate.

5.1. Road Characteristics

The road disturbance is changed from 0.02 m to 0.035 m to observe system behaviour for different controllers .

The real-time experiment is examined for different road disturbance for LQR (R=0.01), LQR (R=0.001) and MPC. The road disturbance is taken as 0.02 m during the project. For this comparison, the road disturbance is changed into 0.035 m.

When the responses are analysed, the system with LQR (R=0.01) has better response in terms of control signal while the system with LQR (R=0.001) has better transient response without oscillations and the system with MPC has better response in terms of integral square error (ISE) and integral time square error (ITSE) values of the upper plate position. In the Table 1 and Table 2, the green color represents the relatively best value of errors among three controllers when the reference is 0.02 m, and the blue color represents the best value when the reference is 0.035 m.

Table 1. Comparison of LQR (R=0.01), LQR(R=0.001), and MPC for ref=0.02 for nominal system.

	LQR (R=0.01)	LQR (R=0.001)	MPC
ISE	0.0033	0.0029	0.0028
ITSE	0.0329	0.0286	0.0282
IAE for Control Signal	21.5846	39.1070	88.6940
ISE for Control Signal	111.2853	349.5482	918.2082

When the Table 1 and Table 2 are examined, it can be said that MPC has better performance in terms of ISE and ITSE values which causes larger cost due to larger control signal values. Moreover, LQR (R=0.01) causes less cost since the control

Table 2. Comparison of LQR (R=0.01), LQR(R=0.001), and MPC for ref=0.035 for nominal system.

	LQR (R=0.01)	LQR (R=0.001)	MPC
ISE	0.0101	0.0093	0.0091
ITSE	0.1009	0.0938	0.0928
IAE for Control Signal	35.9205	66.9958	148.1477
ISE for Control Signal	267.4477	914.3725	3093.1843

signal is limited with larger R. Also, LQR (R=0.001) has better performance considering the transient responses in experiments. Although LQR (R=0.001) has similar responses with MPC and in some cases even better responses from MPC, the LQR algorithm creates a static system by placing the system poles at optimal positions, thus once it is designed and set, the system remains same. MPC has better performance since it has a dynamic behavior which can solve problems such as sudden disturbances. However, it causes higher cost due to the control signal.

5.2. Parameter Variations

The plant is changed by adding an extra mass which is 1.2 kg to the upper plate, and results are observed for all controllers. When the modified plant by adding extra weights is examined, it is observed that both of the LQR controllers provide better responses than MPC which can be seen from Figure 13 and Figure 14. The system with LQR (R=0.001) gives a better response than the system with LQR (R=0.01) since smaller R parameter enables LQR to generate a larger control signal. Moreover, the system response with MPC is the worst among three controllers because MPC algorithm requires an accurate plant to predict future horizons and generate an appropriate control signal for that plant. When the plant model is irrelevant from the actual plant, MPC cannot control the system as it can be observed from Figure 13. However, when the plant model is changed with a mass of 0.3 kg instead of 1.2 kg, the control becomes visibly better as can be seen in Figure 14.

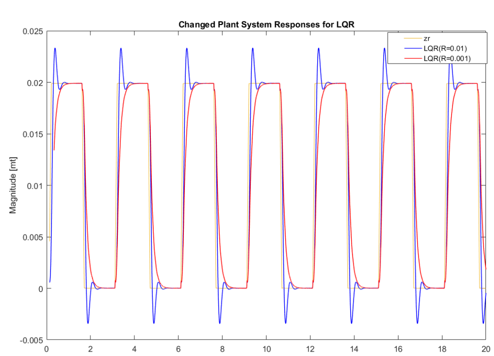


Figure 13. Real-time response for LQR with changed plant.

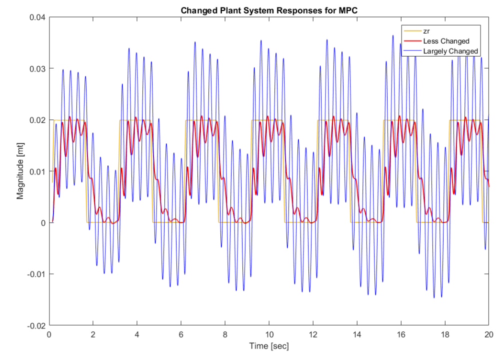


Figure 14. Real-time response for MPC with changed plant.

5.3. Varying Load Profile

To examine the disturbance rejection of the system with each controller, disturbance is applied to the upper plate. To obtain more accurate results, an experiment is designed. An object which weighs 0.2 kg is chosen as a disturbance mass, and the active suspension experiment set is adjusted to start 7 cm above from ground at each trial. After that, a step-like road disturbance is applied to the system and waited until the system settles. After approximately three seconds from settling, the object is released from 3 cm above from upper plate. Then, the object is removed from the upper plate after approximately 4 seconds later. After the disturbance comparison experiment set up, the system responses for LQR (R=0.01), LQR (R=0.001), and MPC are analysed and the data which is used for comparison is gathered from MATLAB. The systems are compared with the ability to preserve their first settling position. Therefore, integral absolute error (IAE) and ISE values are calculated by taking the difference between the system response and settling value after the system settles. For all three controllers, upper plate position is changed through the ground when the disturbance object is released, and after removing the object from upper plate, upper plate's position is changed and it is different from the initial settling position. It is observed that when MPC is used, the upper plate position has not changed as much as the system with LQR controllers considering initial settling position. As it can be seen from the Table 3 and Figure 16, MPC has a visibly better performance for both IAE and ISE with a higher cost of the control signal.

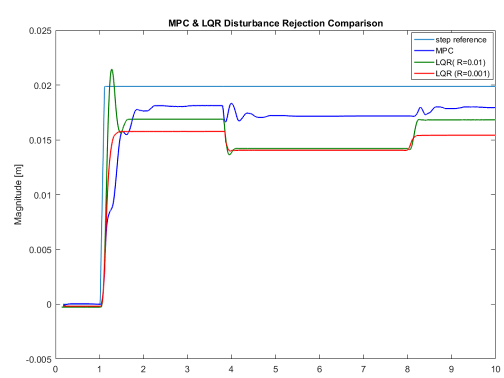


Figure 15. Real-time disturbance rejection comparison for three controllers.

In this experiment, the settling time at the beginning is not considered as an aspect of comparison. Furthermore, the responses against larger mass disturbances for both square-wave and step-like road disturbances can be seen from Figure 18 and Figure 15.

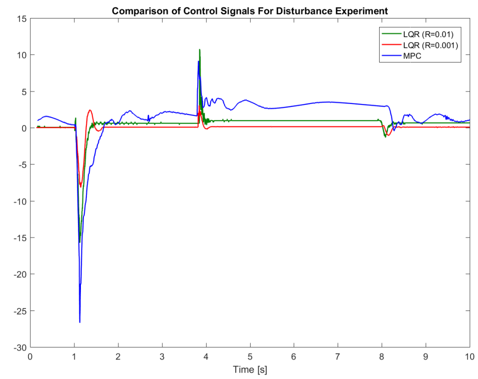


Figure 16. Disturbance experiment control signal comparison for three controllers.

It can be observed that MPC is more powerful in terms of keeping the system in the range of its settling positions. In Figure 12, upper mass has oscillations for MPC. However, upper mass preserve its position successfully as it can be seen from Figure 15.

Table 3. Disturbance experiment results.

	LQR(R=0.01)	LQR(R=0.001)	MPC
(IAE)	0.0307	0.0125	0.0041
ISE (ISE)	11.6375	7.8876	4.5463
Abs Con.Sig.	2.6516	9.2277	25.2984
ISE Con.Sig.	7.8794	29.3048	116.2787

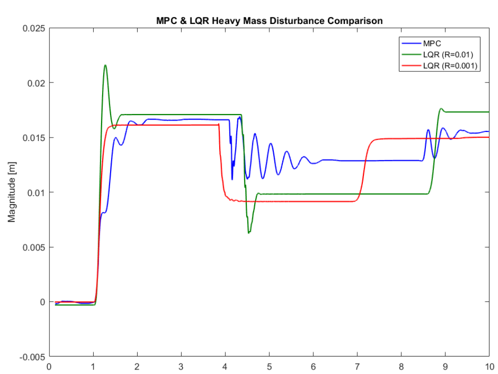


Figure 17. Real-time disturbance (heavy) rejection comparison for three controllers.

In Figure 18, although the upper mass oscillations are visibly more than LQR controller systems, the upper mass stayed in the range of open loop response against road disturbances.

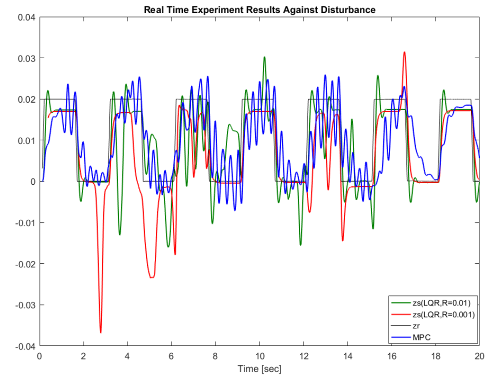


Figure 18. Real-time response against disturbance for three controllers.

6. Conclusion

In conclusion, the active suspension system differs from other suspension types by having an actuator which enables it to implement force directly to the system rather than static behavior. Control of active suspension system is a disturbance rejection problem. In this study, the disturbances are generated as square wave which simulates the speed bumps and pits on the road surface. After the system's mathematical model is examined it is controlled with LQR and MPC respectively.

LQR is selected to control the system since it is an optimal control method which is more convenient for designing controllers. Two different LQR controllers are designed and they differ from each other only in terms of R which affects the aggressiveness of control signal. It is observed that both of the LQR controllers are successful against disturbances in both simulation and real-time experiments in terms of minimizing the oscillations.

After that, MPC is used due to its dynamic behavior and increasing popularity in the industry. When the responses are observed it is clear that, MPC is one of the successful methods to control such systems.

Finally, LQR and MPC are compared in terms of performance and control expense specifically such as control cost, total error, responses against the disturbances, and responses to model changes since LQR and MPC methods are similar to each other in terms of theoretical background. To obtain more scientific results, the disturbance comparison was made by setting up an experiment that is based on putting an object on the upper plate. After that, the experiment has executed on Quanser active suspension experiment set-up. Considering the results, model predictive control is found as the best control method in terms of disturbance rejection, stability around balance point, and response time against sudden changes of working environment since it has a dynamic behavior. Linear quadratic regulator is more successful against plant model changes although it is a static control method. Also, systems with LQR and MPC has similar responses but LQR has significantly smaller control expense and requires much less computational power.

Moreover, a number of improvements can be suggested for the system. For instance, LQR design matrices Q and R can be calculated using optimizing search methods such as the genetic algorithm to obtain better results. Also, the plant model is vital for MPC structure, thus it can be calculated by measuring the parameters of the system such as weight.

7. References

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