Real Frequency Design of Pi and T Matching Networks with Complex Terminations

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Abstract

In this paper, real frequency design of Pi and T matching networks with complex terminations is studied. Generally the generator and load termination impedances are given as measurement values. So they can be regarded as a resistor and a reactive element connected in series. To be able to design a Pi network, this series impedance models must be transformed to parallel models. But for T network designs, the assumed series models can be utilized, there is no need a transformation process. Then Pi or T matching networks can be designed via Q based method which is well defined in the literature.

1. Introduction

Narrowband impedance matching methods [1-8] have a very important role in the design of matching networks. Besides they can solve narrowband matching problems, also the designed narrowband networks can be used as an initial guess for broadband matching problems [9-11].

The maximum power transfer is the most important issue in the design of matching networks; which can be realized if conjugate matching is obtained. Therefore the input and output impedance of the designed LC matching network must be the conjugate of the termination impedances.

Two-element L networks as seen in Fig. 1 are the simplest and most widely used narrow band impedance matching circuits and can easily be designed via the following equations [1]:

$$Q_S = Q_P = \sqrt{\frac{R_L}{R_G} - 1}\ , \tag{1a}$$

$$X_S = Q_S \cdot R_G\ , \tag{1b}$$

$$X_P = R_L / Q_P \tag{1c}$$

where Q_S is the quality factor of the series section, Q_P is the quality factor of the parallel section, R_L is the parallel load resistance, X_P is the parallel matching reactance, R_G is the series generator resistance, X_S is the series matching reactance.

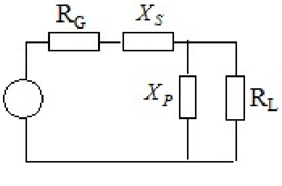

Fig. 1. A typical two-element L matching network.

Here X_P and X_S will be either a capacitor or an inductor but they must be opposite type of components. If a capacitor is connected as X_P, then an inductor must be connected as X_S, and vice-versa.

If the termination impedances are purely resistive, then Q based method described above is used to design narrowband L matching networks. But usually the termination impedances are complex. Then absorption and resonance methods can be used.

The conjugate matching condition can be met by using two-element L matching networks [1]. But the loaded Q of the circuit is fixed by the given termination impedances. But it has been shown in the literature that if the three-element Pi or T matching networks seen in Fig. 2 are used, the designer can select the loaded Q of the circuit. Also Pi or T networks have a wider matchable region [6-8] than L networks.

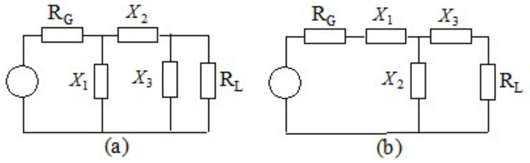

Fig. 2. Three-element Pi and T matching networks.

Pi and T networks can be regarded as two “back-to-back” L networks with a virtual resistor between two L networks. So these two L networks match the generator and load impedances to the virtual resistance. So each L network in Pi or T networks can be designed by using Q based method described above.

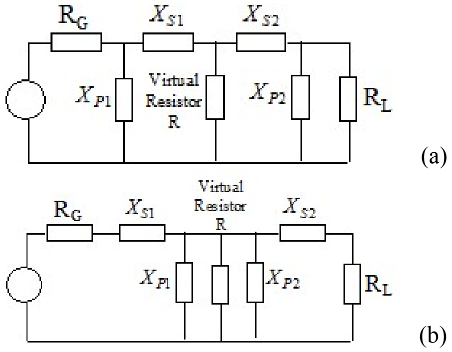

Fig. 3. a) The Pi network as two back-to back L networks., b) The T network as two back-to-back L networks.

For Pi matching networks as seen in Fig. 3a, the virtual resistor and X_{S1} form a series section, while X_{P1} and R_G form a parallel section. Similarly the virtual resistor and X_{S2}

form a series section, and X_{P2} and R_L form a parallel section. For T matching networks (Fig. 3b), the virtual resistor and X_{P1} form a parallel section, while X_{S1} and R_G form a series section. On the other hand, the virtual resistor and X_{P2} form a parallel section, and X_{S2} and R_L form a series section.

There are lots of studies about this kind of matching networks in the literature. The Q based method is studied systematically to design Pi matching networks in [4,5]; design formulae based on the loaded Q are analytically obtained. A method for determining the matching domain of impedance matching networks is presented in [6,13]. The studies are focused on the Pi networks, but the proposed method is applicable to the T networks.

The analysis and design considerations for lumped element matching networks operating at high efficiency are presented in [14]. In this work, only low-pass and high-pass single stage L matching networks are studied, since Pi and T matching networks have lower efficiency than an equivalent L matching network [15].

In [16], a tunable impedance matching network is applied to achieve very widely tunable antennae. In this work, a T matching network is selected since Pi and T networks are confined to carry out high selectivity and a wide scope of tunable impedance dynamic ranges.

2. The Proposed Approach

Usually the generator and load impedances are given as measurement values at the interested frequency. Suppose they are measured as

$$Z_G = R_G + jX_G, \tag{2a}$$

$$Z_L = R_L + jX_L. \tag{2b}$$

These impedances can be regarded as a resistor (R_G or R_L) and a reactive element connected in series (Fig. 4), whose reactance is X_G or X_L at the matching frequency.

Suppose a Pi network will be designed. As seen in Fig. 3a, the series sections (the virtual resistor- X_{S1} section and the virtual resistor- X_{S2} section) are already exist. So the generator impedance- X_{P1} section and the load impedance- X_{P2} section must form the parallel sections. Then it is clear that both of the given termination impedance values must be modeled as a parallel configuration. For instance, a complex termination impedance ($Z = R + jX$) can be modeled as a resistor (R_p) and a reactive element (X_p) connected in parallel as seen in Fig. 4.

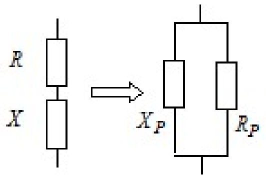

Fig. 4. Series to parallel transformation.

The new element values seen in Fig. 4 can be calculated as $R_P = R(Q^2+1)$ and $X_P = X\left(\dfrac{Q^2+1}{Q^2}\right)$ where $Q = X/R$, respectively.

Now we have the necessary series and parallel sections in each L networks, which can be designed via Q based method. But the explained series to parallel transformation is not required for T matching network design since the termination impedances and X_{S1}, X_{S2} reactances already form the series sections of the L networks, and the virtual resistor, X_{P1} and X_{P2} form the parallel sections as seen in Fig. 3b.

In the next section, the given examples will illustrate the utilization of the proposed approach.

3. Examples

Suppose at $1GHz$, the generator and load impedances are measured as $Z_G = 50 + j6.2832\Omega$, $Z_L = 71.6957 - j45.0477\Omega$, respectively. A Pi matching network with $Q = 5$ is desired.

The given generator impedance value can be modeled as a resistor ($R_{G,P} = 50.7896\Omega$) connected in parallel with an inductor ($L_{G,P} = 64.326nH$), and the given load impedance value can be modelled as a resistor ($R_{L,P} = 100\Omega$) connected in parallel with a capacitor ($C_{L,P} = 1pF$).

Since $R_{L,P} > R_{G,P}$, first calculate X_{S2} and X_{P2} values. The virtual resistor value can be found as

$R = \dfrac{R_{L,P}}{Q^2+1} = \dfrac{100}{5^2+1} = 3.8462$. Then

$$X_{P2} = \frac{R_{L,P}}{Q} = \frac{100}{5} = 20\Omega$$

$$X_{S2} = Q \cdot R = 5 \cdot 3.8462 = 19.2308\Omega$$

Now let us calculate the Q of the generator side L network as follows:

$Q_{new} = \sqrt{\dfrac{R_{G,P}}{R} - 1} = \sqrt{\dfrac{50.7896}{3.8462} - 1} = 3.4936$. Then

$$X_{P1} = \frac{R_{G,P}}{Q_{new}} = \frac{50.7896}{3.4936} = 14.5379\Omega$$

$$X_{S1} = Q_{new} \cdot R = 3.4936 \cdot 3.8462 = 13.4371\Omega$$

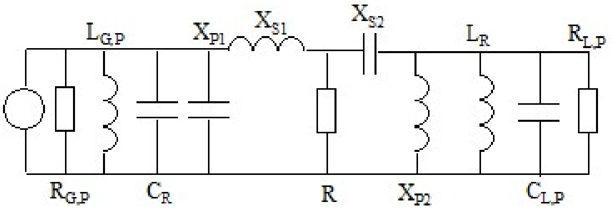

Fig. 5. Calculated element values for Pi network.

As seen in Fig. 5, we need to connect a parallel capacitor ($C_R = 0.39378pF$) to resonate the generator inductor ($L_{G,P} = 64.326nH$). So X_{P1} must be a capacitor and X_{S1} must be an inductor. Also we need a parallel inductor ($L_R = 25.33nH$) to resonate the load capacitor ($C_{L,P} = 1pF$). So X_{P2} must be an inductor and X_{S2} must be a capacitor. Therefore the component values can be computed as $X_{P1} = 10.948pF$, $X_{S1} = 2.1386nH$, $X_{S2} = 8.2760pF$,

$X_{P2} = 3.1831 nH$. After combining X_{P1} and C_R , X_{S1} and X_{S2} , X_{P2} and L_R , the element values (X_1 , X_2 and X_3 , respectively) are obtained as seen in Fig. 6. Transducer power gain curve of the matched system is given in Fig. 9.

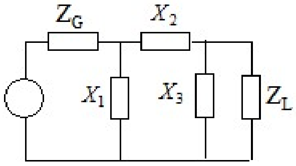

Fig. 6. Designed Pi matching network ($X_1 = 11.341 pF$, $X_2 = 27.47 pF$, $X_3 = 2.8278 nH$).

Now let us design a T network. The given generator and load impedance values can be modelled as a resistor ($R_G = 50\Omega$ and $R_L = 71.6957\Omega$) connected in series with an inductor ($L_G = 1nH$) and a capacitor ($C_L = 3.533 pF$), respectively.

Since $R_G < R_L$, first calculate X_{S1} and X_{P1} values. The virtual resistor value can be found as

$R = R_G(Q^2 + 1) = 50(5^2 + 1) = 1300\Omega$. Then

$$X_{P1} = \frac{R}{Q} = \frac{1300}{5} = 260\Omega$$

$$X_{S1} = Q \cdot R_G = 5 \cdot 50 = 250\Omega$$

Now let us calculate the Q of the load side L network as follows:

$Q_{new} = \sqrt{\frac{R}{R_L} - 1} = \sqrt{\frac{1300}{71.6957} - 1} = 4.1391$. Then

$$X_{P2} = \frac{R}{Q_{new}} = \frac{1300}{4.1391} = 314.0776\Omega$$

$$X_{S2} = Q_{new} \cdot R_L = 4.1391 \cdot 71.6957 = 296.7557\Omega$$

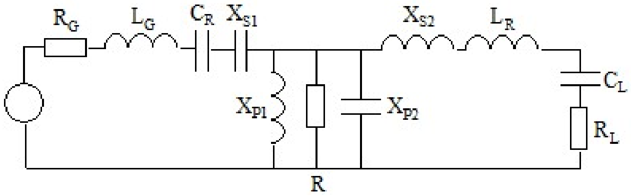

Fig. 7. Calculated element values for T network.

As seen in Fig. 7, we need to connect a series capacitor ($C_R = 25.33 pF$) to resonate the generator inductor ($L_G = 1nH$). So X_{P1} must be an inductor and X_{S1} must be a capacitor. Also we need a series inductor ($L_R = 7.1696 nH$) to resonate the load capacitor ($C_L = 3.533 pF$). So X_{P2} must be a capacitor and X_{S2} must be an inductor Therefore the component values can be computed as $X_{P1} = 41.38 nH$, $X_{S1} = 0.63662 pF$, $X_{S2} = 47.23 nH$, $X_{P2} = 0.50674 pF$. After combining X_{S1} and C_R , X_{P1} and X_{P2} , X_{S2} and L_R , the element values (X_1 , X_2 and X_3 , respectively) are obtained as seen in Fig. 8.

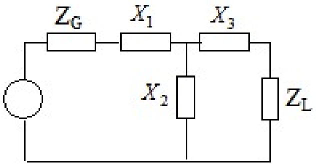

Fig. 8. Designed T matching network ($X_1 = 0.6210 pF$, $X_2 = 240.34 nH$, $X_3 = 54.4 nH$).

Transducer power gain curve of the matched system is given in Fig. 9.

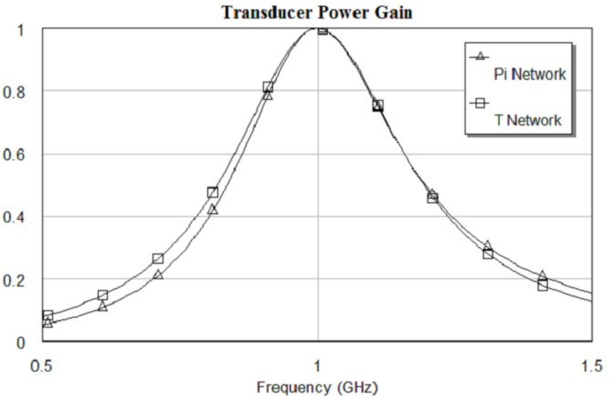

Fig. 9. Transducer power gain curves of the matched systems.

4. Conclusions

Two-element L networks are the simplest and most widely used narrow band impedance matching circuits. If the termination impedances are purely resistive, then Q based method is used to design this kind of networks. Since the conjugate matching condition can be met by using two-element L networks, it is possible to transfer maximum power at the interested frequency. But the loaded Q of the circuit is fixed by the given termination impedances. So if the designer wants to select the Q of the circuit, three-element Pi or T networks must be used.

Generally the generator and load termination impedances are described by means of real frequency measurement results, and these impedances can be modeled as a resistor and a reactive element connected in series. So before stating to design Pi matching networks, this series impedance models must be transformed to parallel models. But in the design of T matching networks, the given series models can be utilized, there is no need a transformation step. Then since Pi or T matching networks can be considered as two “back-to-back” L networks with a virtual resistor between two L networks, they can be designed via Q based method which is well defined in the literature for the design of L networks.

Here after explaining the proposed real frequency design of Pi and T matching networks with complex terminations, two examples are given to show the utilization of the approach.

5. References

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