Outage Performance Analysis of Two-Way AF Relaying over $\alpha - \mu$ Fading Distribution

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Abstract

In this study, we analysis the performance of the two-way amplify-and-forward (AF) relaying scheme over alpha-mu fading channels in wireless communication systems. We evaluate the expression of the cumulative distribution function over alpha-mu fading channels with the approach of the integral region based geometric analysis. Further, we derive the closed form expressions for the system outage probability for both the upper and lower bounds in the considered system model.

1. Introduction

Cooperative relaying which depends on cooperation among distributed single-antenna wireless terminals has emerged recently much attention to prevent a loss in the spectral efficiency due to half-duplex transmission [1-4]. In order to mitigate such a loss and to improve wireless communication performance, two-way relaying schemes have been proposed in the literatures [5-9]. In these schemes two nodes exchange their information bidirectionally in two time slots. There are various cooperative relaying methods, and two of the most well-known are amplify-and-forward (AF) and decode-and forward (DF) protocols [10-14]. Among them, the AF cooperation protocol is considered in this paper, due to its low complexity, in which the relay terminal retransmits a scaled version of the received signal bidirectionally, obtained from the source terminals without any processing.

A set of work employing two-way relay networks is presented in the literature. Performance analysis for two-way AF relaying in N-Nakagami-m fading and cascaded generalized-K fading channels are investigated in the papers [5, 6]. The overall outage and symbol error probability of a two-way AF relaying system are also considered for Rayleigh fading channel in [7, 8]. Another paper presented the results for the performance of an asymmetric two-way AF relaying network in Rician fading environments [9]. The author in [15] investigate the error rate performance of two-way AF relaying communication system over independently but not necessarily identically distributed (i.n.i.d.) cascaded Nakagami-m fading channels. In [16], the authors studied the system outage analysis for multiuser two-way relaying in mixed Rayleigh and Rician fading. The closed form expressions for the outage probability, average symbol error rate, and ergodic capacity are derived for two-way AF system in mixed Rician and Nakagami-m fading environment, presented in [17]. Besides, a dual-hop underlay cognitive relaying network with interference constraints over alpha-mu ($\alpha - \mu$) fading channels is studied in [18]. In this paper, the authors analytically derived the outage probability and sysmbol error rate expressions. On the other hand, the paper in [19] introduced the $\alpha - \mu$ distribution. In this study, the author provided the detailed closed form expressions for the statistical fading parameters. In addition, an approximate formulations for the outage probability of equal-gain receivers over $\alpha - \mu$ fading channels have been provided in [20]. In the same work, the outage probability versus normalized signal-to-interference power ratio is illustrated for different α and μ values

Motivated by the aforementioned studies, first, our paper presents a completely analytical approach in studying the performance of the AF-based cooperative two-way relay networks over $\alpha - \mu$ fading channel. This general fading distribution covers the Nakagami-m, Rayleigh, Weibull, one-sided Gaussian and negative exponential distributions as special cases. Second, and more importantly, we generate a closed form expressions for the system outage probability covering both the upper and lower bounds. To the best of the authors knowledge, our main contributions stated above, are not studied yet in the literature.

The rest of this paper is organized as follows: In Section II, the system model is presented. In Section III, we focus on the derivation of the system outage probability where both links are subject to $\alpha - \mu$ fading distribution. The exact closed form expressions for the overall outage probability for both the upper and lower bounds in the considered system are obtained in this section. Finally, Section V concludes this paper.

2. System Model

We consider a single-relay two-way AF relaying cooperative scenario (as shown in Fig.1) where the source terminals (S_1 and S_2) and a relay terminal (R) operate in half-duplex mode and are equipped with a single pair of transmit and receive antennas. We assume that, the transmission is divided in two phases. In the first phase, the source terminals transmit their signals, x_{S_1} and x_{S_2}, simultaneously to the relay terminal, and there is no direct connection between S_1 and S_2. The received signal at the relay terminal can be given by [5]

$$y_R = h_{S_1}\sqrt{P_{S_1}}x_{S_1} + h_{S_2}\sqrt{P_{S_2}}x_{S_2} + n_R \qquad (1)$$

with the transmitted signal power P_{S_1} and P_{S_2} from the source terminals, S_1 and S_2, respectively. In here, $n_R \sim CN(0, N_0)$ is the additive white Gaussian noise (AWGN) term with zero-mean and variance of N_0 at the relay terminal. The complex channel fading coefficients between $S_1 \rightarrow R$, and $R \rightarrow S_2$ links, whose magnitudes follow $\alpha - \mu$ distribution are denoted

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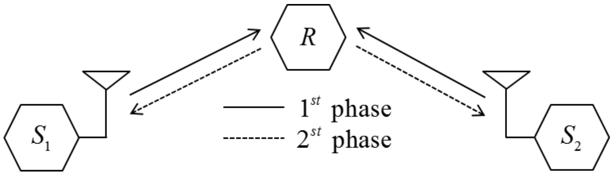

Figure 1. System model for two-way AF relaying

with h_{S_1} and h_{S_2}, respectively. In the second phase, the relay terminal scales the combined signal y_R with the amplifying gain $G = \sqrt{P_R / \left(P_{S_1}\left|h_{S_1}\right|^2 + P_{S_2}\left|h_{S_2}\right|^2\right)}$, and broadcasts to the source terminals. Here, P_R indicates the transmitted power at the relay terminal. It is assumed that both S_1 and S_2 have knowledge about their own sent information and are able to remove the back-propagating self-interference from the superimposed signals. In that case the resultant instantaneous SNRs (Signal-to-Noise Ratio) at the source terminals, S_1 and S_2 can be given by [5, 7]:

$$\gamma_{S_1} = \frac{P_R P_{S_2}\left|h_{S_1}\right|^2\left|h_{S_2}\right|^2 / N_0}{\left(P_R + P_{S_1}\right)\left|h_{S_1}\right|^2 + P_{S_2}\left|h_{S_2}\right|^2}, \tag{2}$$

$$\gamma_{S_2} = \frac{P_R P_{S_1}\left|h_{S_1}\right|^2\left|h_{S_2}\right|^2 / N_0}{\left(P_R + P_{S_2}\right)\left|h_{S_2}\right|^2 + P_{S_1}\left|h_{S_1}\right|^2}, \tag{3}$$

respectively.

3. System Outage Probability Analysis

Without loss of generality, we assume that $\left(\frac{P_{S_1}}{N_0}\right) = \left(\frac{P_{S_2}}{N_0}\right) = \left(\frac{P_R}{N_0}\right) \triangleq \Omega$. The system outage event is occurred when the instantaneous SNR of the bidirectional links between the source and the relay, drops below a predefined threshold, γ_{th}. Then, the outage probability, P_{out} can be expressed as:

$$P_{out} = 1 - Pr\left[\gamma_{S_1} > \gamma_{th}, \gamma_{S_2} < \gamma_{th}\right] \tag{4}$$

where $\gamma_{th} = 2^{2R_{th}} - 1$ and R_{th} is the required information rate for the receiving terminal. In this context, we consider the method developed in [5-7]. We enlarge the integral region from OEMB to OEPMRB (as shown in Fig. 2) for the lower bound system outage probability, then we obtain P_{out}^{LB} as

$$\begin{aligned} P_{out}^{LB} >& 1 - F_{Y_{S_1}}\left(\frac{\Omega}{2\gamma_{th}}\right) F_{Y_{S_2}}\left(\frac{\Omega}{3\gamma_{th}}\right) \\ & - F_{Y_{S_1}}\left(\frac{\Omega}{3\gamma_{th}}\right)\left[F_{Y_{S_2}}\left(\frac{\Omega}{2\gamma_{th}}\right) - F_{Y_{S_2}}\left(\frac{\Omega}{3\gamma_{th}}\right)\right] \end{aligned} \tag{5}$$

where $F_{Y_i}(y_i)$ is the cumulative distribution function (CDF) of Y_i, for $i = S_1, S_2$, and defined by [20]

$$F_{Y_i}(y_i) = \frac{\Gamma\left(\mu, \mu y_i^\alpha / \hat{y}_i^\alpha\right)}{\Gamma(\mu)} \tag{6}$$

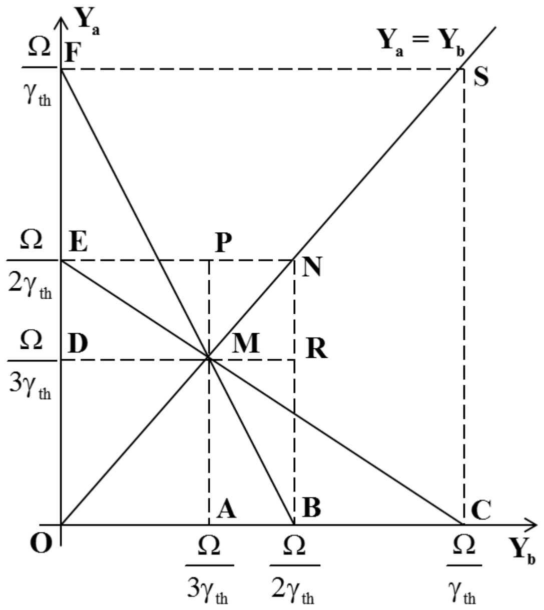

Figure 2. Integral region based geometric analysis to evaluate the overall outage probability [6]

and evaluated as follows, for the specified y_i values:

$$\begin{aligned} F_{Y_{S_1}}\left(\frac{\Omega}{3\gamma_{th}}\right)\Big|_{\alpha-\mu} &= \Gamma\left(\mu, \frac{\mu\left(\frac{\Omega}{3\gamma_{th}}\right)^\alpha}{E\left[\left(\frac{1}{\left|h_{S_1}\right|^2}\right)^\alpha\right]}\right)\Bigg/\Gamma(\mu) \\ F_{Y_{S_2}}\left(\frac{\Omega}{3\gamma_{th}}\right)\Big|_{\alpha-\mu} &= \Gamma\left(\mu, \frac{\mu\left(\frac{\Omega}{3\gamma_{th}}\right)^\alpha}{E\left[\left(\frac{1}{\left|h_{S_2}\right|^2}\right)^\alpha\right]}\right)\Bigg/\Gamma(\mu) \\ F_{Y_{S_1}}\left(\frac{\Omega}{2\gamma_{th}}\right)\Big|_{\alpha-\mu} &= \Gamma\left(\mu, \frac{\mu\left(\frac{\Omega}{2\gamma_{th}}\right)^\alpha}{E\left[\left(\frac{1}{\left|h_{S_1}\right|^2}\right)^\alpha\right]}\right)\Bigg/\Gamma(\mu) \\ F_{Y_{S_2}}\left(\frac{\Omega}{2\gamma_{th}}\right)\Big|_{\alpha-\mu} &= \Gamma\left(\mu, \frac{\mu\left(\frac{\Omega}{2\gamma_{th}}\right)^\alpha}{E\left[\left(\frac{1}{\left|h_{S_2}\right|^2}\right)^\alpha\right]}\right)\Bigg/\Gamma(\mu). \end{aligned} \tag{7}$$

In (6), $\hat{y}_i = \sqrt[\alpha]{E\left(Y_i^\alpha\right)}$, and $E\left(Y_i^\alpha\right) = \frac{\Gamma(\mu + 1)}{\mu\Gamma(\mu)}$ [20]. $\Gamma(.)$ is the Gamma function, $\Gamma(.,.)$ is the incomplete Gamma function [21, 22], $\alpha\,(\alpha > 0)$ is related to the nonlinearity of the environment, and $\mu\,(\mu > 0)$ is associated to the multipath clusters for the $\alpha - \mu$ fading distribution.

In a similar manner with the lower bound, the upper bound system outage probability, P_{out}^{UB} can be written as in the following closed-form expression [5, 7]:

$$P_{out}^{UB} < 1 - F_{Y_{S_1}}\left(\frac{\Omega}{3\gamma_{th}}\right) F_{Y_{S_2}}\left(\frac{\Omega}{3\gamma_{th}}\right). \tag{8}$$

Using the necessary CDF terms in (7), and substitute into (5) and (8), after some algebraic manipulations, P_{out}^{LB} and P_{out}^{UB} can be derived in the following forms:

$$
\begin{aligned}
P_{out}^{LB} > 1 - &\frac{\Gamma\left(\mu, \dfrac{\mu\left(\dfrac{\Omega}{2\gamma_{th}}\right)^{\alpha}}{E\left[\left(\dfrac{1}{|h_{S_1}|^2}\right)^{\alpha}\right]}\right)}{\Gamma(\mu)} \\
\times &\frac{\Gamma\left(\mu, \dfrac{\mu\left(\dfrac{\Omega}{3\gamma_{th}}\right)^{\alpha}}{E\left[\left(\dfrac{1}{|h_{S_2}|^2}\right)^{\alpha}\right]}\right)}{\Gamma(\mu)} - \frac{\Gamma\left(\mu, \dfrac{\mu\left(\dfrac{\Omega}{3\gamma_{th}}\right)^{\alpha}}{E\left[\left(\dfrac{1}{|h_{S_1}|^2}\right)^{\alpha}\right]}\right)}{\Gamma(\mu)} \\
\times &\left[\frac{\Gamma\left(\mu, \dfrac{\mu\left(\dfrac{\Omega}{2\gamma_{th}}\right)^{\alpha}}{E\left[\left(\dfrac{1}{|h_{S_2}|^2}\right)^{\alpha}\right]}\right)}{\Gamma(\mu)} - \frac{\Gamma\left(\mu, \dfrac{\mu\left(\dfrac{\Omega}{3\gamma_{th}}\right)^{\alpha}}{E\left[\left(\dfrac{1}{|h_{S_2}|^2}\right)^{\alpha}\right]}\right)}{\Gamma(\mu)}\right],
\end{aligned}
\tag{9}
$$

$$
\begin{aligned}
P_{out}^{UB} < 1 - \Gamma&\left(\mu, \frac{\mu\left(\dfrac{\Omega}{3\gamma_{th}}\right)^{\alpha}}{E\left[\left(\dfrac{1}{|h_{S_1}|^2}\right)^{\alpha}\right]}\right)\Bigg/\Gamma(\mu) \\
\times \Gamma&\left(\mu, \frac{\mu\left(\dfrac{\Omega}{3\gamma_{th}}\right)^{\alpha}}{E\left[\left(\dfrac{1}{|h_{S_2}|^2}\right)^{\alpha}\right]}\right)\Bigg/\Gamma(\mu),
\end{aligned}
\tag{10}
$$

respectively.

4. Numerical Results

In this section we provide the outage probability for both the lower and upper bounds of a two-way AF relaying scheme for different values of $\alpha - \mu$ fading parameters. For the numerical results, the special functions and transforms, can be evaluated with the software packages, such as Mathematica and Matlab.

Fig 3. shows the system outage probability against the SNR values for varying information rates of R_{th} over $\alpha - \mu$ fading distribution. In this figure, the overall outage probability is compared with one another for different R_{th} values of $\{0.25, 0.5,$ and $1\}$ bps/Hz, respectively. In here, we set $\alpha = 3$ and $\mu = 2$. It can be seen from the figure that the system performance is improved with the increase of R_{th} value.

The outage probability performance is also demonstrated for Nakagami-m $(\alpha = 2)$, Weibull $(\mu = 1)$, and Rayleigh $(\alpha = 2, \mu = 1)$ fading channels for a fixed value of R_{th} = 1 bps/Hz in Fig. 4. This figure shows, the impact of the $\alpha - \mu$ fading parameters on the outage performance is vitally important.

5. Conclusion

In this paper, we have studied on the outage performance analysis of two-way AF Relaying over $\alpha - \mu$ fading channel. By using integral region based geometric analysis, we evaluate some closed-form equations for the cumulative distribution function. Moreover, the exact closed form expressions for both the lower and upper bounds for the system outage probability over $\alpha - \mu$ fading channel are derived. It is worth noting that the potential of the two-way AF relaying method can be further utilized. Other applications, such as bit error rate analysis of this system model would be useful and are considered for the future studies.

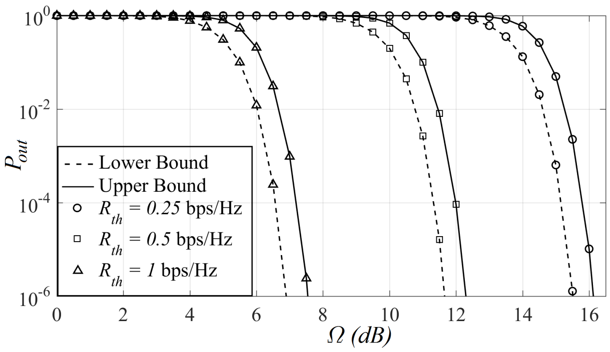

Figure 3. Outage probability performance for two-way AF relaying system with different data rates over $\alpha - \mu$ fading.

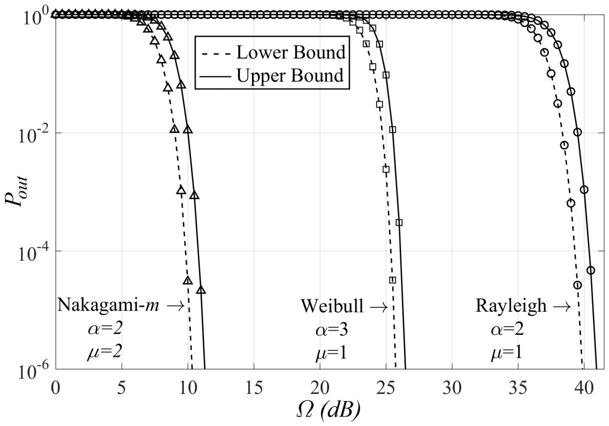

Figure 4. Outage probability performance versus SNR for two-way AF relaying system with R_{th}=1 bps/Hz over $\alpha - \mu$ fading.

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