Sparse Direction-of-Arrival Estimation for Two Sources with Constrained Antenna Arrays

Saleh A. Alawsh[1], Ali H. Muqaibel[2], and Mohammad S. Sharawi[3]

Electrical Engineering Department, King Fahd University of Petroleum & Minerals (KFUPM), Dhahran, Saudi Arabia
{salawsh[1], muqaibel[2], msharawi[3]}@KFUPM.edu.sa

Abstract

Compressive sensing (CS), multiple signal classification (MUSIC), and estimation of signal parameter via rotational invariance techniques (ESPRIT) are among the main used estimation techniques for direction of arrival (DOA). Though, the practical implementation of DOA techniques in handheld wireless devices is limited by the number of antennas and the spacing between them. A robust DOA estimation technique is needed to overcome the different impairments in the communication channel. This paper mainly focuses on DOA estimation of two sources in the presence of practical limitations. A comparison between important DOA estimation algorithms is presented including: Beamforming, Capon, MUSIC, and First-norm singular value decomposition (l_1-SVD).

1. Introduction

A set of antenna/sensor elements arranged in a certain geometry is called an antenna/sensor array. Antenna arrays can enhance antenna directivity, in return enhancing the signal to noise ratio (SNR) as well as providing the system with some control over the maximum radiation power. The beam of the array can be steered towards certain directions which enhances its direction-of-arrival (DOA) capability [1]. Many array configurations were investigated in the literature for DOA estimation. Each configuration has its own advantages and limitations. Very few publications considered the practical limitations in the presence of two sources. There are many DOA algorithms such as Beamforming, Capon, subspace-based techniques, etc. The most widely ones are reviewed and compared.

Wideband source localization using beamforming was considered in [2], [3], [4]. DOA estimation using Capon algorithm was originally developed by [5]. Based on time reversal for multiple-input-multiple-output (MIMO) radars, Capon algorithm was introduced in [6].

Subspace-based techniques such as estimation of signal parameters via rotational invariance technique (ESPRIT) [7]–[10] and multiple signal classification (MUSIC) were considered in [11]–[14]. The authors in [8] modified the ESPRIT algorithm and used the singular value decomposition (SVD) instead of the Eigen value decomposition (EVD) to estimate the signal subspace. As a result, the performance was enhanced even in the presence of interference. A two-stage MUSIC algorithm was proposed in [12]. MUSIC algorithm was also used in [13] and extended in [15] to estimate the DOAs and the power of the far-field sources.

Since the received signal is sparse in some domains, different approaches were developed for DOA estimation based on compressive sensing (CS) [16]–[20] which works at sub-Nyquist sampling rates [16]. In [17], the authors used sparse Bayesian learning when the number of the unknown sources is greater than the number of measurements. The work in [18] estimated the DOA by solving a set of Basis Pursuit De-Noising (BPDN) problems. In addition, grid position refinement was used to reduce the complexity of the BPDN problem.

Narrowband DOA estimation based on the First-norm SVD (l_1-SVD) was investigated in [21]–[23]. All l_1-SVD algorithms use CS first, so they can be considered also as CS based algorithms. In [21], l_1-SVD was proposed for multiple time or frequency samples such that sharp estimate and super-resolution of the DOA is achieved. Reference [23] suggested a modified l_1-SVD algorithm which has an improved performance in the presence of interference.

In mobile handsets, the number of antennas [24] and the inter-element spacing are restricted by the available physical size. Very close antenna elements suffer from the mutual coupling effect [25], [26]. While, the grating lobe problem appears if the spacing increases beyond half-wavelength. Estimation with more antennas requires large storage and increased processing capability. Under the given restrictions, researchers examined different inter-element spacing to optimize the DOA estimation. The authors in [27] suggested antenna spacing of 0.1λ and used impedance matching to avoid the degradation with such close spacing. Others examined different configurations for distributing the antennas within an array while maintaining a minimum distance between the elements [28]. In [19], the minimum adjacent antenna separations were evaluated by exploiting the antenna size as a constraint. The authors in [29], [30] used only two antennas to estimate the DOA. The MUSIC algorithm based system in [29] utilized only one radio frequency to reduce the complexity. A switch was used to change from one element to the other. In [30], preprocessing the received signal and performing data reduction was used. The generalized least squares estimator was used for DOA estimation.

Developing a receiver with DOA estimation capability is restricted by the physical size of the handsets. Based on the literature review, very few researches have considered DOA estimation of two sources with limited number of antennas and limited inter-element spacing which is the main scope of this paper. The performance is investigated for different number of antenna elements and different inter-element spacing using different performance criteria. In [31], a single source was considered and estimated through search based-DOA estimation algorithms. This paper considers two sources and investigates the ability of the algorithms to resolve them in the presence of such limitations.

The rest of this paper is organized as follows. The considered system model is introduced in Section 2. In Section 3, some performance metrics for DOA estimation are presented. Simulation results are presented in Section 4. Finally, the paper is concluded in Section 5.

2. System Model

Let us consider K narrowband active sources $u_i(t)$, $i = 1,..,K$ located in the far-field with θ_i as the DOA impinging on an array of M equally spaced omnidirectional antennas as in [31]. The received corrupted measurements with additive white Gaussian noise, $\mathbf{n}(t)$, at the output of the array can be written as:

$$\mathbf{y}(t) = \mathbf{A}(\boldsymbol{\theta})\mathbf{u}(t) + \mathbf{n}(t), \quad t \in \{1,2,..,T\} \tag{1}$$

where $\mathbf{A}(\boldsymbol{\theta}) = [\mathbf{a}(\theta_1),..,\mathbf{a}(\theta_K)]$ is the steering matrix with steering vector $\mathbf{a}(\theta_i) = \left[1, e^{\frac{j2\pi d\cos(\theta_i)}{\lambda}}, e^{\frac{j4\pi d\cos(\theta_i)}{\lambda}}, .., e^{\frac{j2\pi d(M-1)\cos(\theta_i)}{\lambda}}\right]'$, T is the number of samples or snapshots, d represents the inter-element spacing and λ is the carrier wavelength. The unknown DOAs are represented by $\boldsymbol{\theta} = [\theta_1, \theta_1, \ldots, \theta_K]'$ and $\mathbf{u}(t) = [u_1(t),..,u_K(t)]'$ is the transmitted signals. Given $\mathbf{y}(t)$, we have to find $\widehat{\boldsymbol{\theta}}$, which is an estimate of $\boldsymbol{\theta}$ and the number of active sources, K.

3. Performance Metrics

There are many performance metrics that have been used to assess DOA estimation. The most widely used in the literature are discussed in the following subsections.

A. Root Mean Squared Error (RMSE)

One of the most widely performance metrics used in the literature to assess the DOA estimation is the root mean squared error (RMSE). The RMSE of the estimated DOA is defined as:

$$\text{RMSE} = \sqrt{\frac{1}{K}\sum_{i=1}^{K}\left(\theta_i - \hat{\theta}_i\right)^2} \tag{2}$$

where θ_i, and $\hat{\theta}_i$ are the actual and the estimated DOA, respectively, and K is the number of sources to be localized.

B. Bias

The bias can be defined as the difference between the actual location of a source and its estimated location [21], [32], as:

$$Bias_i = \left(\theta_i - \hat{\theta}_i\right) \tag{3}$$

C. Sources Resolvability

This metric gives an indication on how certain DOA algorithm is capable to resolve two sources and can be calculated based on the following. Two sources are resolvable if the following condition is satisfied [2], [33]:

$$\left|\theta_i - \hat{\theta}_i\right| \leq \frac{\Delta\theta}{2} \tag{4}$$

where $\Delta\theta = |\theta_2 - \theta_1|$ and θ_i and $\hat{\theta}_i$ are the actual and the estimated DOAs of two sources for $i = 1,2$, respectively.

4. Comparative Performance Evaluation

In this section we present some results for DOA estimation using different algorithms in which the effects of the most important parameters presented in the system model are investigated. Two narrowband and uncorrelated sources are assumed to be located in the far-field with discrete uniform DOA angle distribution, $\theta_k \sim U[1^{\circ}, 180^{\circ}]$. A uniform linear array (ULA) is considered with $d = \lambda/2$ and the number of samples is $T = 200$ samples. The search grid is uniform with 1° step size. All these parameters are fixed unless stated otherwise. All the performance measures are calculated based on 500 independent runs that are averaged afterwards.

A. Impact of the SNR

Fig. 1 displays the RMSE versus SNR for different M. The RMSE is very large when $M = 2$ based on all algorithms and even with SNR = 40 dB. The RMSE using MUSIC algorithm when $M = 2$ is larger than others because $K = M$. Consequently, the noise subspace becomes an empty matrix and degradation occurs. Therefore, two antenna elements are not enough to estimate the locations of two sources.

Since Beamforming has a very wide Beamwidth at the estimated angels and it's search based algorithm, no improvement is noticed by increasing M. Apart from Beamforming algorithm, using more antenna elements improve the performance significantly. Increasing the SNR reduces the RMSE as well, though this improvement is negligible beyond SNR = 20 dB using l_1-SVD algorithm. After this SNR, simulation proves that Capon and MUSIC algorithms realize quite better RMSE.

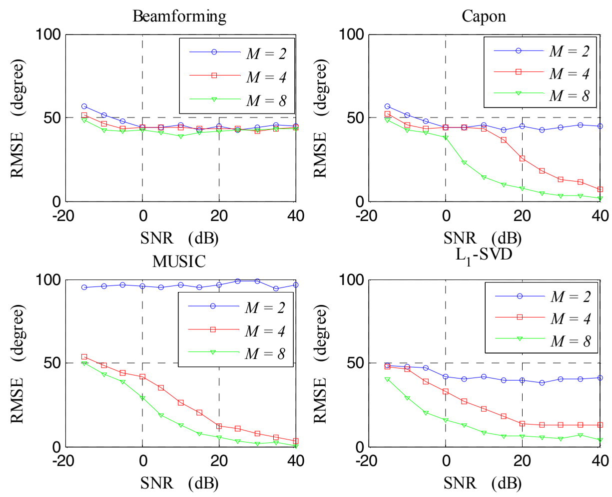

Fig. 1. The RMSE as function of SNR for different M

B. Impact of the Ratio between the Inter-element Spacing and Wavelength

Again, the RMSE is used since it is the most widely used performance measures in the literature. The RMSE is plotted versus $\frac{d}{\lambda} = \frac{1}{8}, \frac{1}{6}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, 1, \frac{4}{3}$ and 2 with SNR =20 dB in Fig. 2. The RMSE for MUSIC and l_1-SVD decreases as $\frac{1}{8} \leq d/\lambda < \frac{1}{3}$ since the mutual effect reduces. On the other hand, the RMSE increases when $\frac{d}{\lambda} > \frac{1}{3}$ because the ambiguity increases due to the grating lobes. We have got almost similar observations as in [31].

C. Impact of the Number of Samples

The effect of the number of samples can be evaluated using the probability of source resolvability. The two sources are adjusted to be located at $\boldsymbol{\theta} = [60,80]^o$ as shown in Fig. 3 using $M = 4,8$ and the SNR = 10, 20 dB. The probability of source resolution is plotted against $T = 2, 20, 50, 100, 150$ and 200 samples. Apart from l_1-SVD algorithm, using only two samples is not enough to achieve a good probability of detection. However, using l_1-SVD the probabilities with only two samples are much greater than all other algorithms, see Fig. 3. Capon algorithm needs large SNR and large number of samples in order to resolve the two sources perfectly. Beamforming algorithm has got the worst performance among all algorithms.

D. Impact of the Sources Separation

The bias as a function of the angular separation between the two sources with $M = 8$ is plotted in Fig. 4. The first source is fixed at 42 degree while the second one is changing. For the case when the SNR = 10 dB (solid lines), 20 dB (dotted lines), we observe some bias for low separations. Though this bias vanishes as Fig. 4 indicates when the SNR = 10 dB at around 16, 14 and 12 degrees for Capon, MUSIC and l_1-SVD algorithms, respectively. On the other hand, the bias increases with separations using Beamforming algorithm since it is a search based algorithm and it has a very wide beamwidth. Apart from beamforming algorithm, when the SNR increases to 20 dB all biases decrease.

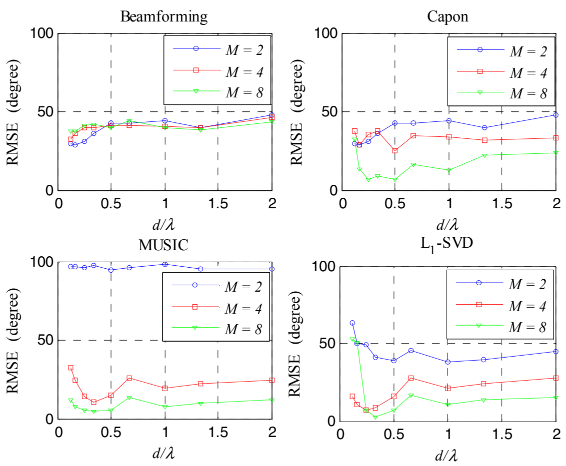

Fig. 2. The RMSE vs d/λ for two sources, SNR = 20 dB, and different M

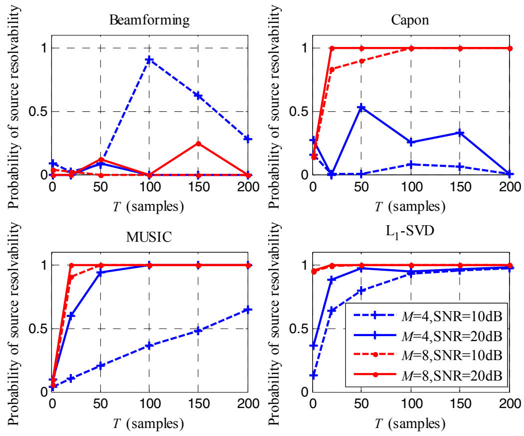

Fig. 3. The probability of source detection as a function of T with SNR=10 dB (dashed lines), SNR=20 dB (solid lines) and $M = 4$, 8 and $\theta = [60,80]^o$

The probability of source resolvability is shown in Fig. 5 versus the SNR. The two sources in Fig. 5(a) are adjusted to be located at $\boldsymbol{\theta} = [60,80]^o$ and we compare $M = 4$ (solid lines) and $M = 8$ (dashed lines). All subplots have the same legend and markers. A probability of 1 is achieved using l_1-SVD at low SNR compared with other algorithms. In order to realize probability of resolvability greater than 0.8, around 7.5 and 10 dB are required using MUSIC and Capon algorithms, respectively, using $M = 8$ elements. Moreover, we have almost the same trend using $M = 4$ elements. Beamforming algorithm can't resolve the two sources.

The two sources in Fig. 5(b) are adjusted to be located at $\boldsymbol{\theta} = [60,100]^o$ and we compare $M = 4$ (solid lines) and $M = 8$ (dashed lines). Again beamforming algorithm merges the two sources and cannot resolve them. Similarly, l_1-SVD is better than MUSIC and Capon algorithms. Comparing the two cases with $M = 4$ and $M = 8$, the differences between them are not as before (smaller) because the sources now are separated by 40 degrees.

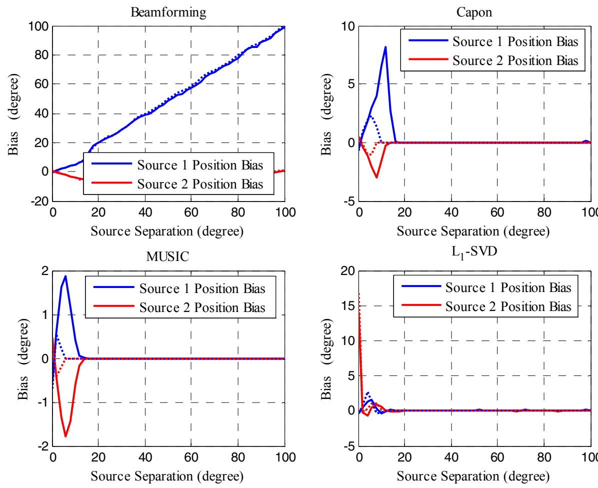

Fig. 4. Bias in localizing two sources as a function of angular separation with SNR =10 dB (solid lines), 20 dB (dotted lines) and $M = 8$

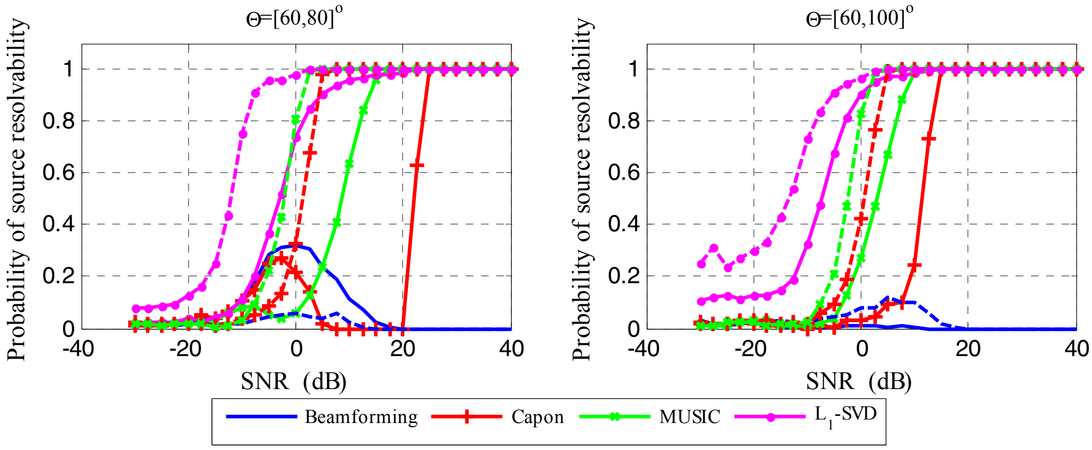

Fig. 5. Probability of source resolvability versus the SNR for $M = 4$ and 8 antenna elements. The solid and the dashed lines represent $M = 4$ and 8 respectively

5. Conclusion

In this work, we presented a comparative study on sparse DOA estimation for two sources with practical antenna arrays. Two practical issues were discussed namely: limited number of antenna elements and limited spacing in between. Different performance measures have been discussed and evaluated. This includes the RMSE, bias and the sources resolvability based on several algorithms. The l_1-SVD algorithm attains super-resolution since the beamwidth at the estimated angle is very narrow. It also utilizes both sparsity and SVD concepts. Thus it can work with a reduced data set and the processing time is reduced dramatically.

Acknowledgment

The authors would like to acknowledge the support provided by the Deanship of Scientific Research (DSR) at King Fahd University of Petroleum & Minerals (KFUPM) for funding this work through project No. IN161015.

References

[1] Mohammad Sharawi, *Printed MIMO Antenna Engineering*. Artech House, 2014.

[2] Y. Tang, X. Ma, W. Sheng, and Y. Han, “A New Transmit Beamforming Algorithm for Subarray MIMO RADAR,” *IEEE Int. Symp. Phased Array Syst. Technol.*, pp. 255–258, Oct. 2013.

[3] M. B. Hawes and W. Liu, “Design of Low-Complexity Wideband Beamformers with Temporal Sparsity,” *2014 9th Int. Symp. Commun. Syst. Networks Digit. Sign*, no. 1, pp. 580–584, Jul. 2014.

[4] M. B. Hawes and W. Liu, “Sparse Array Design for Wideband Beamforming with Reduced Complexity in Tapped Delay-Lines,” *IEEE/ACM Trans. Audio, Speech, Leanguage Process.*, vol. 22, no. 8, pp. 1236–1247, Aug. 2014.

[5] F. Foroozan, A. Asif, and Y. Jin, “Direction Finding Algorithms for Time Reversal MIMO RADARs,” *IEEE Stat. Signal Process. Work.*, pp. 433–436, 2011.

[6] J. Capon, “High-Resolution Frequency-Wavenumber Spectrum Analysis,” *Proc. IEEE*, vol. 57, no. 8, pp. 1408–1418, 1969.

[7] J. He, M. O. Ahmad, and M. N. S. Swamy, “Near-Field Localization of Partially Polarized Sources with a Cross-Dipole Array,” *IEEE Trans. Aerosp. Electron. Syst.*, vol. 49, no. 2, pp. 857–870, 2013.

[8] C. Yu, X. Zhang, Y. Bai, and Z. Du, “DOD-DOA Estimation by Exploiting Signal Cyclostationarity for Bistatic MIMO RADAR,” *IEEE Int. Conf. Signal Process. Commun. Comput. (ICSPCC 2013)*, pp. 1–4, Aug. 2013.

[9] F. Liu and E. Optimization, “An Effective Virtual ESPRIT Algorithm for Multi-target Localization in Bistatic MIMO RADAR System,” *Int. Conf. Comput. Des. Appliations (ICCDA 2010)*, no. Iccda, pp. 412–415, 2010.

[10] H. Jiang, D. Wang, and C. Liu, “Estimation of DOD and 2D-DOA and Polarizations for Bistatic MIMO RADAR,” in *19th Annual Wireless and Optical Communications Conference (WOCC)*, 2010, no. 60901060.

[11] R. Schmidt and X. W. Af, “Multiple Emitter Location and Signal Parameter Estimation,” *IEEE Trans. Antennas Propag.*, no. 3, pp. 276–280, 1986.

[12] J. Liang and D. Liu, “Passive Localization of Mixed Near-Field and Far-Field Sources Using Two-stage MUSIC Algorithm,” *IEEE Trans. Signal Process.*, vol. 58, no. 1, pp. 108–120, Jan. 2010.

[13] J. He, M. N. S. Swamy, and M. O. Ahmad, “Efficient Application of MUSIC Algorithm Under the Coexistence of Far-Field and Near-Field Sources,” *IEEE Trans. Signal Process.*, vol. 60, no. 4, pp. 2066–2070, 2012.

[14] G. H. and S. S. Abeysekera, “A Comparison on DOA Parameter Identifiability for MIMO and Phased-Array RADAR,” in *9th International Conference on Information, Communications and Signal Processing (ICICS)*, 2013, pp. 1–5.

[15] Guohong Liu and Xiaoying Sun, "Efficient Method of Passive Localization for Mixed Far-Field and Near-Field Sources," *IEEE Antennas Wirel. Propag. Lett.*, vol. 12, no. Grant 61171137, pp. 902–905, 2013.

[16] K. Han, Y. Wang, B. Kou, and W. Hong, "Parameters Estimation Using a Random linear Array and Compressed Sensing," *3rd Int. Congr. Image Signal Process.*, no. 1, pp. 3950–3954, Oct. 2010.

[17] O. Balkan, K. Kreutz-delgado, and S. Makeig, "Localization of More Sources Than Sensors via Jointly-Sparse Bayesian Learning," *IEEE Signal Process. Lett.*, vol. 21, no. 2, pp. 131–134, 2014.

[18] C. Liu, Y. V Zakharov, and T. Chen, "Broadband Underwater Localization of Multiple Sources Using Basis Pursuit De-Noising," *IEEE Trans. Signal Process.*, vol. 60, no. 4, pp. 1708–1717, 2012.

[19] W. Liu and M. B. Hawes, "Compressive Sensing-Based Approach to the Design of Linear Robust Sparse Antenna Arrays with Physical Size Constraint," *IET Microwaves, Antennas Propag.*, vol. 8, no. 10, pp. 736–746, Jul. 2014.

[20] A. Manikas, Y. I. Kamil, and and Marc Willerton, "Source Localization Using Sparse Large Aperture Arrays," *IEEE Trans. Signal Process.*, vol. 60, no. 12, pp. 6617–6629, 2012.

[21] D. Malioutov, M. Çetin, and A. S. Willsky, "A Sparse Signal Reconstruction Perspective for Source Localization With Sensor Arrays," *IEEE Trans. Signal Process.*, vol. 53, no. 8, pp. 3010–3022, 2005.

[22] Z. Wang, C. Wang, H. Zhang, Y. Tang, and M. Liu, "SAR Tomography via Sparse Representation of Multiple Snapshots and Backscattering Signals — The L1-SVD Approach," *IEEE Geosci. Remote Sens. Symp.*, pp. 1325–1328, Jul. 2014.

[23] L. Guolong and H. Bo, "New Method of DOA Estimation in the Presence of Interference," *IEEE 11th Int. Conf. Electron. Meas. Instruments*, no. 2, pp. 650–654, Aug. 2013.

[24] M. Martínez-Vázquez, "Considerations for the Design of Antennas Embedded in Mobile Communications Devices," *2010 Loughbrgh. Antennas Propag. Conf. LAPC 2010*, no. November, pp. 36–40, 2010.

[25] H. J. Chaloupka and X. Wang, "On the Properties of Small Arrays with Closely Spaced Antenna Elements," *IEEE Antennas Propag. Soc. Symp. 2004.*, vol. 3, no. 3, pp. 2699–2702, 2004.

[26] C. Lai and S. Chen, "Design Consideration of Closely Spaced Polarization- and Pattern-Diversity Antenna Pair," *IEEE Antennas Propag. Soc. Int. Symp.*, no. 2, pp. 2–3, 2012.

[27] F. a Bhatti, G. B. Rowe, K. W. Sowerby, and C. R. C. M. Silva, "Blind Signal Detection Using a Linear Antenna Array : An Experimental Approach," vol. 63, no. 3, pp. 1135–1145, 2014.

[28] S. Pazos, M. Hurtado, and C. H. Muravchik, "DOA Estimation Using Random Linear Arrays Via Compressive Sensing," *IEEE Bienn. Congr. Argentina*, pp. 219–222, 2014.

[29] A. Gorcin and H. Arslan, "A Two-Antenna Single RF Front-End DOA Estimation System for Wireless Communications Signals," *IEEE Trans. Antennas Propag.*, vol. 62, no. 10, pp. 5321–5333, 2014.

[30] J. Sheinvald and M. Wax, "Direction Finding with Fewer Receivers via Time-Varying Preprocessing," *IEEE Trans. Signal Process.*, vol. 47, no. 1, pp. 2–9, 1999.

[31] S. A. Alawsh, A. H. Muqaibel, and M. S. Sharawi, "DOA Estimation in MIMO Systems With Compressive Sensing for Future Handsets," *2015 IEEE Jordan Conf. Appl. Electr. Eng. Comput. Technol.*, pp. 1–6, 2015.

[32] Y. Tian, X. Sun, and S. Zhao, "DOA and Power Estimation using a Sparse Representation of Second-order Statistics Vector and $\ell 0$ -norm Approximation," *ELSERVIER, Signal Process.*, vol. 105, pp. 98–108, 2014.

[33] A. Khabbazibasmenj, S. A. Vorobyov, A. Hassanien, and M. W. Morency, "Transmit Beamspace Design for Direction Finding in Colocated MIMO RADAR with Arbitrary Receive Array and Even Number of Waveforms," in *Conference Record of the Forty Sixth Asilomar Conference on Signals, Systems and Computers (ASILOMAR)*, 2012, pp. 1307–1311.