Sparse Direction of Arrival Estimation Using Sparse Arrays Based on Software-Defined-Radio Platform

Saleh A. Alawsh[1], Omar A. Al Khazragi[2], Ali H. Muqaibel[3], and Samir N. Al-Ghadhban[4]

King Fahd University of Petroleum and Minerals
Dhahran 31261, Saudi Arabia
{salawsh[1], muqaibel[3], samir[4]}@KFUPM.edu.sa, omarkhazragy[2]@gmail.com

Abstract

Direction of arrival (DOA) estimation techniques estimate the angle from which the signal is received. Using multiple antennas, the best DOA estimate is found by comparing the signals from these antennas and their phase shifts. Optimizing the arrangements of these antennas can greatly increase the accuracy of the system and decrease the cost of a Radio Direction Finding (RDF) system. In this paper, a Software-Defined-Radio (SDR) platform is implemented for sparse DOA estimation based on sparse array configurations. The results show that it is possible to estimate the location of more targets without increasing the number of antennas using sparse arrays.

1. Introduction

Direction of arrival (DOA) is specified by the angle from which the signal is received. Measuring or estimating this angle is one of the most common approaches to find the location of targets which is useful in many applications. These applications range from wireless communications, such as sonar, navigation, radar, radio astronomy [1], and localization of illegal repeaters [2], to military applications such as locating a radio communication device of an enemy [3]. In these applications, the target to be located does not have to be an active target, as it can be a passive target that reflects the signal produced by the DOA estimation system. Some systems can estimate the location of single target while others can estimate more.

DOA estimation requires expensive hardware as well as capable processing units that can estimate the angle within the desired time. The use of Software-Defined-Radio (SDR) allows researchers to implement DOA estimation systems with relative ease. This is because such devices can be controlled using computers and the received signals can be easily analyzed. One type of these devices is the Universal Software Radio Peripheral designed by National Instruments (NI-USRP). This specific type of SDR can be controlled using LabVIEW.

The angle is estimated using information from the amplitude and/or the phase of the signal received by an antenna, or more commonly, an array of antennas. The number of antennas, usually referred to as elements, in the array affects the resolution and accuracy of the estimation [3]. Furthermore, the shape and size of the antenna also affect the range of frequencies for proper DOA estimation [3]. The elements of the array can be placed linearly, rectangularly, or even in a circle [4]. Another factor needs to be taken into consideration is the inter-element spacing between elements, which can be uniform as in the uniform linear array (ULA), or nonuniform as in the interesting sparse systems.

Different configurations have been developed, and each has its advantages and disadvantages. The most common arrangement of antennas in the literature is the ULA. This is because it is very effective with super resolution techniques [1]. For instance, Abdessamad, et al. in [2] used a ULA in which they placed four NI-USRPs to estimate the angle in order to locate illegal repeaters. It is also possible to implement an array with limited number of antennas. Such a system was presented by Redondo, et al. in [5], where they used two antennas to estimate the DOA in three different environments. The purpose of using fewer antennas is reducing the cost of the system. The system was only capable of properly estimating angles greater than 30°. In addition, the system estimated the real angle and its reflection. Such a system can estimate the location of one target at most. This is because a ULA of M elements can only estimate $M-1$ targets.

Instead of using a ULA, some researchers used a sparse antenna array. In sparse arras, the inter-element spacing is not uniform, and it can be represented as an arrangement of ULAs but with missing elements. One of these arrangements is shown in [6], where Vaidyanathan, et al. used two ULAs with the numbers of antennas in the two arrays being co-prime. The system was able to estimate the correlations, signal spectra, and DOA at a significantly higher resolution compared with ULA. Another example is presented in [7], where a sparse array was used to construct a virtual ULA from the data collected but with fewer antennas. The number of estimated sources was more than the number of the array elements, which is a very important advantage of sparse arrays over ULAs. Despite the possibility of many array arrangements [8], [9], most of the research done in this area is done using a ULA with different number of antennas as can be seen in the works presented in [10]-[17].

For a given array, the received signals are processed to estimate the DOA. Choosing a proper DOA estimation algorithm is crucial to get the best result for the desired application. One of the most commonly used algorithms in literature is MUltiple SIgnal Classification (MUSIC), which is an Eigen-structure which assumes orthogonality between the noise and the signal subspaces [5]. Amin, et al. showed in [4] that the simulation done using MUSIC delivered improved performance because the noise was reduced significantly. Redondo, et al. showed in [5] that using MUSIC algorithm results in an efficient estimation. Because of all these advantages, MUSIC algorithm is widely used [1], [12], [13], [18]. Capon algorithm was compared with MUSIC algorithm in [5]. It has a good estimation capability though the estimated sources should be uncorrelated.

Some studies in literature relied on software simulations, some relied on hardware implementation, and other combined the two approaches. In [4], for example, Amin, et al. used only simulations done on MATLAB and LabVIEW. On the other hand, Tayem, et al. first simulated the system and then implemented it on a Peripheral Component Interconnect (PCI) eXtensions for Instrumentation (PXI) platform designed by National Instruments in [12]. The advantage of building the system after simulation is to make sure it can operate in imperfect

situations where the presence of noise and multipath might affect the results.

From the discussion above, it is clear that there is a need for a simple, practical, and experimentally-tested system. This paper uses SDR to implement a DOA-estimation system and tests its performance. In our research, we use several SDRs (NI USRP-2950R), and a PXI platform to implement the system based on coprime array structure. The advantage of such array is that we can achieve results similar to those obtained from a ULA with more elements, which reduces the cost of the system. We use LabVIEW to control the USRPs, process the received signals, and perform DOA estimation via sparse reconstruction, MUSIC, and Capon algorithms. Finally, we compare the results we obtained with others based on ULAs.

This paper is organized as follows. Section II presents the general model for DOA estimation system based on a ULA, before addressing sparse antenna arrays. In Section III, the considered DOA algorithms are discussed in details. Section IV shows the experimental setup and the LabVIEW code, while Section V discusses the results. Conclusions are presented in Section VI.

2. System Model For DOA Estimation

We validate the system by building a ULA. Then the arrangement is changed to form a sparse-array and the performance improvement is quantified. The model shown in Fig. 1 consists of a ULA with M antennas and d targets which are assumed to be far-field sources. The assumption allows us to treat the received signals as plane waves, which means, the DOA is the same for all elements. To realize this, we placed the sources at a distance larger than $2D^2/\lambda$, where D is the length of the array and λ is the carrier wavelength. The inter-element spacing between antennas is denoted as Δ with $\Delta \leq \lambda/2$. The DOA of the kth source is denoted as θ_k. We define the steering matrix $\mathbf{A}$ as follows [20]:

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ e^{j\mu_1} & e^{j\mu_2} & \cdots & e^{j\mu_d} \\ \vdots & \vdots & \ddots & \vdots \\ e^{j(M-1)\mu_1} & e^{j(M-1)\mu_2} & \cdots & e^{j(M-1)\mu_d} \end{bmatrix}, \tag{1}$$

$$\mathbf{A} = [\mathbf{a}(\mu_1) \quad \mathbf{a}(\mu_2) \quad \cdots \quad \mathbf{a}(\mu_d)], \tag{2}$$

where $\mu_k = -\frac{2\pi}{\lambda}\Delta\sin(\theta_k)$ and $\mathbf{a}(\mu_k) = \left[1, e^{j\mu_k}, e^{j(M-1)\mu_k}\right]^T$.

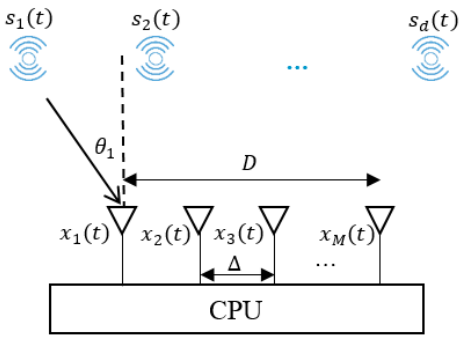


Fig. 1. A typical DOA estimation system

We define a new vector, $\mathbf{x}(t) = [x_1(t), x_2(t), \ldots, x_M(t)]^T$, to be a vector that contains all the M received signals as:

$$\mathbf{x}(t) = \mathbf{A}[s_1(t), s_2(t), \ldots, s_d(t)]^T + \mathbf{n}(t), \tag{3}$$

where $s_k(t)$, is the signal transmitted by the kth source and $\mathbf{n}(t)$ is a vector of uncorrelated Gaussian additive noises with zero mean. The spatial covariance matrix of the received noised signals, $\mathbf{R_{xx}}$, is defined as [20]:

$$\mathbf{R_{xx}} = E[\mathbf{x}(t)\mathbf{x}^H(t)] \tag{4}$$

The sparse array to be formed is called coprime array, which is built using two ULAs [21]. Let M and N be the number of elements in the first and the second ULAs, respectively where M and N are coprime integers. The inter-element spacing of the first subarray is $N\lambda/2$, and for the second subarray it is $M\lambda/2$. The two arrays start from zero, so they share the first antenna, which means the total number of elements equals $M + N - 1$. We may write μ_k for such an array as:

$$\mu_k = -\frac{2\pi}{\lambda} p_i \sin(\theta_k), \tag{5}$$

where p_i is the position of each antenna for $i = 1,2, \ldots, M + N - 1$. The received data is fed to any of the available algorithms to perform DOA estimation.

3. DOA Estimation Algorithms

Of all the available DOA estimation algorithms, we focus on three. The first two algorithms are used for ULA because of their superior performance. The third algorithm suits the sparse estimation case.

3.1 MUSIC

The MUSIC algorithm works best in the cases where the noises in all signals are uncorrelated. Otherwise, the performance of the algorithm drops significantly. MUSIC algorithm estimates the DOA based on the following formula:

$$P_M(\theta) = \frac{1}{|\mathbf{a}^H(\theta)\mathbf{V_n}\mathbf{V^H}_{\boldsymbol{n}}\mathbf{a}(\theta)|}, \tag{6}$$

$$\mathbf{V_n} = [q_{d+1}, \cdots, q_M], \tag{7}$$

where $P_M(\theta)$ is the average output power, $\mathbf{V_n}$ in (7) is the noise subspace and q is the eigenvector corresponding to the smallest eigenvalues [20].

3.2 Capon

Capon algorithm requires the received signals to be uncorrelated. This is because it considers one signal at a time while assuming the rest are interferences. To estimate the DOA, we may write average power of Capon algorithm as:

$$P_C(\theta) = \frac{1}{\mathbf{a}^H(\theta)\mathbf{R}^{-1}{}_{\mathbf{xx}}\mathbf{a}(\theta)}, \tag{8}$$

Herein, the inverse of the covariance matrix should exist.

3.3 Compressive Sensing

For the sparse array, we use compressive sensing (CS). The first step of estimating the angle is by solving the l_0-norm minimization problem stated as:

$$\hat{\mathbf{r}}^{\circ} = \arg\min_{\mathbf{r}^{\circ}} \|\mathbf{r}^{\circ}\|_0 \text{ subject to } \|\mathbf{z} - \mathbf{B}^{\circ}\mathbf{r}^{\circ}\|_2 < \varepsilon \tag{9}$$

where $\mathbf{r}^{\circ}$ is the sparse entries into the search grids, $\mathbf{B}^{\circ}$ is the sensing matrix constructed as in [21], $\mathbf{z} = \text{vec}(\mathbf{R}_{\mathbf{xx}})$ is the vectorized covariance matrix, and ε is a bound specified by the user. The positions as well as the values of the non-zero entries of all $\mathbf{r}^{\circ}$ elements, except the last one, represent the estimated DOAs and powers. One method for solving such problems is the Lasso algorithm, whose objective function is shown in (10), where λ_t is a penalty parameter [21].

$$\hat{\mathbf{r}}^{\circ} = \arg\min_{\mathbf{r}^{\circ}} \left[\frac{1}{2}\|\mathbf{z} - \mathbf{B}^{\circ}\mathbf{r}^{\circ}\|_2 + \lambda_t\|\mathbf{r}^{\circ}\|_1\right]. \tag{10}$$

4. Experimental Setup

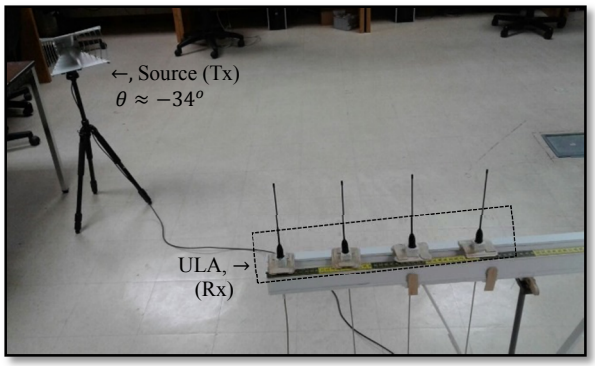
We built two DOA-estimation systems. The first system uses a ULA with four antennas while the second system uses a sparse array. In the former, two SDRs (NI USRP 2950R) are used to estimate the DOA. Two antennas are connected to each of these USRPs using their Rx ports. A synchronization device, called the NI-OctoClock-G CDA-2990, is connected to the USRPs to make sure they start receiving data at the exact same instant of time. This is important to ensure that the differences in the phases of the received signals are caused only by the difference in their positions. To ensure synchronization of the frequencies of both USRPs, we connected their REF IN ports to the OctoClock. The transmitted antenna is connected to another USRP using the Tx port, which can be used for both receiving and transmitting signals. It is also connected to the Octoclock. The carrier frequency of the signal is set to 1.2 GHz. We placed the receiving antennas with inter-element spacing equal to half the wavelength of the signal, which makes the space between two antennas 12.5 cm, to avoid spatial aliasing. The transmitter was at least 1.5 m away from the array in all different tested angles, to make sure that the signal is received as a plane wave, see Fig. 2. Both the array and the transmitter were 1 m above the ground. The experiments were conducted in the Telecommunication Research Lab (TRL) at King Fahd University of Petroleum & Minerals (KFUPM).

As shown in Fig. 2, the array uses monopole antennas (Vert 400: 144 MHz, 400 MHz, 1200 MHz tri band vertical antenna). The transmitter, on the other hand, is a broadband horn antenna (Type JXTXLB-10180 and bandwidth of 1-18 GHz), which helps us to direct the signal toward the receiving antennas and reduce the multipath effects.

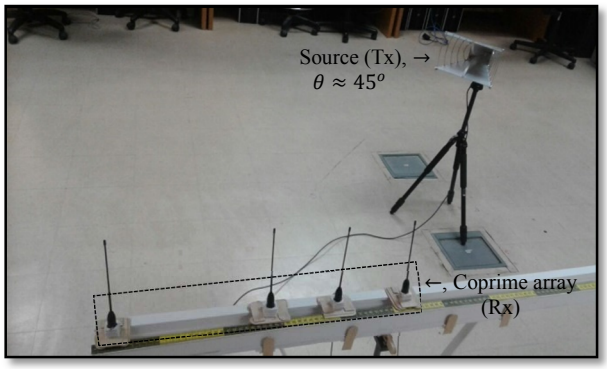
The described system, including the receivers (Rx) and the transmitter (Tx), is controlled through a developed LabVIEW code. The first part of the code defines the transmitter. A single-tone signal or multiple tones can be used as the unmodulated message. Selected tones have the following frequencies: 11 kHz, 23 kHz, and 39 kHz. The second part is for the receiving USRPs. Before running the code, we place the transmitter at a reference angle 0° and perform phase calibration. The corresponding received signals before and after phase calibration are shown in Fig. 3. This is achieved by comparing the phases of the received signal by all antennas, and eliminating the differences in the phases. The first antenna is considered as a reference. The received phases before and after calibration are shown in Fig. 4. After phase calibration, the signals are passed through a filter and then they are fed into DOA estimation algorithms. To get smooth plots, we plot the average of the last 20 estimated angles. The second experiment is performed using coprime array which has two subarrays with $M = 2$ and $N = 3$. The first one has three antennas spaced by $M\lambda/2 = 25$ cm ($2\lambda/2$), as Fig. 2(b) illustrates. The second one has two antennas spaced by $N\lambda/2 = 37.5$ cm ($3\lambda/2$). The first antenna of both subarrays is common.

5. Results And Discussion

First, we have tested the system capability to estimate different angles at different directions for both configurations. A single transmitter with three-tone at approximately -34° is assumed and the signals are received with a ULA. Fig. 5 (a) and (b) show the normalized spectra of Capon and MUSIC algorithms respectively. The same transmitter is located then at approximately 45° from a coprime array where the normalized spectrum of Lasso algorithm is demonstrated in Fig. 5 (c). The beam width at the estimated angle is significantly sharper than those produced by MUSIC and Capon. The estimated angles are approximately -30.75^o, -30.75^o, and 41.75^o for Capon, MUSIC and Lasso algorithms respectively. Although, Capon and MUSIC have similar estimated angles, MUSIC algorithm has better resolution, i.e. smaller beam width at the estimated angle.

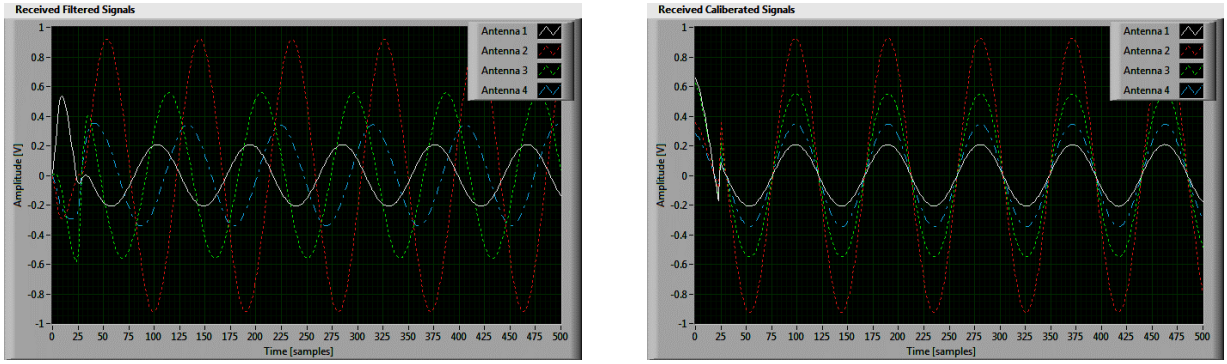


(a)



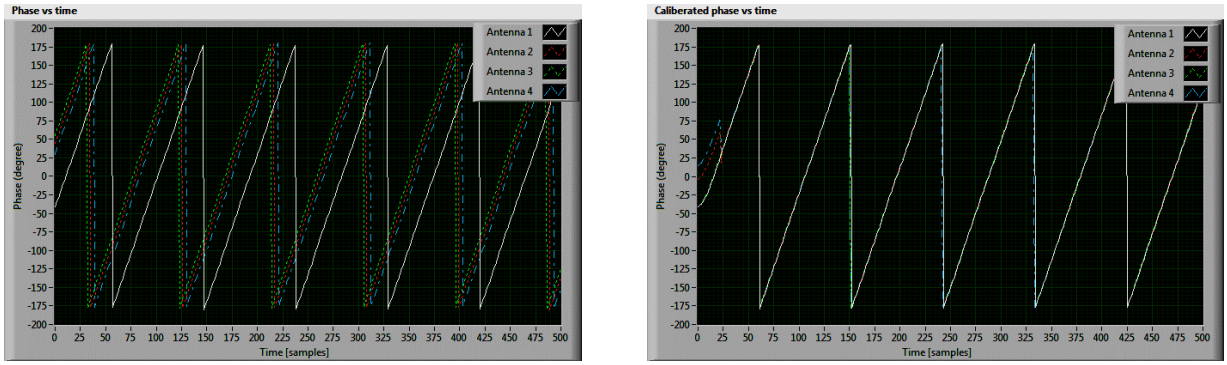
(b)

Fig. 2. DOA estimation setup (a) ULA and (b) coprime array



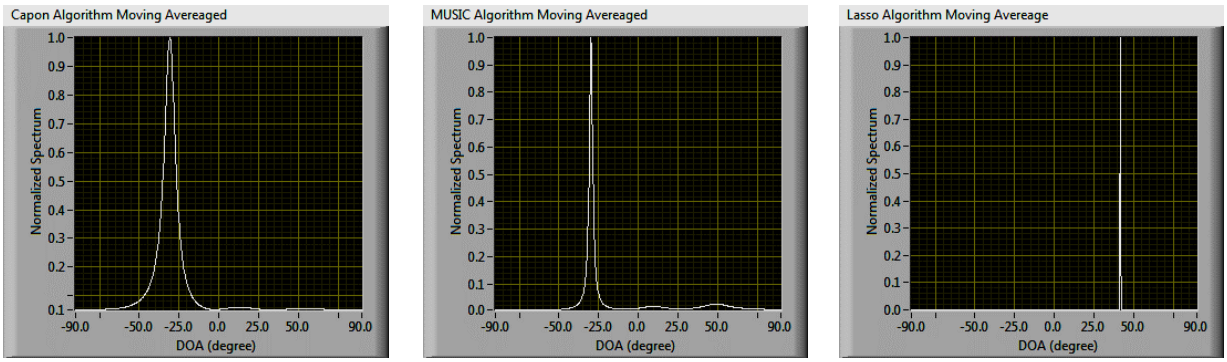
(a) (b)

Fig. 3. The received signals (a) before and (b) after phase calibration



(a) (b)

Fig. 4. The phase of the received signals when the transmitted at zero degree (a) before and (b) after calibration



(a) (b) (c)

Fig. 5. The normalized spectrum of (a) MUSIC, (b) Capon, and (c) Lasso algorithms

Second, we have tested more angles at diverse directions from the arrays. The results obtained from the ULA and the coprime array are presented in Fig. 6 where we plot the estimated angles versus the real angles for a single transmitter with one and three tones MUSIC and Lasso algorithms. The error in the estimated angles becomes negligible as we get closer to 0°. Angles near $+90^o$ are not estimated accurately, though this is not the case at -90°. The main cause of such error is most probably related to the surrounding environment. The system was affected by strong multipath components because of the steel ceil in the lab. Even though we use the fewest number of antennas possible to construct the coprime array, the results are acceptable. Sparse arrays have the capability of locating more targets than the number of antennas. Another advantage of the sparse-array setup is reducing the mutual coupling effect where the antennas are spaced by more than a half-wavelength.

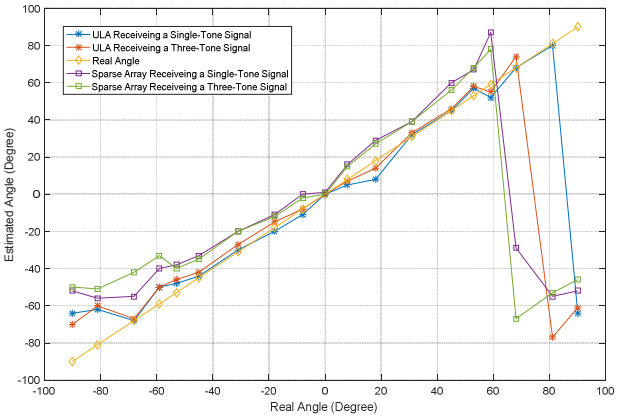


Fig. 6. Estimated angles versus real angles based on ULA and coprime array for a single transmitter with single and three tones

6. Conclusion

In this paper, we have used a sparse array to construct a DOA-estimation system using SDRs (NI USRP-2950R). The advantage of such a system is the flexibility and the ability to locate more targets than is possible using ULAs, and reducing mutual coupling of the receiving antennas. We took into account the issues of synchronizing the USRPs and calibrating the phases of the received signals. The system was tested in a laboratory with dense multipath environment. We also built a ULA DOA-estimation system to compare the results of both systems. This is because of the rough environment in which the system was tested. MUSIC, Capon, and Lasso algorithms have been implemented for estimation. The results show that CS produced sharper beams in the normalized spectrum than those produced by the other methods.

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