Design of Sequences with Low Autocorrelation Sidelobes using Genetic Algorithms

Osman Tayfun Bişkin[1] and Olcay Akay[1]

[1] Dokuz Eylül University, Dept. of Electrical and Electronics Eng.,
Tınaztepe Campus, 35160, Buca / İZMİR TURKEY
tayfun.biskin@ogr.deu.edu.tr, olcay.akay@deu.edu.tr

Abstract

Unimodular constant modulus sequences with desirable autocorrelation function properties are widely used in radar and communication systems. One of the performance metrics that determine the goodness of a designed sequence is the integrated sidelobe level (ISL) of the autocorrelation function of the sequence. Studies in the literature have focused on minimizing the metric of ISL. In this study, we utilize genetic algorithm (GA) to minimize ISL in the frequency domain. The proposed algorithm is initialized by either a random sequence or the Golomb sequence whose autocorrelation is known to have good properties. By this way, a radar transmit signal with minimum ISL is designed using GA. Finally, performance of GA is compared against the already existing cyclic-new (CAN) algorithm. The simulations indicate that minimization of ISL using GA produces better results than the CAN algorithm. Hence, GAs could alternatively be used to design radar transmit sequences by minimizing ISL in the frequency domain.

1. Introduction

Sequences with good autocorrelation function properties are widely used in radar systems as transmit signals and in communication systems for synchronization purposes. Therefore, having desirable autocorrelation properties is an important issue in designing a radar transmit sequence. There are some performance metrics for determining the goodness of a designed sequence. One of them is called integrated sidelobe level (ISL) of the autocorrelation function of the designed sequence. A low ISL value is an indicator that the designed sequence is of desirable type.

Let $\{x_n\}_{n=1}^{N}$ denote a unimodolar constant modulus sequence so that it satisfies the following constraint

$$|x_n| = 1, \quad n = 1, \dots, N. \tag{1}$$

Aperiodic autocorrelation function of x_n can be calculated as

$$r_k = \sum_{n=k+1}^{N} x_n x_{n-k}^* = r_{-k}^*, \quad k = 0, \dots, N-1. \tag{2}$$

The ISL metric can be defined [1–3] in terms of r_k as

$$\text{ISL} = \sum_{k=1}^{N-1} |r_k|^2. \tag{3}$$

Merit factor (MF) [1] is another metric related to ISL as

$$\text{MF} = \frac{N^2}{2(\text{ISL})} = \frac{|r_0|^2}{\sum\limits_{\substack{k=-(N-1)\\ k\neq 0}}^{N-1} |r_k|^2}. \tag{4}$$

Scientists have long been working extensively to design sequences having low ISL (i.e. high MF) values. Both analytic and computational methods have been proposed to come up with such sequences. Following some binary codes (such as Barker codes) suggested early, the so-called polyphase codes are designed in later years [4]. Some well-known polyphase codes have closed form expressions such as Frank [5], Chu [6], and Golomb [7] sequences. Some optimization methods such as genetic algorithms (GAs), heuristic search, and stochastic optimization method have also been used to design waveforms with good autocorrelation properties [2]. Moreover, it is shown that it is possible to design sequences having low ISL [1, 8] or low weighted-ISL (WISL) [1] values by using the so-called cyclic algorithms (CAs). Power spectral density (PSD) fitting approach is another method proposed for sequence design with low WISL [9]. In [1], CA-new (CAN) algorithm has been proposed to minimize the ISL value of a sequence based on the definition of autocorrelation function in the frequency domain. However, CAN minimizes an ISL-related metric which is a quadratic approximation of the exact ISL.

In this study, GA is employed to synthesize unimodular signal sequences with low ISL and the results are compared against CAN. In Section 2, the optimization problem for minimizing ISL is introduced and the CAN algorithm is briefly summarized. In Section 3, GAs are reviewed. In Section 4, simulation results and MF performances of GAs and CAN are given. Finally, in Section 5, our concluding remarks are given.

2. Problem Formulation and Existing Methods

Designing a unimodular constant modulus sequence with good autocorrelation sidelobe properties amounts to solving a constrained optimization problem. Thus, subject to $|x_n| = 1$ for $n = 1, \dots, N$, one minimizes ISL to obtain a sequence $\{x_n\}_{n=1}^{N}$. The CAN algorithm was previously proposed to solve this optimization problem iteratively.

2.1. CAN

Autocorrelation function can be defined [10] in the frequency domain as follows

$$\left|\sum_{n=1}^{N} x_n e^{-j\omega n}\right|^2 = \sum_{k=-(N-1)}^{N-1} r_k e^{-j\omega k} \triangleq \Phi(\omega), \tag{5}$$

where $\Phi(\omega)$ represents power spectral density of x_n and $\omega \in [0, 2\pi]$. Then, the metric of ISL in (3) can be rewritten by using the definition given in (5) as [1,8]

$$\text{ISL} = \frac{1}{4N}\sum_{p=1}^{2N}\left[\Phi\left(\omega_p\right) - N\right]^2 \tag{6}$$

where $\{\omega_p\}_{p=1}^{2N}$ is defined as

$$\omega_p = \frac{2\pi}{2N}p, \quad p = 1, \ldots, 2N. \tag{7}$$

ISL can also be written using (5) and (6) as follows

$$\text{ISL} = \frac{1}{4N}\sum_{p=1}^{2N}\left[\left|\sum_{n=1}^{N} x_n e^{-j\omega_p n}\right|^2 - N\right]^2. \tag{8}$$

Thus, at this point, the optimization problem of minimizing ISL can be expressed as the minimization of the expression below

$$\sum_{p=1}^{2N}\left[\left|\sum_{n=1}^{N} x_n e^{-j\omega_p n}\right|^2 - N\right]^2. \tag{9}$$

Note that the above cost function in (9) is quartic with respect to x_n. Therefore, an almost equivalent minimization problem having a cost function which is quadratic in x_n was suggested in [1,8,11,12] as

$$\min_{\{x_n\}_{n=1}^{N};\{\psi_p\}_{p=1}^{N}} \sum_{p=1}^{2N}\left|\sum_{n=1}^{N} x_n e^{-j\omega_p n} - \sqrt{N}e^{-j\gamma_p n}\right|^2. \tag{10}$$

To solve the constrained obtimization problem above, let

$$\mathbf{a}_p^H = \left[e^{-j\omega_p} \quad e^{-j2\omega_p} \quad \ldots \quad e^{-j2N\omega_p}\right] \tag{11}$$

where $(\cdot)^H$ denotes the Hermitian operation. $\mathbf{A}^H$ below can be defined as $2N \times 2N$ unitary fast Fourier transform (FFT) matrix

$$\mathbf{A}^H = \frac{1}{\sqrt{2N}}\begin{bmatrix}\mathbf{a}_1^H \\ \mathbf{a}_2^H \\ \vdots \\ \mathbf{a}_{2N}^H\end{bmatrix}. \tag{12}$$

Then, the expression in (10) can be more compactly written as

$$\left\|\mathbf{A}^H\mathbf{z} - \mathbf{v}\right\|^2 \tag{13}$$

where $\mathbf{z} = \left[x_1 \quad x_2 \quad \ldots \quad x_N \quad 0 \quad 0 \quad \ldots \quad 0\right]_{2N\times 1}^T$ and

$$\mathbf{v} = \frac{1}{\sqrt{2}}\left[e^{j\gamma_1} \quad e^{j\gamma_2} \quad \ldots \quad e^{j\gamma_{2N}}\right]^T. \tag{14}$$

For a given sequence, $\mathbf{x}$, the vector, $\mathbf{v}$, can be found by minimizing the expression in (13) with respect to $\{\gamma_p\}$. Let $\mathbf{h} = \mathbf{A}^H\mathbf{z}$ denote the FFT of $\mathbf{z}$. Then, minimization of (13) with respect to $\{\gamma_p\}$ for a given sequence, $\mathbf{x}$, produces

$$\gamma_n = \arg\left(h_p\right), \quad p = 1, \ldots, 2N. \tag{15}$$

Similarly, by letting $\mathbf{g} = \mathbf{A}\mathbf{v}$, minimization of (13) with respect to sequence, $\mathbf{x}$, for a given $\mathbf{v}$ yields

$$x_n = e^{j\arg(g_n)}, \quad n = 1, \ldots, N. \tag{16}$$

By this way, the CAN algorithm [1] minimizes the approximate ISL-related metric in (10) instead of the exact ISL metric in (9).

3. Genetic Algoritm (GA)

In this study, the exact quartic cost function in (9) is minimized using GA to find a sequence with minimum ISL. GA is one of the global optimization algorithms designed by taking inspiration from the natural selection mechanism in biology. In biology, the most adaptable generations are able to be alive after the ongoing natural selection mechanism years by years. In the same way, the most probable solution of an optimization problem eliminates the alternative solutions after execution of the GA for that problem. Generally, GAs are used when the analytic solution of the optimization problem cannot be found easily. Another advantage of GAs is that they are less likely to converge to a local minimum. Therefore, GAs are able to both improve the performance of systems and solve complex optimization problems [13–19]. The basic concepts relevant to GAs are explained as follows:

- *Population*: The set which may include the possible solutions of the problem.
- *Individual*: Each element in the population set is named as an individual.
- *Generation*: The process of reproduction of the individuals which are included in the population.
- *Parent*: Individuals which are used in the reproduction process.
- *Child*: An individual arising from two parents after reproduction process.
- *Initialization*: The process to create the initial population of the algorithm.
- *Selection*: The process of determining the appropriate parents in order to give a child.
- *Crossover*: Changing the chromosome of individuals from generations to generations (see Fig. 1).

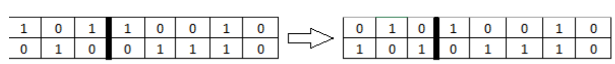

Fig. 1. An example of crossover.

- *Mutation*: A random change which occurs in the chromosome of individuals (see Fig. 2).

Fig. 2. An example of mutation.

4. Numerical Examples

GA is used to find the minimum value of the exact cost function in (9). Implementation of GA is performed via MATLAB 2017a software using "Global Optimization Toolbox".

There are many parameters affecting performance of GA and the optimum values of these parameters are found by experimenting. In our simulations, the quartic objective function in (9) is an input parameter. The population size, maximum number of generations, crossover and mutation operations explained in the previous section are also presented as the other inputs. We decided the population size to be 150 and the maximum generation number is assigned as 20000. We use both crossover and mutation operations in order to increase the diversity in the population. 5 % of the population are selected as elite individuals in each generation. They maintain their chromosomes without any mutation or crossover.

Simulations were performed by employing two different initialization sequences. The first initialization sequence $\mathbf{x} = [e^{j\varphi_1}, \ldots, e^{j\varphi_{2N}}]$ is formed by selecting $\{\varphi_p\}_{p=1}^{N}$ as independent random variables uniformly distributed in $[-\pi, \pi]$. GA was repeated 10 times and CAN was repeated 100 times when the algorithms were initialized by random sequences. The average value of the obtained MFs are presented in Table 1. Secondly, the algorithms were initialized with Golomb sequence [7] and the resulting MFs can be seen in Table 2. Simulations were performed for sequence lengths of $N = 9, 25, 32, 64, 100$.

Fig. 3. shows the average MF values with respect to sequence length for random initialization. It can be understood from the figure that GA performs better in terms of MF for all the simulated sequence lengths. Secondly, GA and CAN were initialized with Golomb sequence. MFs of the resulting designed sequences with respect to sequence length can be seen in Fig. 4. We can see that when CAN and GA are initialized with such a sequence having good correlation properties as Golomb, their performances increase in terms of MF. As opposed to Fig. 3, MFs obtained by CAN and GA are fairly close to each other. However, MFs of GA are slightly better than CAN for sequence lengths of 64 and 100.

Correlation levels (in dB) of Golomb sequence and the sequences designed by CAN and GA for $N = 100$ are shown in Fig. 5, where the correlation level is defined as [1]

$$\text{Corr. Lev.} = 20\log_{10}\frac{|r_k|}{|r_0|}, k = -(N-1), \ldots, N-1. \quad (17)$$

We can see from Fig. 5 that the lowest correlation levels are obtained by the GA.

Table 1. MF values for GA, CAN, and Golomb when GA and CAN were initilized by random sequences.

Algorithms	N (sequence length)				
	9	25	32	64	100
GA	35,14	14,68	13,52	16,99	16,87
CAN	17,78	11,31	11,68	13,80	14,81
Golomb	5,37	8,20	9,18	12,77	15,87

Table 2. MF values for GA, CAN and Golomb when GA and CAN were initilized by Golomb sequence.

Algorithms	N (sequence length)				
	9	25	32	64	100
GA	38,23	20,33	14,95	47,94	57,10
CAN	38,02	25,01	16,91	46,67	56,47
Golomb	5,37	8,20	9,18	12,77	15,87

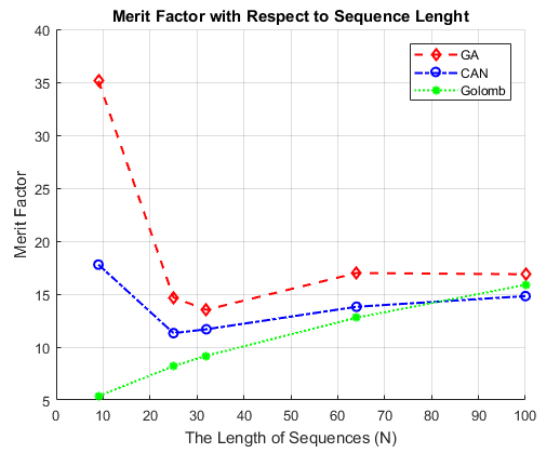

Fig. 3. Average MF versus sequence length. GA and CAN were initialized by random sequences.

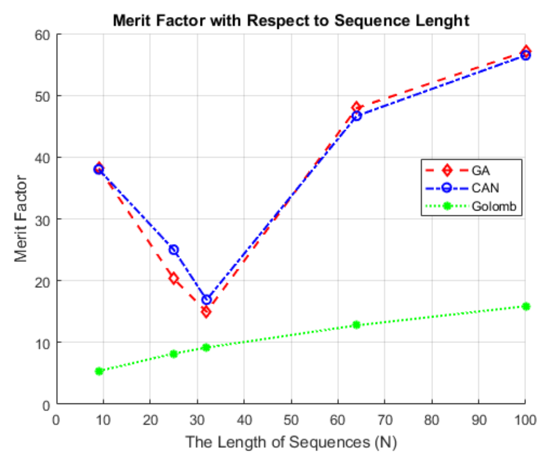

Fig. 4. MF versus sequence length. GA and CAN were initialized by Golomb sequence.

5. Conclusions

In this study, we propose to use GA in order to design unimodular constant modulus sequences attaining minimum ISL values. Then, we compare performance of GA against that of the CAN algorithm in terms of MF. The CAN algorithm is proposed for designing a sequence with good correlation properties and it minimizes an approximate quadratic ISL-related metric instead of the exact quartic ISL metric. Therefore, our basic aim here was to minimize the original quartic metric of ISL using GA and compare its performance against CAN. Although, the evaluation time required for termination of CAN is shorter than GA, GA performs better than CAN in terms of the resulting MF. The difference between these two algorithms on minimizing the ISL-related metric in (10) and the exact ISL metric in (9) become more evident when they are initialized by random sequences. In addition, MFs of the sequences designed by GA are higher than those designed by the CAN algorithm for larger sequence lengths. These results indicate that sequences designed by minimizing the exact metric of ISL using GA can attain higher MFs than those designed by the CAN algorithm. However, execution time of CAN is shorter than GA.

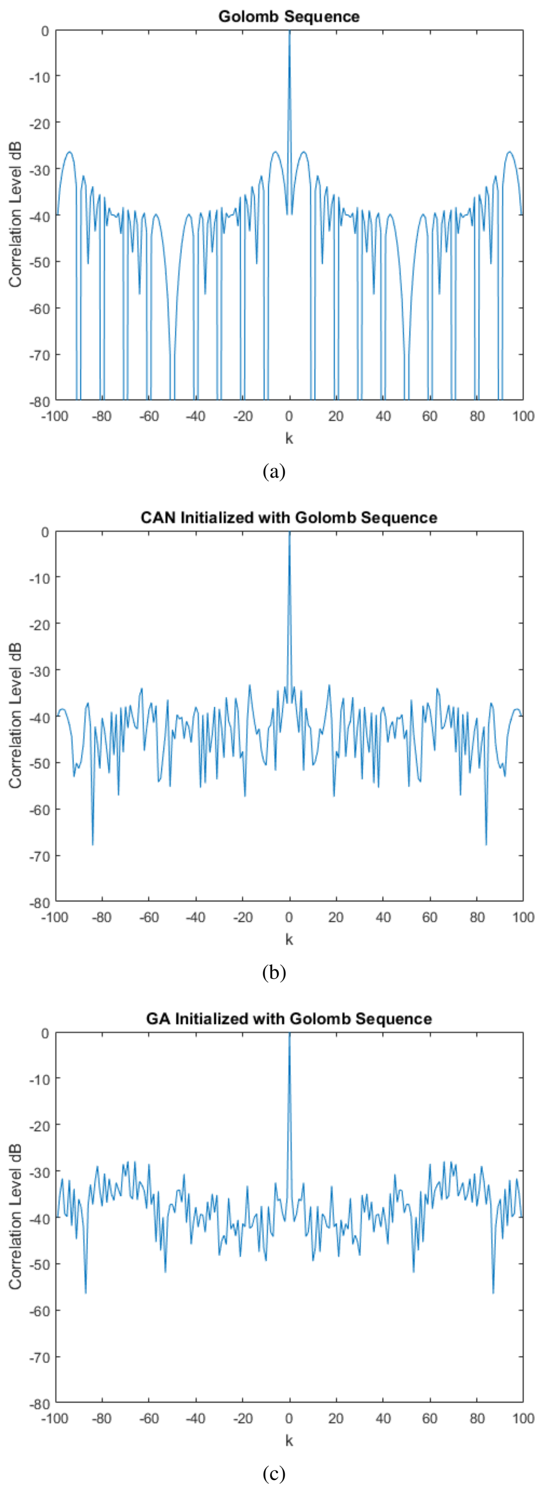

Fig. 5. Correlation levels ($N = 100$). (a) Golomb sequence, (b) Designed CAN sequence, (c) Designed GA sequence.

As future work, we plan to come up with the analytic solution of the minimization problem in (9) and use the simulation results obtained by GA in this study as a benchmark.

6. References

[1] P. Stoica, H. He and J. Li, "New algorithms for designing unimodular sequences with good correlation properties", *IEEE Trans. Signal Process.*, vol. 57, no. 4, pp. 1415-1425, 2009.

[2] J. Song, P. Babu and D. P. Palomar, "Optimization methods for designing sequences with low autocorrelation sidelobes," *IEEE Trans. Signal Process.*, vol. 63, no. 15, pp. 3998-4009, 2015.

[3] J. Song, P. Babu and D. P. Palomar, "Optimization methods for sequence design with low autocorrelation sidelobes," in *ICASSP, IEEE International Conference on Acoustics, Speech and Signal Processing Proceedings*, Brisbane, Australia, pp. 3033-3037, 2015.

[4] N. Levanon and E. Mozeson. *Radar Signals*. John Wiley & Sons, Inc., Hoboken, New Jersey, USA, 2004.

[5] R. L. Frank, "Polyphase codes with good nonperiodic correlation properties," *IEEE Trans. Inf. Theory* , vol. 9, no. 1, pp. 43-45, 1963.

[6] D. C. Chu, "Polyphase codes with good periodic correlation properties," *IEEE Trans. Inf. Theory*, vol. 18, no. 4, pp. 531-532, 1972.

[7] N. Zhang and S. W. Golomb, "Polyphase sequence with low autocorrelations," *IEEE Trans. Inf. Theory*, vol. 39, no. 3, pp. 1085-1089, 1993.

[8] W. Roberts, H. He, J. Li and P. Stoica, "Probing waveform synthesis and receiver filter design," *IEEE Signal Process. Mag.*, vol. 27, no. 4, pp. 99-112, 2010.

[9] H. Wu, Z. Song, H. Fan and Q. Fu, "Designing sequence with low sidelobe levels at specified intervals based on PSD fitting," *Electronics Letters*, vol. 51, no. 1, pp. 99-101, 2015.

[10] P. Stoica and R. L. Moses. *Spectral Analysis of Signals*. Prentice-Hall, Upper Saddle River, NJ, 2005.

[11] J. Li, P. Stoica and X. Zheng, "Signal synthesis and receiver design for MIMO radar imaging," *IEEE Trans. Signal Process.*, vol. 56, no. 8, pp. 3959-3968, 2008.

[12] P. Stoica, J. Li, X. Zhu and B. Guo, "Waveform synthesis for diversity-based transmit beampattern design," in *IEEE Workshop on Statistical Signal Processing Proceedings*, Madison, WI, USA, pp. 473-477, 2007.

[13] D. S. Weile and E. Michielssen, "Genetic algorithm optimization applied to electromagnetics: A review," *IEEE Trans. Antennas and Propagation*, vol. 45, no. 3, pp. 343-353, 1997.

[14] C. T. Capraro, I. Bradaric, G. T. Capraro and T. K. Lue, "Using genetic algorithms for radar waveform selection", in *IEEE Radar Conf. RADAR'8*, Rome, Italy, pp. 1-6, 2008.

[15] G. Sun, J. Wang, Z. Zhang, M. Tao and F. Zhou, "System parameter optimisation for moving target detection and imaging in multi-band synthetic aperture radar based on genetic algorithm", *IET Radar, Sonar, Navig.*, vol. 10, no. 1, pp. 146-154, 2016.

[16] A. Martone, K. Ranney and K. Sherbondy, "Genetic algorithm for adaptable radar bandwidth",in *IEEE Radar Conf. (RadarConf)*, Philadelphia, PA, USA, pp 1-6, 2016.

[17] B. Smith-Martinez, A. Agah and J. M. Stiles, “A genetic algorithm for generating radar transmit codes to minimize the target profile estimation error”, *J. Intell. Syst.*, vol. 22, no. 4, pp. 503-525, 2013.

[18] G. Lellouch, A. Mishra and M. Inggs, “Convex optimization for optimal PMEPR and mismatched filter design in OFDM radar, in *IEEE Radar Conference Proceedings*, Johannesburg, South Africa, pp. 37-41, 2015.

[19] G. Lellouch, A. Mishra and M. Inggs, “Design of OFDM radar pulses using genetic algorithm based techniques, *IEEE Trans. Aerosp. Electron. Syst.*, vol. 52, no. 4, pp. 1953-1966, 2016.