Solving Economic Load Dispatch Problem Using Vortex Search Algorithm

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Abstract

The Vortex Search (VS) algorithm is a recently developed metaheuristic algorithm inspired by a vortex pattern which can be modeled as a number of the nested circles. This algorithm basically adjusts step sizes automatically according to changed values of the radius of circles to improve solutions. In this work, the VS algorithm is applied to solve the economic load dispatch (ELD) problem in the 20-unit test system by considering the system constraints and also the performance of this algorithm has been analyzed in terms of total generation costs and power losses. Simulation results have been compared to the different algorithms used before in the literature. The obtained results demonstrate that VS algorithm is capable of solving the ELD minimization problem efficiently and finding the output power of all the generation units properly.

1. Introduction

To generate the lowest cost energy, determination of the output of the generating units related to operational and transmission constraints, known as "Economic Dispatch". ELD is one of the most challenging problem in the power system operations due to a highly non-linear cost surface of the problem. Traditionally, the cost function involves allocation of different power generation units, and it is used to find optimal combination of generating units while satisfying the system constraints.

Many different optimization techniques have been applied to solve ELD problems. Some of these techniques have known as classical optimization methods, such as linear programming, newton-based methods and quadratic programming [1-3]. Such classical methods are susceptible to initial points and may easily get stuck in local optimum points. To overcome these drawbacks, metaheuristic algorithms have been developed. These algorithms inspired by the idea of natural evolution are easy to implement, robust in finding solutions and have a wide range of applications in different research areas that make them become popular. In general, processes of these algorithms in optimization depend on trying different variations of design variables and using the information obtained to improve the solutions while satisfying all the equality, inequality and side constraints. The algorithms based on metaheuristic approach have been frequently used for solutions of ELD problems like GA (Genetic Algorithm) [4], DE (Differential Evolution) [5], PSO (Particle Swarm Optimization) [6], FA (Firefly Algorithm) [7], ABC (Artificial Bee Colony Algorithm) [8] and so on. Although the metaheuristic algorithms mentioned above have robust structures to achieve solutions, they do not guarantee to find a global optimum solution and their control parameters need to be adjusted according to the changed problems which is a time-consuming procedure.

Here, the recently developed metaheuristic algorithm, namely the Vortex Search (VS) algorithm [9], which has no additional problem-specific parameters, can be applicable to the optimization problems without control parameters tuning. The VS algorithm inspired by a vortex pattern achieves solutions by dynamically changing a vortex circle. On the other hand, examining the literature, the superiority of the algorithm employed in this study was emphasized by using the analog filter designs and numerical problems [9, 10], but as far as we know, there are no published studies which analyze the performance of the VS algorithm on the ELD problems. In this study, the proposed algorithm attempts to fill a gap in the literature by applying for solution of the complex ELD problem in the 20-unit test system. Simulation results show that the VS algorithm has better performance than the compared algorithms existed in the literature in terms of total generation costs and power losses.

The rest of the paper is formed as follows: In Section 2, 'Problem Formulation for Economic Load Dispatch': gives the formulation of the ELD problem; in Section 3 and Section 4, 'Vortex Search Algorithm and Its Implementation to the ELD Problem': explain the general structure of the proposed algorithm and implementation of the VS algorithm for the ELD problem used'; in Section 5, 'Numerical Results': shows the performance of the VS algorithm in the 20-unit test system, in Section 6, 'Conclusion': gives the conclusions related to this study.

2. Problem Formulation for Economic Load Dispatch

2.1. Formulation of the Fuel Cost

The objective of ELD problem depends on minimizing the total fuel cost F_T in Eq. 1 while satisfying constraints, such as ramp rate limits, prohibited operating zones and valve point effects. The objective function can be formulated as below:

$$F_T = \min \sum_{i=1}^{N} F_i(P_{Gi}) \tag{1}$$

where N is number of generating units and $F_i(P_{Gi})$ is fuel cost of i^{th} generating unit.

In common ELD problems, fuel cost function can be written as a form of quadratic function to be in Eq. 2.

$$F(P_{Gi}) = \alpha_i + \beta_i P_{Gi} + \gamma_i P_{Gi}^2 \quad (2)$$

The quadratic fuel cost formula doesn't express the general ELD fully, because it doesn't include ripple effects that are results of opened valves of large thermal generators, called valve point effect. The objective function can be formed by considering valve point effect that is given in Eq. 3:

$$F(P_{Gi}) = \alpha_i + \beta_i P_{Gi} + \gamma_i P_{Gi}^2 + \left| e_i x \sin(f_i x (P_i^{\min} - P_i) \right| \quad (3)$$

2.2. Constraints for the Equations of Economic Load Dispatch

The output of the generation unit must be inside the boundaries defined as the maximum and minimum values of the generation unit with the below inequality:

$$P_{Gi}^{\min} \le P_{Gi} \le P_{Gi}^{\max} \quad i=1,\ldots,N \quad (4)$$

where $P_{Gi}^{\max}$ and $P_{Gi}^{\min}$ are the maximum and the minimum powers of the i^{th} unit, respectively. Eq. 5 is referred as the power balance equation below:

$$\sum_{i=1}^{N} P_{Gi} = P_D + P_{loss} \quad (5)$$

where the total generated power P_{Gi} should be equal to the summation of total power demand P_D and transmission loss. P_{loss} represented in Eq. 6 by using B factor as below:

$$P_{loss} = \sum_{j=1}^{N} \sum_{i=1}^{N} P_j B_{ji} P_i + \sum_{i=1}^{N} B_{0j} P_j + B_{00} \quad (6)$$

where B_{00} is the loss coefficient constant , B_{0j} and B_{ji} are loss coefficients of the P_{loss} formula.

3. Vortex Search Algorithm

The VS is an effective technique defined to be a single-solution based metaheuristic algorithm which evolves solutions iteratively. It can be used to solve a wide range of the problems at different types due to its simplicity and robust structure [9, 10]. The processes of this algorithm can be divided into four main parts which are generating the initial solution, generating the candidate solutions, replacement of the current solution and the radius decrement process. These parts can be summarized in the following subsections.

3.1. Generating the initial solution

VS algorithm generates initial solutions by modelling a number of nested circles. For two dimensional optimization problems, the initial center μ_0 is found at first in Eq. 7 as follows:

$$\mu_0 = \frac{upperlimit + lowerlimit}{2} \quad (7)$$

where *upperlimit* and *lowerlimit* represent the side boundaries of the problem.

3.2. Generating the candidate solutions

By using Gaussian distribution, a number of neighbor solutions $C_t(s)$ are created randomly in *d* dimensional search space, where *t* is the iteration index which is selected as 0 at the beginning of the optimization process. The value of Σ, which represents a covariance matrix, is calculated by using Eq. 8.

$$\Sigma = \sigma^2 \left[I \right]_{dxd} \quad (8)$$

where σ^2 is the variance of the distribution and *I* is the *dxd* identity matrix. The initial standard deviation σ_0 can be described in Eq. 9 below:

$$\sigma_0 = \frac{max(upperlimit) - min(lowerlimit)}{2} \quad (9)$$

The value of σ_0 can also be defined as the initial radius of the outer circle r_0 for a two dimensional optimization problem.

3.3. Replacement of the current solution

The best solution $s' \in C_0(s)$ is determined and memorized to update the current circle center μ_0. The center value of the circle is utilized to achieve candidate solutions. After that, boundary control mechanism is used to determine whether the candidate solutions are inside the corresponding upper and lower bounds. If the candidate solutions exceed the boundaries, they can be shifted into boundaries by using Eq. 10.

$$s_k^i = \begin{cases} rand.(upperlimit^i - lowerlimit^i) + lowerlimit^i, & s_k^i < lowerlimit^i \\ s_k^i, & lowerlimit^i \le s_k^i \le upperlimit^i \\ rand.(upperlimit^i - lowerlimit^i) + lowerlimit^i, & s_k^i > upperlimit^i \end{cases} \quad (10)$$

where $rand$ denotes a uniformly distributed random number, $k=1,...,n$ and $i=1,...,d$. After locating the solutions into the appropriate boundaries, the memorized best solution s' is chosen as the new center of the second circle. Afterwards, VS applies the generation phase of the second step to generate a new set of solutions $C_1(s)$ by decreasing the radius. To complement the second step, VS uses the selection phase to evaluate the new set of solutions $C_1(s)$, and then selects a solution $s' \in C_1(s)$. If the selected solution has better objective function value than the best solution found so far, this solution will be the new best solution and the center of the new circle. These processes are repeated until termination criteria are met.

3.4. The radius decrement process

The VS algorithm employs the inverse incomplete gamma function to decrease the value of the radius at each iteration, according to following equation**:**

$$r_t = \sigma_0.(1/x).gammaincinv(x, a_t) \tag{11}$$

where x is a fixed value of 0.1, r_t denotes the radius of the circle at iteration index t and $gammaincinv$ is the inverse incomplete gamma function. The shape parameter a_t can be found by using Eq. 12 below:

$$a_t = a_0 - \frac{t}{maxItr} \tag{12}$$

where a_0 is chosen as to be a_0=1, t and $maxItr$ represent the iteration index and the maximum number of iterations, respectively. The radius r_t is updated during iterations to improve the solutions.

4. Implementation of the VS Algorithm to the ELD Problem

This section summarizes the optimization processes of the VS based ELD problem, as follows:

Step 1) Define the parameters of the 20-unit system, lower and upper boundaries of the units, maximum number of iterations and number of candidate solutions.

Step 2) Generate initial solutions representing a potential solution of the 20-unit test system mentioned to be $C_0(s)$ in Section 3.

Step 3) Check all the constraints. Any violated variable of $C_t(s)$ is directly set to given limit to satisfy the inequality constraints of the units in Eq. 4. For the equality constraints given in Eq. 5, if any variable of $C_t(s)$ exceeds the constraints, add the penalty term to the objective function as given below:

$$F = F(P_{Gi}) + k(\sum_{i=1}^{N} P_{Gi} - P_D - P_{loss}) \tag{13}$$

where k is the penalty factor.

Step 4) Calculate the total generation costs of the units.

Step 5) Improve the candidate solutions of $C_t(s)$ by decreasing the radius r_t according to Eq. 11.

Step 6) Memorize the best solution found so far.

Step 7) If the iteration has reached the maximum number of iterations, stop the optimization process; otherwise, increase the iteration by one and go to Step 3.

5. Numerical Results

In this section, the VS algorithm is applied for the 20-unit test system with loss coefficients to show the effectiveness of the algorithm. The input data of the system are adopted from [14] which are described in Table 1 and the load demand is taken as 2500 MW.

Table 1. Data for 20-Unit System

Unit	α **($/MW)**	β **($/MW)**	γ **($/MW)**	P_{Gi}^{min}	P_{Gi}^{max}
1	1000	18.19	0.00068	150	600
2	970	19.26	0.00071	50	200
3	600	19.8	0.0065	50	200
4	700	19.1	0.005	50	200
5	420	18.1	0.00738	50	160
6	360	19.26	0.00612	20	100
7	490	17.14	0.0079	25	125
8	660	18.92	0.00813	50	150
9	765	18.27	0.00522	50	200
10	770	18.92	0.00573	30	150
11	800	16.69	0.0048	100	300
12	970	16.76	0.0031	150	500
13	900	17.36	0.0085	40	160
14	700	18.7	0.00511	20	130
15	450	18.7	0.00398	25	185
16	370	14.26	0.0712	20	80
17	480	19.14	0.0089	30	85
18	680	18.92	0.00713	30	120
19	700	18.47	0.00622	40	120
20	850	19.79	0.00773	30	100

The convergence characteristic of the VS algorithm is depicted in Fig. 1. The performance of the proposed algorithm in the 20-unit test system is compared with the other algorithms/methods reported in the literature, which are BBO [15], BSA [11], CBA [13], GAMS [12], GSA [12], HM [14] and Lambda-iteration method [14], in terms of total generation cost and power loss. The comparative results of the VS algorithm are tabulated in Table 2. All the units, the cost and the loss values obtained by the VS algorithm are given in Table 3. It should be noted that the valve point effect is not considered but transmission loss is considered for this system.

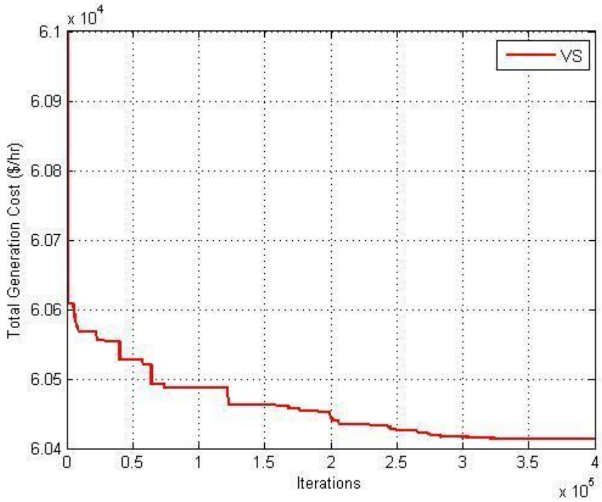

Fig. 1. Convergence characteristic of the VS algorithm for 20-unit test system

As can be shown in Table 2, the minimum total generation cost obtained by the VS algorithm is 60413.224 $/hr, which is lower than the compared algorithms. In the same table, when analyzing the power losses, it is clearly seen that the VS algorithm outperforms the other algorithms by achieving minimum power loss of 13.094 MW for the load demand of 2500 MW. On the other hand, Fig. 1 indicates that VS has an effective convergence ability which provides to explore the search space during the first half of the iterations and to exploit the solutions at the rest of the iterations.

Table 2. Comparison of total generation cost and power loss of the VS algorithm with the other reported results

Method	Total Generation Cost ($/hr)	Power Loss (MW)
VS	60413.224	13.094
CBA [13]	62456.632	91.963
GSA [12]	62456.633	91.965
GAMS [12]	62456.633	91.967
HM [14]	62456.634	91.966
BSA [11]	62456.692	91.893
Lambda-iteration [14]	62456.639	91.967
BBO[15]	62456.779	92.101

The generating unit values found by the VS algorithm illustrated in Table 3 also satisfy the side boundaries of the 20 generating units in this test system. The results reveal that the VS algorithm achieves better optimality than the other compared algorithms.

Table 3. ELD results of the VS algorithm

Unit	Generator Output (MW)
P1	600
P2	160.590
P3	50
P4	50.196
P5	92.881
P6	23.812
P7	124.999
P8	50.001
P9	111.996
P10	53.805
P11	264.473
P12	413.018
P13	120.535
P14	72.239
P15	98.941
P16	36.797
P17	30.646
P18	46.022
P19	82.135
P20	30
Total Power Output (MW)	2513.094
Power Loss (MW)	13.094
Total Generation Cost ($/hr)	60413.224

6. Conclusions

In this paper, the VS algorithm has been applied to solve ELD problem for the 20-unit test system. The structure of the algorithm is based on adaptive step size strategy by decreasing radius of circle which can be symbolized as a vortex pattern. The system has been analyzed in terms of total generation cost and power loss. The VS algorithm performs better than the other competitors in terms of minimizing total generation cost, especially; power loss is reduced greatly as compared to the other reported studies. The proposed approach also provides a good balance between exploration and exploitation that result in high accuracy when searching for the solutions. In future, the VS algorithm may be used for solutions of other power system optimization problems such as dynamic economic dispatch, generation scheduling and reactive power flow.

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