

Fuzzy PID Type STR based on SVR for Nonlinear Systems

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Abstract

In this study, generalized self-tuning regulator (STR) based on support vector regression (SVR) which was previously introduced [1] is deployed to control a continuously stirred tank reactor (CSTR) system. The parameters of the Fuzzy PI D controller utilized in the controller block are tuned via SVR based parameter estimator and system model blocks. The performance of the controller has been evaluated by simulations carried out on the CSTR system.

1. Introduction

Nonlinearity rarifies the control and predictability of system dynamic behavior. In order to attain successful control performance, the controller should act nonlinearly as the system does. For this purpose, nonlinear characteristics can be interfused to the conventional controller structures by introducing adaptation skills to the controller parameters via artificial intelligence and optimization techniques.

Support vector regression (SVR) which is among effective machine learning techniques, have recently been deployed to adjust the parameters of conventional controller structures. SVR based adaptive controller techniques have supplanted neural network (NN) and fuzzy logic (FL) model based adjustment methods in recent years since SVR assures the global minima of the optimization problem and hence by using SVR based structures system models can be identified accurately.

In this paper, strong characteristics of fuzzy control technique and SVR methodology are embodied in a Fuzzy PID Type STR based on SVR for nonlinear systems. Two separate SVR structures are employed in the self-tuning regulator (STR) architecture, one for estimating the system model and the other for approximating the controller parameters. Both SVR's are tuned in online manner. The performance evaluation of Fuzzy PID Type STR based on SVR has been carried out on a continuously stirred tank reactor (CSTR) system.

The paper is organised as follows: Our proposed generalized STR structure based on SVR and a description of Fuzzy PID Type STR based on SVR are given in Section II. The performance evaluation of the controller done on the CSTR system and simulation results are given in Section III. The paper ends with a brief section on conclusions and comments.

2. Generalized STR based on SVR

2.1. Generalized STR structure based on SVR

The adjustment mechanism for the proposed STR structure based on online SVR is illustrated in Fig. 1. The mechanism consists of two separate SVR structures: SVR_{estimator} is deployed

to calculate the controller parameters and SVR_{model} is utilized to predict the future behavior of the controlled system. Since SVR has multi input single output (MISO) structure, it is required to employ a separate SVR_{estimator} for each approximated controller parameter. So, the number of the SVR_{estimator} structures utilized in parameter estimator block is equal to the number of the controller parameters to be approximated. As an example, three SVR_{estimator} structures should be deployed for a PID type STR to forecast K_p, K_i, and K_d parameters.

Training, prediction and control stages are carried out consecutively in online manner in SVR_{estimator} and SVR_{model} blocks. As can be seen from Fig. 1, the controller parameters are approximated via SVR_{estimator} as:

$$\theta_m(\Pi_{mc}) = f_{\text{estimator}_m}(\Pi_{mc}) \\ = \sum \alpha_{mk} K_{\text{estimator}_m}(\Pi_{mk}, \Pi_{mc}) + b_{\text{estimator}_m} \quad (1)$$

$$m \in \{1, 2, \dots, p\}$$

where Π_{mc} denotes the current input of m^{th} estimator, $K_{\text{estimator}_m}$ is the kernel matrix, α_{mk} , Π_{mk} and $b_{\text{estimator}_m}$ indicate the parameters of the m^{th} estimator, $f_{\text{estimator}_m}$ is the regression function to be optimized in training [1]. Thus, the control signal produced by controller block is acquired as:

$$u_n = g_{\text{controller}}([\theta_1(\Pi_{1c}) \ \theta_2(\Pi_{2c}) \dots \theta_m(\Pi_{mc})], \mathbf{X}_c) \quad (2)$$

where $g_{\text{controller}}$ is the control law computed as the output of the controller, θ_m indicates the m^{th} parameter of the controller and \mathbf{X}_c is the current input vector of the controller. After the control signal is computed, it is applied to the SVR_{model} to observe the possible behavior of the system against adjustment of controller parameters. Ideally, it is anticipated that \hat{y}_{n+1} will converge to y_{n+1} in the long run. The system output is predicted as follows via SVR_{model}

$$\hat{y}_{n+1} = f_{\text{model}}(\mathbf{M}_c) = \sum_{j \in \text{SV}} \lambda_j K_{\text{model}}(\mathbf{M}_j, \mathbf{M}_c) + b_{\text{model}} \quad (3)$$

where f_{model} and K_{model} are the regression function and the kernel matrix of the system model respectively, \mathbf{M}_j 's are support vectors, \mathbf{M}_c is current input, and λ_j , and b_{model} are the parameters of the system model to be adjusted. Depending on the approximated tracking error ($\hat{e}_{t_{n+1}}$), the model parameters of parameter estimator are optimized so as to force the system

output to follow the reference signal. Then, the optimal controller parameters are obtained via trained parameter estimator, and the recomputed control signal is applied to the real system to obtain the actual output (y_{n+1}). Thus, the training data pair (\mathbf{M}_c , y_{n+1}) for SVR_{model} can be obtained.

2.2. Fuzzy PID Type STR based on SVR

The structure of an incremental fuzzy PID controller [2] is delineated in Fig. 2, where e_n denotes the tracking error and $u_{\text{FLC}_{\text{PID}_n}}$ is the output of fuzzy PID controller at time index n. The inputs of the fuzzy controller are scaled tracking error (e_{f_n}) and derivative of tracking error (\dot{e}_{f_n}) which are defined as:

$$e_{f_n} = K_{e_n} e_n, \quad \dot{e}_{f_n} = K_{de_n} [e_n - e_{n-1}] \quad (4)$$

The control signal ($u_{\text{FLC}_{\text{PID}_n}}$) and the components of the controller in Fig. 2 are computed as [3][4]:

$$\begin{aligned} u_{\text{FLC}_{\text{PD}_n}} &= \Psi_n f_{\text{FLC}_{\text{PD}}} (e_{f_n}, \dot{e}_{f_n}) \\ u_{\text{FLC}_{\text{PI}_n}} &= u_{\text{FLC}_{\text{PI}_{n-1}}} + \Delta u_{\text{FLC}_{\text{PI}_n}}, \quad \Delta u_{\text{FLC}_{\text{PI}_n}} = \beta_n f_{\text{FLC}_{\text{PI}}} (e_{f_n}, \dot{e}_{f_n}) \\ u_{\text{FLC}_{\text{PID}_n}} &= f_{\text{FLC}_{\text{PID}}} (e_{f_n}, \dot{e}_{f_n}, \Psi_n, \beta_n) = u_{\text{FLC}_{\text{PI}_n}} + u_{\text{FLC}_{\text{PD}_n}} \end{aligned} \quad (5)$$

where K_{e_n} , K_{de_n} are input scaling coefficients, Ψ_n and β_n denote output scaling coefficients for the PD and PI parts of the fuzzy PID controller to be optimized, $f_{\text{FLC}_{\text{PD}}}$ is the fuzzy controller, respectively. By taking the transient and steady state requirements of the controlled system into consideration, the derivative and integral parts of the fuzzy controller can be combined with varying weights via input-output scaling coefficients in order to provide more accurate tracking performance. In our simulations, triangular shaped membership functions with cores $\{-1, -0.4, 0, 0.4, 1\}$ as in [3] are employed for both inputs e_{f_n} and \dot{e}_{f_n} as shown in Fig. 3 where Γ_{z_n} indicates the z^{th} fired fuzzy rule. The fired fuzzy rule (Γ_{z_n}) is formulated as

$$\Gamma_{z_n} = f_{\text{rules}} (e_{f_n}, \dot{e}_{f_n}, k_{z1}, k_{z2}) = S_z + P_z - 1, \quad z \in \{1, 2, 3, 4\} \quad (6)$$

where $S_z = \frac{1}{1 + \exp(-k_{z1} e_{f_n})}$, $P_z = \frac{1}{1 + \exp(-k_{z2} \dot{e}_{f_n})}$ and $k_{11} = k_{12} = 4$, $k_{21} = k_{22} = 5$, $k_{31} = k_{32} = 6$, $k_{41} = k_{42} = 7$.

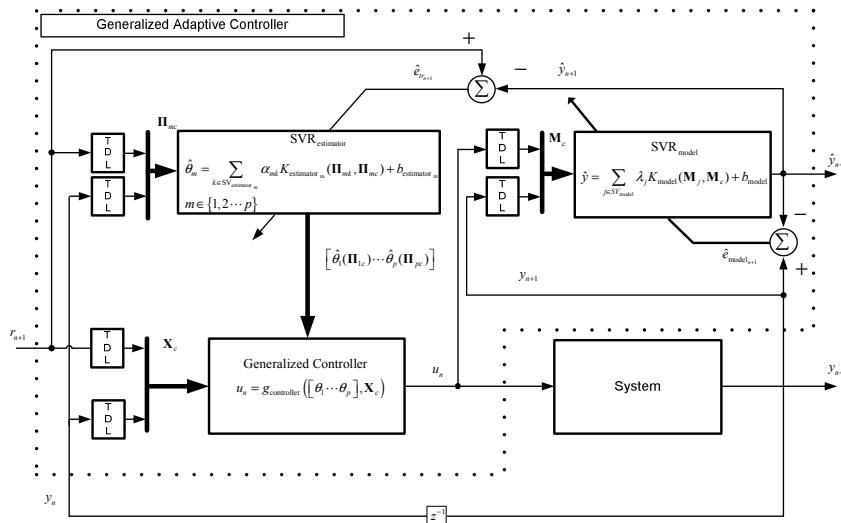


Fig. 1. Generalized self-tuning regulator based on online SVR

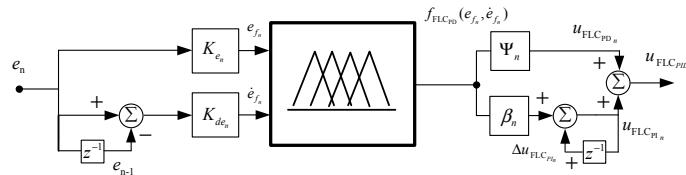


Fig. 2 Fuzzy PID controller

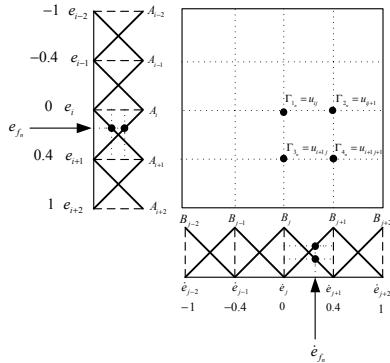


Fig. 3 The membership functions for inputs and rule base

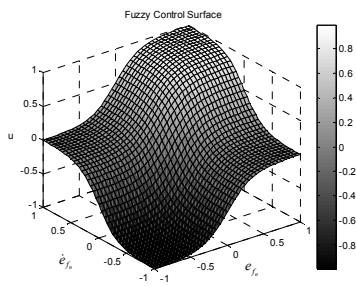


Fig. 4 Fuzzy control surface

The fuzzy rule surface depicted in Fig. 4 is associated with two sigmoid functions as in (6). Center of gravity method has been used as the defuzzification method. Thus, as a result of defuzzification step, the output of the fuzzy controller can be attained as [3]

$$f_{\text{FLC}_{\text{PD}}}(e_{f_n}, \dot{e}_{f_n}) = \frac{\sum_{z=1}^4 w_{z_n} \Gamma_{z_n}}{\sum_{z=1}^4 w_{z_n}} = \sum_{z=1}^4 w_{z_n} \Gamma_{z_n}, \quad \sum_{z=1}^4 w_{z_n} = 1 \quad (7)$$

where w_{z_n} and Γ_{z_n} denote the z^{th} firing strength and fired rule, and are given as follows

$$\begin{aligned} w_{1_n} &= A_i(e_{f_n}) B_j(\dot{e}_{f_n}), \quad w_{2_n} = A_{i+1}(e_{f_n}) B_j(\dot{e}_{f_n}) \\ w_{3_n} &= A_i(e_{f_n}) B_{j+1}(\dot{e}_{f_n}), \quad w_{4_n} = A_{i+1}(e_{f_n}) B_{j+1}(\dot{e}_{f_n}) \end{aligned}$$

and

$$\begin{aligned} A_i(e_{f_n}) &= \frac{e_{i+1} - e_{f_n}}{e_{i+1} - e_i}, \quad A_{i+1}(e_{f_n}) = \frac{e_{f_n} - e_i}{e_{i+1} - e_i} \\ B_j(\dot{e}_{f_n}) &= \frac{\dot{e}_{j+1} - \dot{e}_{f_n}}{\dot{e}_{j+1} - \dot{e}_j}, \quad B_{j+1}(\dot{e}_{f_n}) = \frac{\dot{e}_{f_n} - \dot{e}_j}{\dot{e}_{j+1} - \dot{e}_j} \end{aligned}$$

are the corresponding membership function values in response to e_{f_n} and \dot{e}_{f_n} . $\Gamma_n = [\Gamma_1 \ \Gamma_2 \ \Gamma_3 \ \Gamma_4] = [u_{ij} \ u_{ij+1} \ u_{i+1j} \ u_{i+1j+1}]$ indicate the fired rules in the rule surface since four fuzzy rules are fired at a time depending on the defined input-membership functions[1]. The controller parameter vector $\boldsymbol{\theta} = [K_{e_n} \ K_{de_n} \ \Psi_n \ \beta_n]^T$ is approximated via SVR_{estimator} as follows:

$$\boldsymbol{\theta} = \begin{bmatrix} K_{e_n} \\ K_{de_n} \\ \Psi_n \\ \beta_n \end{bmatrix} = \begin{bmatrix} \sum_{k \in \text{SV}_{\text{estimator}_e}} \alpha_{ek} K_{\text{estimator}_e}(\Pi_{ek}, \Pi_{ec}) + b_{\text{estimator}_e} \\ \sum_{k \in \text{SV}_{\text{estimator}_{de}}} \alpha_{dek} K_{\text{estimator}_{de}}(\Pi_{dek}, \Pi_{dec}) + b_{\text{estimator}_{de}} \\ \sum_{k \in \text{SV}_{\text{estimator}_{\Psi}}} \alpha_{\Psi k} K_{\text{estimator}_{\Psi}}(\Pi_{\Psi k}, \Pi_{\Psi c}) + b_{\text{estimator}_{\Psi}} \\ \sum_{k \in \text{SV}_{\text{estimator}_{\beta}}} \alpha_{\beta k} K_{\text{estimator}_{\beta}}(\Pi_{\beta k}, \Pi_{\beta c}) + b_{\text{estimator}_{\beta}} \end{bmatrix} \quad (8)$$

Thus, the control signal ($u_{\text{FLC}_{\text{PID}_n}}$) applied to the system is computed as :

$$\begin{aligned} u_{\text{FLC}_{\text{PID}_n}} &= f_{\text{FLC}_{\text{PID}}}(\boldsymbol{\theta}, \mathbf{X}) \\ &= u_{\text{FLC}_{\text{PID}_n}} + (\beta_n + \Psi_n) f_{\text{FLC}_{\text{PD}}}(K_{e_n} x_1, K_{de_n} x_2) \\ \mathbf{X} &= \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} e_n \\ e_n - e_{n-1} \end{bmatrix} \end{aligned} \quad (9)$$

where $\boldsymbol{\theta}$ denotes the controller parameters and \mathbf{X} indicates the input of the controller. The proposed mechanism merges the strong characteristics of fuzzy control technique and SVR methodology in the structure of Fuzzy PID Type STR based on SVR for nonlinear systems.

3. Simulation Results

The performance of the Fuzzy PID type STR has been assessed by simulations performed on a nonlinear CSTR system illustrated in Fig. 5.

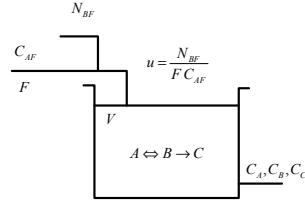


Fig. 5 CSTR system

CSTR is a widely utilized chemical reactor system in industry, mainly used to produce polymers, pharmaceuticals, and other various chemical products [5]. CSTR which is also referred as a vat or backmix reactor [6][7] is a chemical reactor in which isothermal, liquid-phase, successive multicomponent chemical reactions can be successfully performed [8][9].

Consider a chemical reaction system given in (10) where the inlet reactants (A,B) are mixed in a vessel with constant volume via an agitator and return the product C.

$$A \rightleftharpoons B \rightarrow C \quad (10)$$

There are two reaction sites: first one is between A-B, and second is among B-C. It is aimed to regulate the concentration of product C by adjusting the molar feed rate of reactant B. The dynamics of the reaction which is a third order highly non-linear time varying system are expressed with the following set of differential equations by Kravaris and Palanki[8]

$$\begin{aligned}\dot{x}_1(t) &= 1 - x_1(t) - D_{a_1}x_1(t) + D_{a_2}x_2^2(t) \\ \dot{x}_2(t) &= -x_2(t) + D_{a_1}x_1(t) - D_{a_2}x_2^2(t) - D_{a_3}d_2(t)x_2^2(t) + u(t) \\ \dot{x}_3(t) &= -x_3(t) + D_{a_3}d_2(t)x_2^2(t)\end{aligned}\quad (11)$$

where $x_1(t)$, $x_2(t)$ and $x_3(t)$ represent the states obtained from the concentrations of reactant A, middle reactant B and product C, respectively, $d_2(t)$ represents the activity of the second reaction site on the bifunctional catalyst, $D_{a_1} = 3$, $D_{a_2} = 0.5$, $D_{a_3} = 1$, $u(t)$ is the control signal, $x_3(t)$ is the output of the system and $d_2(t)$ is the time-varying parameter of the system, the nominal value of which is $d_2(t) = 1$ as given in [9][10][11]. The performance of the system has been examined for three different cases: 1) Nominal case: when there is no noise and parametric uncertainty in the system 2) Measurement noise case: 30 dB measurement noise is added to the output of the system 3) Parametric uncertainty: time-varying parameter is introduced to the system. $\mathbf{M}_c = [u_{n-1} \cdots u_{n-n_u} y_n \cdots y_{n-n_y}]^T$ is used as the input feature vector for SVRmodel where $n_u = n_y = 2$. The inputs feature vectors of the SVRestimator's for fuzzy PID controller for all three cases are assigned as $\Pi_{ke} = [P_n \ y_n]^T$, $\Pi_{kde} = [I_n \ r_n]^T$, $\Pi_\psi = [P_n \ D_n]^T$, $\Pi_\beta = [y_n \ u_{n-1}]^T$.

3.1. Nominal Conditions

First, the simulations are performed without any noise applied to the system and with all parameters fully known. The tracking performance of the controller for step and sinusoidal reference signals, control signals computed by the controller and alternation of the controller parameters for Fuzzy PID Type STR are given in Fig. 6-8. The time-varying parameter, $d_2(t)$ is taken as its nominal value $d_2(t) = 1$. It can be observed from Fig. 6 that the system tracks the reference signals accurately for fuzzy PID controller. The tuned controller parameters of fuzzy PID are depicted in Fig. 7.

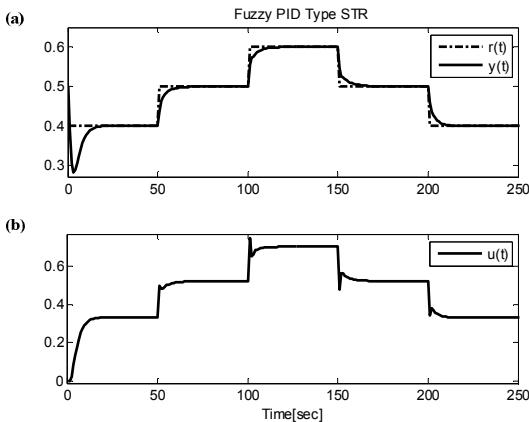


Fig. 6 System Output(a), Control Signal(b)

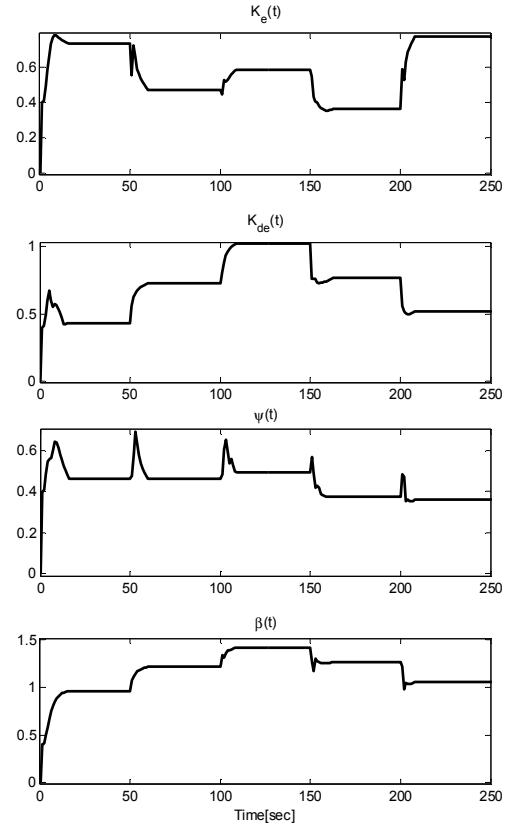


Fig. 7 Fuzzy PID Parameters

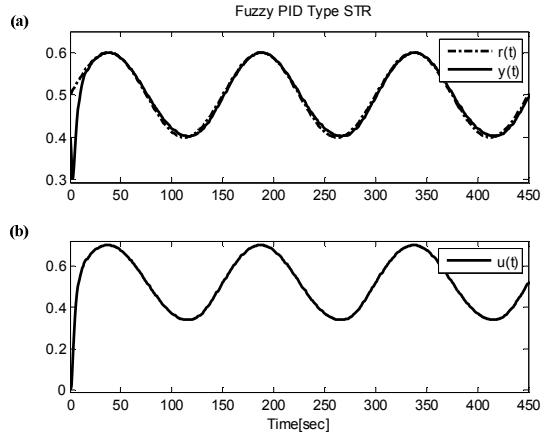


Fig. 8 System Output(a), Control Signal(b)

3.2. Mesurement Noise

The robustness and performance of the controller with respect to measurement noise is evaluated by adding a zero mean Gaussian noise with 30 dB SNR to the measured output of the system.

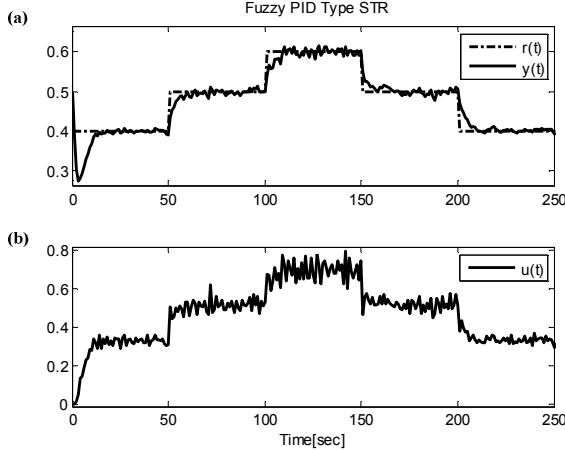


Fig. 9 System Output(a), Control Signal(b)

The tracking performance and control input of controller with Gaussian measurement noise added to the system is depicted in Fig. 9.

3.3. Parametric Uncertainty

The robustness and performance of the controller are examined in terms of parametric uncertainty, by varying a system parameter. In our simulation, $d_2(t)$ is chosen as the time varying parameter and it changes slowly in the vicinity of its nominal value with $d_2(t) = 1 + 0.1\sin(0.048\pi t)$.

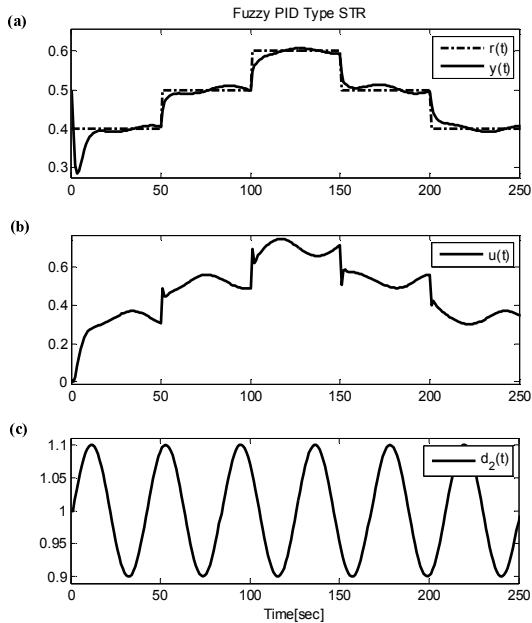


Fig. 10 System Output(a), Control Signal(b) , time-varying parameter (c)

The disturbance rejection performance of the controller is illustrated in Fig. 10. When the control signals in Fig. 6 are compared with Fig. 10, it can be concluded that the uncertainty in system parameter is successfully rejected by the controller.

4. Conclusion

In this paper, nonlinear CSTR system has been controlled by a Fuzzy PID Type STR based on SVR. The adjustment mechanism consists of two separate SVR structures; SVR_{estimator} and SVR_{model} which are concurrently utilized to compute the controller parameters and estimate the system model. The SVR_{estimator} structure utilized here and proposed first in [1], is capable of tuning the controller parameters without an explicit knowledge of the desired parameter values. The results indicate that the closed-loop system can be successfully compelled to track the reference signal with small transient and steady-state errors. In future works, new SVR based STR's are planned to be developed for nonlinear systems, by employing different types of parameterized controller types in the STR block.

7. References

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