Solution to Short Term Hydrothermal Scheduling Problem for a Power System Area Including Limited Energy Supply Thermal Units by Using Modified Subgradient Algorithm Based on Feasible Values

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Abstract

In this paper, the modified subgradient algorithm based on feasible values (F-MSG) is applied to a short term hydrothermal coordination problem for a power system including limited energy supply thermal units. In the proposed method, all the constraints such as units' power generation limits, transmission line capacities, bus voltage magnitude limits, hydraulic units' minimum and maximum reservoir volume limits and hydraulic units' starting and final reservoir water volumes are added into the optimization model. Actual transmission losses are inserted into the optimization model as equality constraints via load flow equations. It is also assumed that limited energy supply thermal units are fueled under *take-or-pay* agreement. The proposed method is tested on 16 bus test system which includes three normal thermal plants, two limited energy supply thermal plants and four serial-parallel hydraulically coupled hydro plants and better results are obtained in terms of optimal fuel cost values.

1. Introduction

Short term hydrothermal coordination problem (STHCP) can be mathematically formulated as a constrained non-linear optimization problem where the total thermal generation cost is tried to be minimized during the operation period by satisfying all possible physical, hydraulic and electric constraints. The operation period can range from one day to a week and is divided into subintervals where the system load values are known and assumed to remain constant [1].

Under take-or-pay (T-O-P) fuel contract, a minimum value of the total fuel amount to be spent by the limited energy supply thermal units during the operation period is determined in advance. If the utility company fails to use this minimum amount, it agrees to pay the cost of the minimum amount[2].

In the literature, STHCP was solved by various solution methods. Some of these methods use the gradient method [1], the spot price of electricity algorithm [2], the genetic algorithm [3], the differential evolution method [4] and the partical swarm optimization method [5]. In addition to these, a survey on the methods applied to solve the STHCP can be found in [6].

The modified subgradient algorithm based on feasible values (F-MSG) was developed by Kasimbeyli et al [7]. The main idea of the F-MSG algorithm is to find feasible and infeasible values of the problem by using augmented sharp LaGrange function. In the F-MSG algorithm, first an upper bound for the cost function value is specified in advance and then the algorithm tries to find a solution where the cost function is less than or equal to the upper bound and also all constraints are satisfied. If it finds it (feasible total cost), the upper bound is decreased a certain amount, otherwise (infeasible total cost) the upper bound is increased a certain amount. The amount of decrease or increase on the upper bound for the next iteration depends on if any feasible or infeasible total cost value was obtained in the previous iterations. This process continues until absolute value of the change in the upper bound is less than a predefined tolerance value [7].

F-MSG algorithm has been already applied to non-convex economic dispatch problem [8-9]. Furthermore, power dispatch problem including limited energy supply thermal units [10-11], non-convex pumped-storage hydraulic unit scheduling problem [12-13] and short term hydrothermal coordination problem [14] were solved via F-MSG method. To our knowledge, the proposed algorithm has not been applied to the problem considered in this paper so far.

2. Problem Formulation

The STHCP considered in this paper can be mathematically formulated as given below.

$$\min\ F_T=\sum_{j=1}^{j_{max}}\sum_{i\in \boldsymbol{N}_T} F_i(P_{Gi,j})\,t_j \tag{1}$$

Subject to

$$P_{Gi,j}-P_{Load\,i,j}-\sum_{k\in \boldsymbol{N}_{Bi}} p_{ik,j}=0,\ \ i=1,2,\cdots \qquad \cdots \tag{2}$$

$$Q_{Gi,j}-Q_{Load\,i,j}-\sum_{k\in \boldsymbol{N}_{Bi}} q_{ik,j}=0,\ \ i=1,2,\cdots \qquad \cdots \tag{3}$$

$$P_{Gi}^{min}\le P_{Gi,j}\le P_{Gi}^{max},\qquad i\in\{\boldsymbol{N}_T\cup \boldsymbol{N} \qquad \cdots \tag{4}$$

$$Q_{Gi}^{min}\le Q_{Gi,j}\le Q_{Gi}^{max},\qquad i\in\{\boldsymbol{N}_T\cup \boldsymbol{N} \qquad \cdots \tag{5}$$

$$p_{l,j}\le p_l^{max},\qquad l\in \boldsymbol{L},\ \ j=1,2,\cdots \tag{6}$$

$$U_i^{min}\le U_{i,j}\le U_i^{max}, i=1,2,\cdots \qquad {}^{r}ef, vc,\ j=1,\cdots \tag{7}$$

$$C_{spent}-C_{tot}=0,\quad C_{spent}=\sum_{j=1}^{j_{max}}\sum_{T\in \boldsymbol{N}_{LS}} C_T(P_{Gi,j})\,t_j \tag{8}$$

$$V_{i,j}=V_{i,j-1}+(r_{i,j}-q_{GHi,j}-s_{i,j})t_j+\sum_{m\in \boldsymbol{N}_{Hi}}(q_{GHm}+s_m)\,t_j, \quad i\in \boldsymbol{N}_H,\ j=1,\cdots\ j_{max} \tag{9}$$

$$q_{Hi}^{\min}\le q_{GH}\left(P_{Gi,j}\right)\le q_{Hi}^{\max},\quad i\in \boldsymbol{N}_H, j=1,\cdots \tag{10}$$

$$V_i^{\min}\le V_{i,j}\le V_i^{\max},\qquad i\in \boldsymbol{N}_H,\quad j=1,\cdots \tag{11}$$

$$V_{i,0}=V_i^{start},\quad V_{i,j\max}=V_i^{end},\quad i\in \boldsymbol{N}_H \tag{12}$$

The meanings of the symbols used in this paper are given in the List of Symbols section.

2.1. Determination of Line Flows and Power Generations

In order to express the objective function in terms of independent variables of our optimization model, line flows should be written in terms of bus voltage magnitudes and phase angles, off-nominal tap settings, susceptance values of SVAR systems (see equations (1), (2) and (3)). The active and reactive power generations of the i^{th} unit connected to bus i in the j^{th} subinterval can be calculated as below by using the active and reactive power flows in the j^{th} subinterval (for expression of $p_{ik,j}$ and $q_{ik,j}$ interms of independent variables of our optimization model please refer to reference [8]) and equations (2) and (3):

$$P_{Gi,j} = P_{Load\,i,j} + \sum_{j \in N_{Bi}} p_{ik,j} \quad \text{and} \quad Q_{Gi,j} = Q_{Load\,i,j} + \sum_{k \in N_{Bi}} q_{ik,j} \quad (13)$$

Also, the total active transmission loss of the system can be calculated as;

$$p_{loss\,ik,j} = p_{ik,j} + p_{ki,j} \quad \text{and} \quad P_{LOSS} = \sum_{i \in N} \sum_{k \in N, k \neq i} p_{ik,j} \quad (14)$$

2.2. Converting Inequality Constraints into Equality Constraints

Since the F-MSG algorithm requires that all constraints need to be expressed in equality constraint form, the inequality constraints in the optimization model should be converted into the corresponding equality constraints [7]. The double sided inequality $x_i^- < x_{i,j} < x_i^+$ can be written as the following two inequalities;

$$h_i^+(x_{i,j}) = (x_{i,j} - x_i^+) \le 0 \;,\; h_i^-(x_{i,j}) = (x_i^- - x_{i,j}) \le 0 \quad (15)$$

The above inequalities can be rewritten as continuous equality forms by the followings, respectively [8]:

$$h_i^{eq+}(x_{i,j}) = \max\left\{0, (x_{i,j} - x_i^+)\right\}, h_i^{eq-}(x_{i,j}) = \max\left\{0, (x_i^- - x_{i,j})\right\} \quad (16)$$

If the inequality $x_i^- \le x_{i,j} \le x_i^+$ is satisfied, then it is obvious that $(x_{i,j} - x_i^+) \le 0$, $(x_i^- - x_{i,j}) \le 0$ and $\max\left\{0, (x_{i,j} - x_i^+)\right\} = 0$, $\max\left\{0, (x_i^- - x_{i,j})\right\} = 0$. So inequality constraints in (15) can be represented by the corresponding equality constraints in (16). In this paper the inequality constraints, given in section 2, are converted into the corresponding equality constraints in this manner without adding any extra independent variable into the optimization model.

3. The Modified Subgradient Algorithm Based on Feasible Values

The independent (decision) variables of the method are made up voltage magnitudes and phase angles of the buses (except reference bus), the tap settings of the off-nominal tap ratio transformers and the susceptance values of the SVAR systems in the network. The method uses an augmented LaGrange function that is called as sharp LaGrange function. The F-MSG algorithm proposed to solve the dispatch problem of each subinterval based on the modified subgradient method based on feasible values is given in reference [8-9] and [14] in detailed manner. The reader should refer to those references to examine the F-MSG algorithm.

3.1. The Proposed Solution Technique for the STHCP

F-MSG algorithm can be applied to each subinterval of the considered problem one by one to reduce the solution time and complexity of the problem, However, the net water usage constraints of the hydraulic units and the total fuel consumption constraint of the limited energy supply thermal units cannot be controlled in this manner. Therefore, we proposed the following iterative method to solve STHCP described in this paper.

INITIAL STEP

- Take ITER=0
- Apply F-MSG to STHCP of each subinterval one by one without considering any hydraulic and limited energy fuel consumption constraints given by eq (8-12).
- At the solution point, calculate the followings and go to step 1.

$$F_{OPT}^{*(ITER)} = \sum_{j=1}^{j_{max}} \sum_{i \in \boldsymbol{N}_T} F_i(P_{Gi,j}) t_j \;,\; P_{LOSS,j}^{\;*} = P_{LOSS,j} \;,\quad j = 1,2,...,j_{\max}$$

STEP-1

- ITER=ITER+1
- Solve the following sub-problem by using $F_{OPT}^{*(ITER)}$ and $P_{LOSS,j}^{\;*}$ values obtained in the previous step

$$\min \left\{ \left(\sum_{j=1}^{j_{max}} \sum_{i \in \boldsymbol{N}_T} F_i(P_{Gi,j}) t_j \right) - F_{OPT}^{*(ITER-1)} \right\}$$

Subject to

$$\sum_{i \in N_T} P_{Gi} + \sum_{k \in N_H} P_{Gk} - P_{LOSS,j}^{\;*} - P_{LOAD,j} = 0 \,,\quad j = 1,2,...,j_{\max}$$

$$P_{Gi}^{min} \le P_{Gi,j} \le P_{Gi}^{max}, \qquad i \in \{\boldsymbol{N}_T \cup \boldsymbol{N} \qquad \cdots$$

$$C_{spent} - C_{tot} = 0,\; C_{spent} = \sum_{j=1}^{j_{max}} \sum_{T \in \boldsymbol{N}_{LS}} C_T(P_{Gi,j}) t_j$$

$$V_i^{\min} \le V_{i,j} \le V_i^{\max}, \qquad i \in \boldsymbol{N}_H, \quad j = 1, \cdots$$

$$V_{i,j} = V_{i,j-1} + (r_{i,j} - q_{GHi,j} - s_{i,j}) t_j + \sum_{m \in \boldsymbol{N}_{Hi}} (q_{GHm} + s_m) t_j \,,$$
$$i \in \boldsymbol{N}_H, j = 1, \cdots \; j_{max}$$

$$q_{Hi}^{\min} \le q_{GH}\left(P_{Gi,j}\right) \le q_{Hi}^{\max}, \quad i \in \boldsymbol{N}_H, j = 1, \cdots$$

$$V_{i,0} = V_i^{start}, \quad V_{i,j\max} = V_i^{end}, \quad i \in \boldsymbol{N}_H$$

- At the solution point, take

$P_{Gi,j}^{\;**} = P_{Gi,j}, \qquad i \in \boldsymbol{N}_H,\; j = 1,2,\cdots$ and go to step 2.

STEP-2

- Take $P_{Gi}^{min} = P_{Gi}^{max} = P_{Gi,j}^{\;**}, i \in \boldsymbol{N}_H,\; j = 1,2,\cdots$
- Apply F-MSG to STHCP of each subinterval one by one with considering all constraints given by equations (2)-(7). At the solution point, calculate the followings

$$F_{OPT}^{*(ITER)} = \sum_{j=1}^{j_{max}} \sum_{i \in \boldsymbol{N}_T} F_i(P_{Gi,j}) t_j \;,\; P_{LOSS,j}^{\;*} = P_{LOSS,j} \;,\quad j = 1,2,...,j_{\max}$$

- If ITER=1, then go to step 1. Else calculate $DECR = F_{OPT}^{*(ITER)} - F_{OPT}^{*(ITER-1)}$
- If $DECR < 0$, then go to step 1. Otherwise stop. $F_{OPT}^{*(ITER-1)}$ is the optimal solution to STHCP.

In the initial step, F-MSG algorithm is applied to each subinterval of the considered problem one by one without considering any hydraulic constraints in order to obtain the lowest possible fuel cost value of the considered system. This lowest cost value is taken as starting point for the proposed method. Since hydraulic constraints were not considered in the initial step, the active power generations should be recalculated in order to obtain a new solution whose total cost value is closer to starting point's total cost and also all the hydraulic constraints are satisfied [14]. This is performed in step 1 by solving the sub-problem over the whole operation period. *In step 1, active power generations of the units are taken as decision variables.* Although all hydraulic constraints are satisfied in this new solution point, it is still not an exact solution to STHCP since any constraints related with the exact reactive power generation consumption balance, exact reactive power generation limits and security constraints (line flow and bus voltage magnitude constraints) are not considered in the sub-problem of step 1. Therefore, power generations of thermal units are recalculated in order to find an exact solution to STHCP in step 2 [14]. This time, F-MSG can be applied in each subinterval one by one without having any violations on the hydraulic constraints since the active generations of the hydraulic units remain constant at the values obtained in step 1. *Note that the voltage magnitudes and phase angles of the buses (except the reference bus), tap settings of the off-nominal tap ratio transformers and susceptance values of the SVAR systems in the network are taken as decision variables* in step 2 [14].

In the proposed method, step 1 and step 2 can be considered as one iteration. The solutions obtained at the end of each iteration are the actual solutions to STHCP since the whole model described by equations (1)-(12) are considered in step 2. Therefore, we carry out these iterations until there is not any further decrease on the optimal total cost value.

4. Numeric Example

The proposed dispatch technique is tested on 16 bus test system which has three normal thermal (connected to bus 1, 8, 15) and two limited energy supply gas-fired thermal (connected to bus 4, 5) and four serial-parallel hydraulically coupled hydro plants [2]. Cost rate functions for thermal units are taken as convex functions. A 24-hr operation period having six equal subintervals is considered. Please refer to [2] for the system's single-line diagram, line, bus, load and generator data. Reservoir storage limits, starting and ending water volumes, total water amounts to be used by hydraulic units and hydraulic relationship among the hydraulic units, the minimum total gas amount to be burned by the limited energy supply thermal units can be also found in [2]. Active power transmission capacities for all lines are taken as 1.50 p.u. Bus 1 is chosen as the reference bus, and its voltage magnitude and phase angle are taken as 1.05 p.u. and 0.0 rad, respectively. The lower and upper limits of all bus voltage magnitudes are taken as 0.90 p.u. and 1.10 p.u., respectively. The simulation program was coded in MATLAB 6.1 and GAMS 21.5 with a Conopt-type solver.

First of all, we applied the initial step described in section 3.2 and found the lowest possible fuel cost value of the dispatch problem as 171501.5038 R. The solution point data obtained in the initial step are given in table 1 and table 2, respectively.

As it seen from table 1, in each subinterval most part of the load demand is supplied by the hydraulic units which are connected to bus 10, 12, 14 and 16. It is also obvious that the ending reservoir volume constraints given by eq (12) are not satisfied (see table 2). Note that the hydraulic units should have the following ending reservoir volumes (as acre-ft) $V_{10}^{end} = 48000$, $V_{12}^{end} = 46600$, $V_{14}^{end} = 40600$, $V_{16}^{end} = 50600$ [2].

Table 1. Active power generations of the units obtained in the initial step (all the values are in pu).

	Subinterval (j)					
	1	2	3	4	5	6
$P_{G1,j}$	1.6858	2.4865	2.4489	2.1175	2.6043	2.2038
$P_{G4,j}$	0.4501	0.4500	0.4521	0.8582	0.5407	0.4500
$P_{G5,j}$	0.5820	0.6293	0.8510	0.9917	1.0565	0.4850
$P_{G8,j}$	0.5000	0.5000	0.5000	0.8903	0.5000	0.6899
$P_{G15,j}$	0.4500	0.4500	0.4500	0.4500	0.4500	0.4500
$P_{G10,j}$	1.1859	1.3500	1.3500	1.3447	1.3500	0.9848
$P_{G12,j}$	0.4990	0.7985	0.9472	0.9230	0.8611	0.8611
$P_{G14,j}$	0.5420	0.6939	0.7988	0.8088	0.8580	0.6456
$P_{G16,j}$	1.0472	1.1539	1.2278	1.2895	1.2206	1.0070
P_{LOSS}	0.1419	0.2121	0.2758	0.2738	0.2913	0.1770

Table 2. Starting and ending reservoir water volumes of hydraulic units obtained in the initial step (all values are given in acre-ft).

Hydraulic Unit (i)	$V_{i,0}$	$V_{i,6}$
10	50000.00	41527,78
12	45000.00	49265,34
14	46600.00	38455,80
16	40000.00	54704,65

Then we began first iteration by applying step 1. In step 1, we found an intermediate solution in which all the hydraulic constraints are met. The solution point data obtained in the first step are given in table 3 and table 4, respectively.

Table 3. Active power generations of the units obtained in step 1 of the first iteration (all the values are in pu).

	Subinterval (j)					
	1	*2*	*3*	*4*	*5*	*6*
$P_{G1,j}$	2.3304	2.3304	2.3304	2.3304	2.3304	2.3304
$P_{G4,j}$	1.5783	1.5783	1.5783	1.5783	1.5783	1.5783
$P_{G5,j}$	0.7229	0.7229	0.7229	0.7229	0.7229	0.7229
$P_{G8,j}$	0.5017	0.5017	0.5017	0.5017	0.5017	0.5017
$P_{G15,j}$	0.4500	0.4500	0.4500	0.4500	0.4500	0.4500
$P_{G10,j}$	0.3257	1.0052	0.6772	1.0876	0.9272	0.8464
$P_{G12,j}$	0.0853	0.3405	1.2226	0.9559	0.6345	0.1160
$P_{G14,j}$	0.0566	0.5264	0.4503	1.0213	1.1281	0.2156
$P_{G16,j}$	0.8911	1.0567	1.0924	1.0258	1.1682	1.0158
P_{LOSS}	0.1419	0.2121	0.2758	0.2738	0.2913	0.1771

However, this solution does not consider any constraints related with reactive power generation consumption balance, reactive power generation limits and security constraints. Therefore, we applied step 2 in order to find an exact solution in which all constraints given in section 2 are satisfied. The solution point data obtained in step 2 are given in table 5 and table 6, respectively.

Table 4. Starting and ending reservoir water volumes of hydraulic units obtained in step 1 of the first iteration (in acre-ft).

Hydraulic Unit (i)	$V_{i,0}$	$V_{i,6}$
10	50000.00	48000.00
12	45000.00	46600.00
14	46600.00	40600.00
16	40000.00	50600.00

Table 5. Active power generations of the units obtained in step 2 of the first iteration (all the values are in pu).

	Subinterval (j)					
	1	*2*	*3*	*4*	*5*	*6*
$P_{G1,j}$	1.9111	1.9304	2.0191	2.0269	2.0493	1.8931
$P_{G4,j}$	1.5783	1.5783	1.5783	1.5783	1.5783	1.5783
$P_{G5,j}$	0.7229	0.7229	0.7229	0.7229	0.7229	0.7229
$P_{G8,j}$	0.8616	0.7997	0.7932	0.8030	0.7639	0.8633
$P_{G15,j}$	0.5148	0.5280	0.4500	0.4500	0.4500	0.5163
$P_{G10,j}$	0.3257	1.0052	0.6772	1.0876	0.9272	0.8464
$P_{G12,j}$	0.0853	0.3405	1.2226	0.9559	0.6345	0.1160
$P_{G14,j}$	0.0566	0.5264	0.4503	1.0213	1.1281	0.2156
$P_{G16,j}$	0.8911	1.0567	1.0924	1.0258	1.1682	1.0158
P_{LOSS}	0.1472	0.1881	0.2561	0.2717	0.2724	0.1676

Table 6. Starting and ending reservoir water volumes of hydraulic units obtained in step 1 of the first iteration (in acre-ft).

Hydraulic Unit (i)	$V_{i,0}$	$V_{i,6}$
10	50000.00	48000.00
12	45000.00	46600.00
14	46600.00	40600.00
16	40000.00	50600.00

Then we carry out further iterations. Total cost values obtained at the end of each iteration are given in table 7. As it seen from table 7, the total cost value is increased at the end of the fourth iteration to 169988.4329 R from 168873.1136 R, so we stopped the iterations and, the solution obtained at the end of third iteration is the optimal solution to STHCP. The optimal solution data are given in table 8 and table 9, respectively.

Table 7. Total fuel cost values obtained at the end of each iteration.

Iteration No	*Total Fuel Cost Values (R)*
1	170908.6222
2	170008.4521
3	**168873.1136**
4	169988.4329

The amount of fuel spent by each limited fuel supply unit in all subintervals at the solution point are given in table 10. From table 10, the total amount of gas spent by the limited energy supply thermal units is found to be $C_{spent} = 44500.00$ *ccf*. This shows the fuel constraint given by eq (8) is also satisfied at the solution point. Note that $C_{tot} = 44500$ *ccf* according to T-O-P contract [2].

Table 8. Active power generations of the units obtained in the optimal solution point (all the values are in pu).

	Subinterval (j)					
	1	*2*	*3*	*4*	*5*	*6*
$P_{G1,j}$	1.5840	1.7841	2.0727	1.9101	1.9127	2.4924
$P_{G4,j}$	1.0934	1.3347	1.3015	1.4300	1.7525	0.9525
$P_{G5,j}$	0.9605	0.9675	0.9692	0.9901	1.0602	0.9681
$P_{G8,j}$	0.5206	0.7852	0.6090	0.7938	0.7554	1.2708
$P_{G15,j}$	0.4500	0.4715	0.4500	0.4500	0.4500	0.8601
$P_{G10,j}$	0.5671	1.0056	1.0555	1.1202	0.9107	0.2101
$P_{G12,j}$	0.1785	0.3331	0.8937	0.8717	1.0541	0.0237
$P_{G14,j}$	0.5713	0.7414	0.5401	1.0577	0.4640	0.0236
$P_{G16,j}$	1.0050	1.0584	1.1084	1.0342	1.0525	0.9915
P_{LOSS}	0.1305	0.1817	0.2502	0.2579	0.2621	0.1928

Table 9. Starting and ending reservoir volume of hydraulic units obtained in the optimal solution point (in acre-ft).

Hydraulic Unit (i)	$V_{i,0}$	$V_{i,6}$
10	50000.00	48000.00
12	45000.00	46600.00
14	46600.00	40600.00
16	40000.00	50600.00

Table 10. Amount of the fuel spent by each limited fuel supply unit in all subintervals at the solution point (in *ccf*).

	Subinterval (j)					
	1	*2*	*3*	*4*	*5*	*6*
$C_{spent\,4,j}$	3564.8	4120.6	4074.6	4384.7	5188.9	3250.0
$C_{spent\,5,j}$	2955.4	3030.1	3278.8	3647.7	3813.8	3190.6

In the literature, the same dispatch problem was also solved by using a spot price of electricity algorithm (SPOEA). The proposed method gives a total fuel cost that is 711 R lower than the one supplied by the SPOEA [2].

To show the effect of T-O-P fuel contract, we also solved the same problem with the assumption that the fuel constraint does not exist. Therefore, we did not consider the fuel constraint in equation (8) and we applied the proposed method via F-MSG algorithm. The total consumed gas amount is found as $C_{spent} = 17472.4315$ *ccf*. Thus, the total fuel cost is calculated as $F_T = 112472.425 + 2\times 44500 = \mathbf{201472.425}\,(R)$. It is clear that this total fuel cost is higher than the one obtained when the fuel constraint in equation (8) is considered (see table 7).

5.Discussion And Conclusion

In this paper, we propose a solution to short term hydrothermal coordination problem for a power system including limited energy supply thermal units by using the F-

MSG algorithm for a lossy power system area. The dispatch technique is tested on the 16 bus test system. The obtained results showed that proposed technique provides *lover total cost value* than the one obtained by the technique given in ref [2].

6. References

[1] Wood AJ, Wollenberg BF. Power Generation Operation and Control, New York: John Wiley & Sons, 1996.

[2] Celal Yaşar, Salih Fadıl and Mustafa Babadagı, "A spot price of electricity algorithm applied to lossy short-term hydrothermal scheduling problem with limited energy supply thermal units", European Transactions On Electrical Power, vol 18, p. 296–312, 2008.

[3] V. Senthil Kumar, M.R. Mohan, "A genetic algorithm solution to the optimal short-term hydrothermal scheduling", Int J Elec Power, Vol. 33, pp. 827–835, 2011.

[4] M. Basu, "Improved differential evolution for short-term hydrothermal scheduling", *Int J Elec Power*, Vol. 58, pp. 91–100, 2014.

[5] P.K. Hota, A.K. Barisal, R. Chakrabarti, "An improved PSO technique for short-term optimal hydrothermal scheduling", *Electr Pow Sys Res*, Vol.79, pp.1047–1053, 2009.

[6] I.A. Farhat, M.E. El-Hawary, "Optimization methods applied for solving the short-term hydrothermal coordination problem", *Electric Power Systems Research*, vol 79, p.1308–1320, 2009.

[7] Kasimbeyli, R., Ustun O., Rubinov, AM., "The modified subgradient algorithm based on feasible values", Optimization, vol.58, (5), pp. 535-561, 2009.

[8] Fadıl, S., Yaıcı, A., Urazel, B., "A solution to security constrained non-convex economic dispatch problem by modified subgradient algorithm based on feasible values", International Journal of Electrical Power and Energy Systems, vol.43, pp.849–858, 2012.

[9] Salih Fadıl, Burak Urazel, Bünyamin Tamyürek, "Application of Modified Subgradient Algorithm Based on Feasible Values to Security Constrained Non-convex Economic Dispatch Problems with Prohibited Operation Zones and Ramp Rates, International Journal of Electronics and Electrical Engineering (IJEEE), Vol. 4, No. 1, February 2016, pp. 1-8, doi: 10.18178/ijeee4.1.1-8

[10] S. Fadıl, B. Urazel, "A Security Constrained Eniviromental/Economic Power Dispatch Technique Using F-MSG Algorithm for a Power System Area Including limited Energy Supply Thermal Units", International Journal of Electrical Power & Energy Systems, Vol. 56, March 2014, pp. 185-197,

[11] S. Fadıl, A. Yazıcı,, B. Urazel, "A security Constrained Economic Power Dispatch Technique Using Modified Subgradient Algorithm Based on Feasible Values and Pseudo Scaling Factor for a Power System Area Including limited Energy Supply Units", Electric Power Components and Systems, Vol.39, No.16, 2011, pp.1748-1768

[12] S.Fadıl, B.Urazel, "A Solution to Security Constrained Non-Convex Pumped-Storage Hydraulic Unit Scheduling Problem by Modified Subgradient Algorithm Based on Feasible Values and Pseudo Water Price", Electric Power Components and Systems, Vol.41, No.2, 2013, pp.111-135.

[13] S. Fadıl, B. Urazel, "Solution to Security Constrained Environmental/Economic Pumped-Storage Hydraulic Unit Scheduling Problem by Modified Subgradient Algorithm Based on Feasible Values and Pseudo Water Price", International Journal of Electrical power & Energy Systems, Vol. 60, September 2014, pp. 399-413.

[14] Salih Fadıl, Burak Urazel, "Solution to Short Term Hydrothermal Scheduling Problem by Modified Subgradient Algorithm Based on Feasible Values", ELECO 2015 9th International Conference on Electrical and Electronics Engineering, November 26-28 2015, Bursa, Türkiye.

7. List of Symbols

R : a fictitious monetary unit.

F_T : total active power generation cost of the system.

$F_i(P_{Gi,j})$: fuel cost rate value of i^{th} unit in the j^{th} subinterval.

$\boldsymbol{N}$: number of buses in the network.

$\boldsymbol{N_T}$, $\boldsymbol{N_H}$: sets containing all buses to which thermal generation units and hydro generation units are connected, respectively.

$\boldsymbol{N_{LS}}$: set containing all buses to which limited energy supply thermal units are connected.

$\boldsymbol{N_{Bi}}$: set that contains all buses *directly* connected to bus *i*.

$\boldsymbol{L}$: set that contains all lines in the network.

$j_{\max}$: total number of subintervals

t_j : length of time interval *j*.

$P_{Gi,j}, Q_{Gi,j}$: active and reactive power generations of the i^{th} unit in the j^{th} subinterval, respectively.

$P_{Load\ i,j}, Q_{Load\ i,j}$: active and reactive loads of the i^{th} bus in the j^{th} subinterval, respectively.

$p_{ik,j}, q_{ik,j}$: active and reactive power flows from bus *i* to bus *k* at bus *i* border in the j^{th} subinterval, respectively.

P_{Gi}^{min}, P_{Gi}^{max} : the lower and upper active generation limits of the i^{th} unit, respectively.

Q_{Gi}^{min}, Q_{Gi}^{max} : the lower and upper reactive generation limits of the i^{th} unit, respectively.

$p_{l,j}$: active power flow on line *l* in the j^{th} subinterval.

p_l^{max} : maximum active power transmission capacity of line l .

U_i^{min}, U_i^{max} : lower and upper voltage magnitude limits of the i^{th} bus.

$U_{i,j}$: voltage magnitude of bus the *i* in the j^{th} subinterval

$V_i^{\min}, V_i^{\max}$: lower and upper reservoir limits of the i^{th} hydraulic unit.

$V_{i,j}$: stored water volume in the reservoir of the i^{th} hydraulic unit at the end of the j^{th} subinterval

$q_{GH}\left(P_{Gi,j}\right)$: discharge rate of the i^{th} hydro unit in the j^{th} subinterval.

$q_{Hi}^{\min}$, $q_{Hi}^{\max}$: lower and upper discharge rate limits of the i^{th} hydro unit.

V_i^{start}, V_i^{end} : specified starting and final stored water volumes in the reservoir of the i^{th} hydraulic unit.

$C_T(P_{GT,j})$: fuel consumption rate for the T^{th} limited energy supply thermal unit in the j^{th} subinterval.

C_{tot} : minimum total fuel amount that should be spent by all limited energy supply thermal units during the operation period according to T-O-P fuel contract *(ton, m^3, ccf, etc.)*.

C_{spent} : amount of the total fuel spent by the all limited energy supply thermal units during the operation period (*ton, m^3,ccf, etc.*)

$C_{spent\ T,j}$: amount of the fuel spent by T^{th} limited energy supply thermal unit in the j^{th} subinterval