Performance of Zero-Forcing MIMO Systems with Signal Space Diversity under Transmit Antenna Correlation

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Abstract

In this work, signal space diversity (SSD) is incorporated into a zero-forcing multiple-input multiple-output system with various numbers of transmit and receive antennas. We assume that the number of antennas at both ends is equal and the channel state information is only available at the receiver. The performance of the proposed system is studied under a slow flat Rayleigh fading channel scenario with correlated transmit antennas and uncorrelated receive antennas. We adopt different realistic transmit antenna correlation models. A number of different component interleaving strategies are compared for these correlation models. It is shown that the error performance of the system can be significantly improved by making use of SSD with only a negligible increase in complexity and no extra use of bandwidth/time resources.

1. Introduction

The potential of high data rate transmission without additional bandwidth or time slots makes multiple-input multiple-output (MIMO) systems a popular research area. With enough scattering, the capacity linearly increases with the minimum of t and r in a single-user MIMO system with t transmitter and r receiver antennas. This is known as spatial multiplexing. Signal space diversity (SSD) (also known as modulation diversity) is a technique used to improve the error performance of the communication systems without the need for extra bandwidth, time slots, and power [1]. SSD also requires almost no additional complexity. It has the potential to increase the diversity level only by utilizing the orthogonal dimensions contained in the transmitted signal. The concept of SSD is first introduced in [1] and is applied in two steps. The first part is transforming the signal constellation such that no two components on the same coordinate axis are identical. By performing this step, any component of any signal point in the constellation gets sufficient information for the symbol identification. With proper rotation of the signal constellation, this first condition can be satisfied [2]. The second part is ensuring that the components of the transmitted symbol are independently affected by the channel. This condition is generally assured by employing component interleaving/deinterleaving blocks [2-4]. When these two conditions are met, the diversity level can be multiplied. For a two dimensional signal constellation, when each of in-phase (I) and quadrature (Q) components is independently affected by the fading through the channel, a dual diversity can be obtained [2-4]. SSD for single-input single-output systems has been studied widely [1-4]. More recently, SSD has been also integrated into MIMO systems [5-10]. Some of these schemes use component interleaving over time domain, where the interleaving depth required can be long and burdensome [7], [9]. On the other hand, in [5], [6], [8], the independence between the fading coefficients affecting the components of a signal point is provided by transmitting the in-phase and quadrature components of each signal point over different transmit antennas. In [10], SSD is incorporated into MIMO systems with maximal ratio combining and transmit antenna selection and the performance is studied under a scenario with correlated receive antennas. The performance of MIMO systems combined with ZF (zero-forcing) precoding and SSD is analyzed in [11] by assuming channel state information (CSI) both at the transmitter and receiver. In this paper, the performance of transmit antenna correlated ZF MIMO systems with SSD is investigated for slow flat Rayleigh fading channels under the assumption of different models of transmit antenna correlation. We assume that the number of antennas at both ends is equal and the CSI is only available at the receiver. For each type of correlation model, bit error rate (BER) performance is illustrated for three, four and five transmit/receive antennas.

Notation: The operators, $|.|$, $\|.\|$, $(.)^H$, j, $\mathbb{R}\{.\}$ and $\mathbb{I}\{.\}$ denote the absolute value (magnitude), Euclidean norm, Hermitian transpose, $\sqrt{-1}$, real part of a complex number, and imaginary part of a complex number, respectively. The matrices and column vectors are denoted by uppercase and lowercase bold letters, respectively.

2. System Model

Fig. 1 shows the block diagram of the proposed system. We consider a single-user MIMO transmission system with t transmit antennas and r receive antennas with $t = r$. The goal is to simultaneously transmit t independently modulated symbols in a parallel fashion from the transmitter to the receiver. In the transmitter, a rotated binary phase shift keying (BPSK) signal constellation is first employed to map the (possibly coded) data bits into the modulated symbols (s_i, ($i \in \{1, 2, \ldots, t\}$)). We use s_{iI} and s_{iQ} to respectively denote the I and Q components of the ith modulated symbol, i.e., s_i. As the rotation angle for the BPSK signal constellation, 45 degree counter-clockwise is used [11]. Then, the component interleaving is applied to the t modulated symbols. The component interleaving is implemented over transmit antennas. Hence, the used component interleaving technique is not as troublesome as the component interleaving in time domain, which requires time domain interleaving depths greater than the channel coherence time. The entries of the transmitted baseband signal $\mathbf{x}$ for a certain interleaving strategy are given by $s_{aI} + j\, s_{bQ}$, ($a, b \in \{1, 2, \ldots, t\}$). Here, the choice of a and b depends on the interleaving strategy used. A set of digital-to-analog conversion (DAC) operations are used to transmit the baseband signal $\mathbf{x}$ from the transmit an-

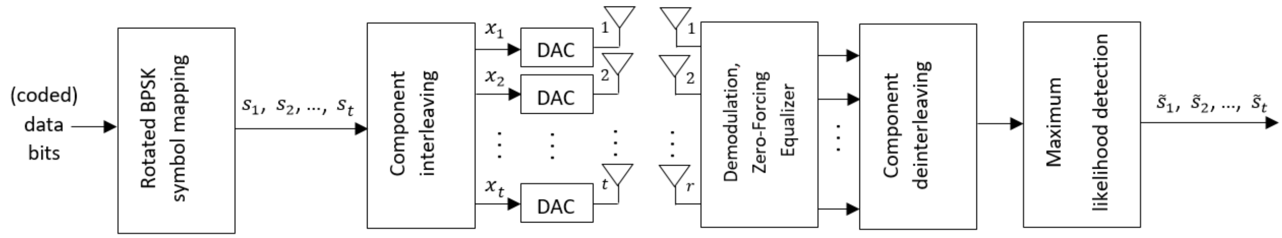

Figure 1. Block diagram of the proposed system.

tennas. The received complex baseband signal **y** is:

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} = \mathbf{h_1}x_1 + \mathbf{h_2}x_2 + ... + \mathbf{h_t}x_t + \mathbf{n} \quad (1)$$

where $\mathbf{H}$ is the r-by-t channel matrix with its (i, k) entry, i.e., $[\mathbf{H}]_{ik} \in \mathbb{C}$, denoting the fading coefficient between the kth transmit antenna and ith receive antenna. Also, $\mathbf{h_1}$, $\mathbf{h_2}$,...,$\mathbf{h_t}$ denote the columns of the matrix $\mathbf{H}$. Additionally, the vector $\mathbf{x} \in \mathbb{C}^{t\times 1}$ denotes the transmitted baseband signal. Also, x_1, x_2,...,x_t denote the elements of the vector **x**. Additionally, $\mathbf{n} \in \mathbb{C}^{r\times 1}$ represents additive white Gaussian noise at the receiver such that $\mathrm{E}\{\mathbf{n}\mathbf{n}^H\} = N_0\mathbf{I}$ with **I** denoting the identity matrix. We assume a slow flat Rayleigh fading scenario with rich scattering and enough antenna spacing only at the receive side. Hence, the scenario of interest involves uncorrelated receive antennas and correlated transmit antennas. We have:

$$\mathbf{H} = \widetilde{\mathbf{H}}\mathbf{R}^{1/2} \quad (2)$$

where the t-by-t matrix **R** is the correlation matrix capturing the effect of the correlation among the transmit antennas. The elements of the matrix $\widetilde{\mathbf{H}}$ are independent and identically distributed (IID) zero-mean complex Gaussian random variables with unit variance. In our work three types of correlation matrix structures **R** are used namely uniform, dual, and exponential correlation models. All of these models have practical applications.

The entries of the uniform correlation matrix are given by $[\mathbf{R}]_{ik} = 1$ for $i = k$ and $[\mathbf{R}]_{ik} = \rho$ for $i \neq k$, where $[\mathbf{R}]_{ik}$ represents the ith row kth column element of **R**. Also, ρ ($0 < \rho < 1$) denotes the correlation coefficient between any two transmit antennas. We have:

$$\mathbf{R} = \begin{bmatrix} 1 & \rho & \rho & \rho & \cdots & \rho & \rho \\ \rho & 1 & \rho & \rho & \cdots & \rho & \rho \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \rho & \rho & \rho & \rho & \cdots & \rho & 1 \end{bmatrix}. \quad (3)$$

For an antenna array, the uniform correlation matrix is applicable when all the transmit antennas are separated with the same distance from each other.

The elements of the dual correlation matrix are given by $[\mathbf{R}]_{ik} = 1$ for $i = k$, $[\mathbf{R}]_{ik} = \rho$ for $|i - k| = 1$, and $[\mathbf{R}]_{ik} = 0$ for the remaining entries. We have:

$$\mathbf{R} = \begin{bmatrix} 1 & \rho & 0 & 0 & \cdots & 0 & 0 \\ \rho & 1 & \rho & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \rho & 1 \end{bmatrix}. \quad (4)$$

For an antenna array, the dual correlation matrix provides a more realistic model as compared to the uniform correlation matrix. The reason behind this can be explained based on the fact that the correlation between two antennas reduces when the distance between them increases and when separated far enough the correlation may be equal to zero.

The entries of the exponential correlation matrix are given by $[\mathbf{R}]_{ik} = \rho^{|i-k|}$. We have:

$$\mathbf{R} = \begin{bmatrix} 1 & \rho & \rho^2 & \rho^3 & \cdots & \rho^{t-2} & \rho^{t-1} \\ \rho & 1 & \rho & \rho^2 & \cdots & \rho^{t-3} & \rho^{t-2} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \rho^{t-1} & \rho^{t-2} & \rho^{t-3} & \rho^{t-4} & \cdots & \rho & 1 \end{bmatrix}. \quad (5)$$

For an antenna array, the exponential correlation matrix provides a more realistic model as compared to the other discussed correlation models. The reason behind this can be explained again by the fact that the correlation between two antennas reduces when the distance between them increases.

The receiver uses ZF equalization to suppress the inter-symbol interference, under the assumption of full CSI being only available at the receiver. For equalization, the received complex baseband signal **y** is projected onto the vector $\mathbf{h}_i^H\mathbf{P}_i^{\perp}$, for all i, where $i \in \{1, 2, \ldots, t\}$ and $\mathbf{P}_i^{\perp}$ denotes the projection matrix onto the null space of the vectors $\mathbf{h}_1, \mathbf{h}_2, ..., \mathbf{h}_{i-1}, \mathbf{h}_{i+1}, \mathbf{h}_{i+2}, ..., \mathbf{h}_t$. This yields:

$$\frac{\mathbf{h}_i^H\mathbf{P}_i^{\perp}}{\left\|\mathbf{h}_i^H\mathbf{P}_i^{\perp}\right\|}\mathbf{y} = \left\|\mathbf{h}_i^H\mathbf{P}_i^{\perp}\right\| x_i + \frac{\mathbf{h}_i^H\mathbf{P}_i^{\perp}}{\left\|\mathbf{h}_i^H\mathbf{P}_i^{\perp}\right\|}\mathbf{n} = \left\|\mathbf{h}_i^H\mathbf{P}_i^{\perp}\right\| s_{aI} + \mathbb{R}\left\{\frac{\mathbf{h}_i^H\mathbf{P}_i^{\perp}}{\left\|\mathbf{h}_i^H\mathbf{P}_i^{\perp}\right\|}\mathbf{n}\right\} + j\left(\left\|\mathbf{h}_i^H\mathbf{P}_i^{\perp}\right\| s_{bQ} + \mathbb{I}\left\{\frac{\mathbf{h}_i^H\mathbf{P}_i^{\perp}}{\left\|\mathbf{h}_i^H\mathbf{P}_i^{\perp}\right\|}\mathbf{n}\right\}\right) \quad (6)$$

for $i \in \{1, 2, \ldots, t\}$. Suppose that I and Q components of the ith symbol s_i $(i \in \{1, 2, \ldots, t\})$ are sent from the αth and βth transmit antennas, respectively. The maximum likelihood detector at the receiver forms the decision variable for the ith symbol (s_i) as: $d_i = \mathbb{R}\left\{\frac{\mathbf{h}_\alpha^H\mathbf{P}_\alpha^{\perp}}{\left\|\mathbf{h}_\alpha^H\mathbf{P}_\alpha^{\perp}\right\|}\mathbf{y}\right\} + j\mathbb{I}\left\{\frac{\mathbf{h}_\beta^H\mathbf{P}_\beta^{\perp}}{\left\|\mathbf{h}_\beta^H\mathbf{P}_\beta^{\perp}\right\|}\mathbf{y}\right\}$ for $i \in \{1, 2, \ldots, t\}$.

3. The scenario of three transmit/receive antennas

For the scenario with three transmit and receive antennas, we have two different interleaving strategies to test. These two strategies are labeled as Strategy 1 and Strategy 2. For the sake of comparison, we also show the original ZF scheme without SSD. This is called as Strategy 3. The baseband transmit vectors

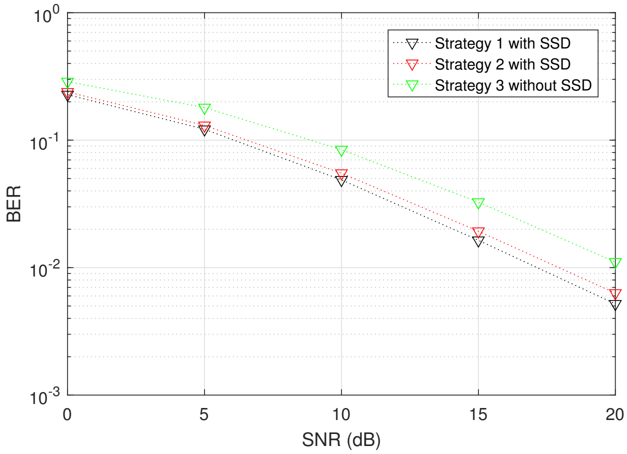

Figure 2. The comparison of the BER performances for the relevant strategies with three transmit and receive antennas under uniform correlation.

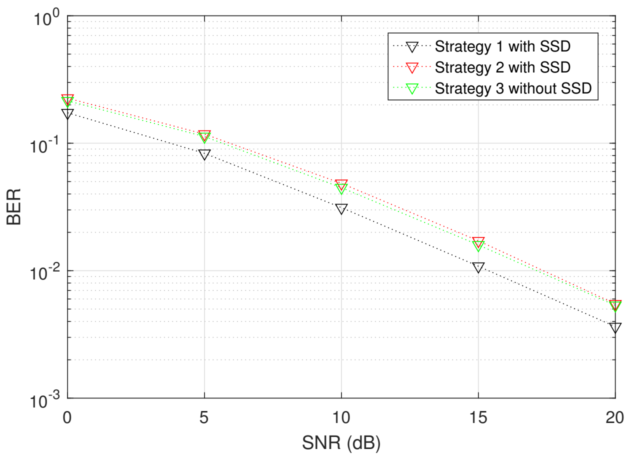

Figure 3. The comparison of the BER performances for the relevant strategies with three transmit and receive antennas under dual correlation.

for these strategies are respectively given as:

$$\mathbf{x} = \begin{bmatrix} s_{1I} + js_{2I} \\ s_{1Q} + js_{3Q} \\ s_{3I} + js_{2Q} \end{bmatrix}, \mathbf{x} = \begin{bmatrix} s_{1I} + js_{2I} \\ s_{3I} + js_{3Q} \\ s_{1Q} + js_{2Q} \end{bmatrix}, \mathbf{x} = \begin{bmatrix} s_{1I} + js_{1Q} \\ s_{2I} + js_{2Q} \\ s_{3I} + js_{3Q} \end{bmatrix}.$$

The correlation coefficient ρ is chosen to be equal to 0.85 in all simulations. It can be seen from Figure 2 that under uniform correlation, Strategy 1 and Strategy 2 (both with SSD) yield better performance as compared to Strategy 3 which has no SSD. For a BER of 10^{-2}, Strategy 1 and Strategy 2 provide about 2.8 and 2 dB gains. It can be seen from Figure 3 that under dual correlation, Strategy 1 outperforms Strategy 2 and Strategy 3. For a BER of 10^{-2}, Strategy 1 provides almost 2.2 dB gain as compared to the other schemes which perform very close to each other for this scenario. It can be seen from Figure 4 that under exponential correlation, Strategy 1 again performs better than Strategy 2 and Strategy 3. For a BER of 10^{-2}, Strat-

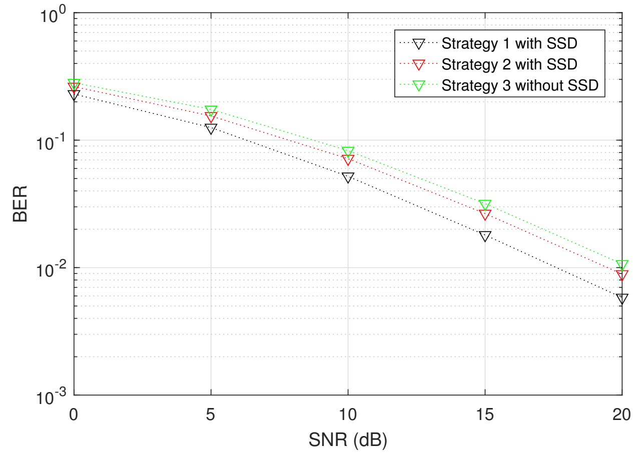

Figure 4. The comparison of the BER performances for the relevant strategies with three transmit and receive antennas under exponential correlation.

egy 1 provides about 2.3 dB gain as compared to Strategy 3. In this case, Strategy 2 attains around 0.6 dB gain as compared to Strategy 3. Hence, Strategy 1 yields the best performance in all the considered correlation models when $t = r = 3$.

4. The scenario of four transmit/receive antennas

There exist three different interleaving strategies for the case of four transmit and receive antennas. These strategies are labeled as Strategy 1, Strategy 2, and Strategy 3. Here, the original ZF scheme without SSD is called Strategy 4. The baseband transmit vectors for these strategies are respectively given as:

$$\mathbf{x} = \begin{bmatrix} s_{1I} + js_{2I} \\ s_{3I} + js_{4I} \\ s_{1Q} + js_{2Q} \\ s_{3Q} + js_{4Q} \end{bmatrix}, \mathbf{x} = \begin{bmatrix} s_{1I} + js_{2I} \\ s_{3I} + js_{4I} \\ s_{3Q} + js_{4Q} \\ s_{1Q} + js_{2Q} \end{bmatrix}, \mathbf{x} = \begin{bmatrix} s_{1I} + js_{2I} \\ s_{1Q} + js_{2Q} \\ s_{3I} + js_{4I} \\ s_{3Q} + js_{4Q} \end{bmatrix},$$

$$\mathbf{x} = \begin{bmatrix} s_{1I} + js_{1Q} \\ s_{2I} + js_{2Q} \\ s_{3I} + js_{3Q} \\ s_{4I} + js_{4Q} \end{bmatrix}.$$

The correlation coefficient ρ is chosen to be equal to 0.85 in all simulations. It can be seen from Figure 5 that under uniform correlation, the Strategies 1, 2 and 3 perform very similarly and all outperforms Strategy 4 by around 3.3 dB gain at a BER of 2×10^{-2}. It can be seen from Figure 6 that under dual correlation, Strategy 3 outperforms all the other strategies. For a BER of 10^{-2}, Strategy 3 produces more than 5.2 dB gain as compared to Strategy 4 without SSD. It can be seen from Figure 7 that under exponential correlation, Strategy 3 performs better than all the other strategies as before. In this case, for a BER of 2×10^{-2}, Strategy 3 yields about 6 dB gain as compared to Strategy 4. It can be deduced from the results that Strategy 3 has a more beneficial component interleaving strategy than the other strategies for the considered correlation models.

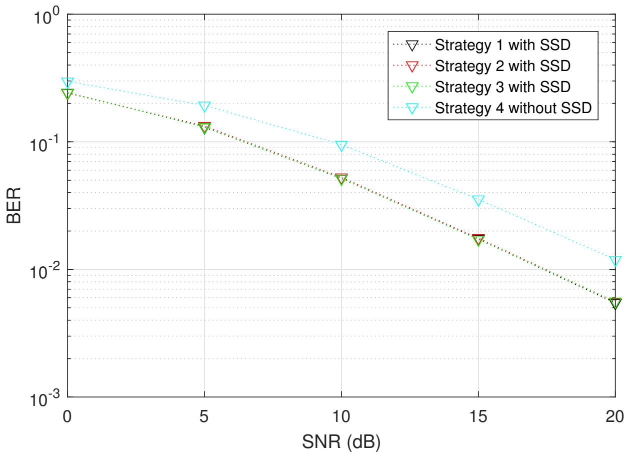

Figure 5. The comparison of the BER performances for the relevant strategies with four transmit and receive antennas under uniform correlation.

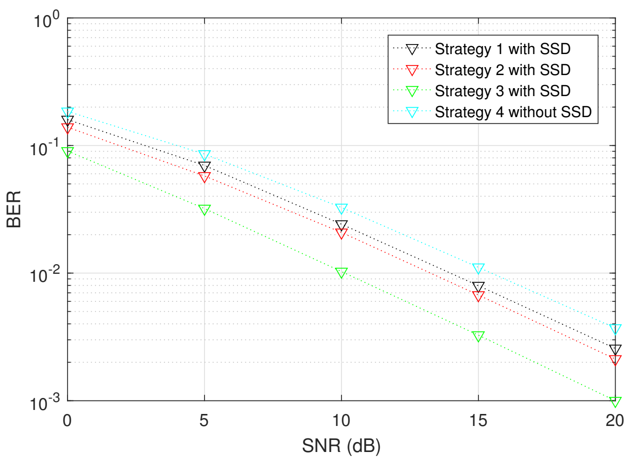

Figure 6. The comparison of the BER performances for the relevant strategies with four transmit and receive antennas under dual correlation.

5. The scenario of five transmit/receive antennas

There are four distinct interleaving strategies for the scenario with five transmit and receive antennas. These strategies are labeled as Strategy 1, Strategy 2, Strategy 3, and Strategy 4. In this case, the original ZF scheme without SSD is represented as Strategy 5. The baseband transmit vectors for these strategies are respectively given as:

$$\mathbf{x} = \begin{bmatrix} s_{1I} + js_{2I} \\ s_{3I} + js_{4I} \\ s_{5I} + js_{5Q} \\ s_{3Q} + js_{4Q} \\ s_{1Q} + js_{2Q} \end{bmatrix}, \mathbf{x} = \begin{bmatrix} s_{1I} + js_{2I} \\ s_{3I} + js_{4I} \\ s_{5I} + js_{5Q} \\ s_{2Q} + js_{4Q} \\ s_{1Q} + js_{3Q} \end{bmatrix}, \mathbf{x} = \begin{bmatrix} s_{1I} + js_{2I} \\ s_{3I} + js_{5I} \\ s_{4I} + js_{5Q} \\ s_{2Q} + js_{3Q} \\ s_{1Q} + js_{4Q} \end{bmatrix},$$

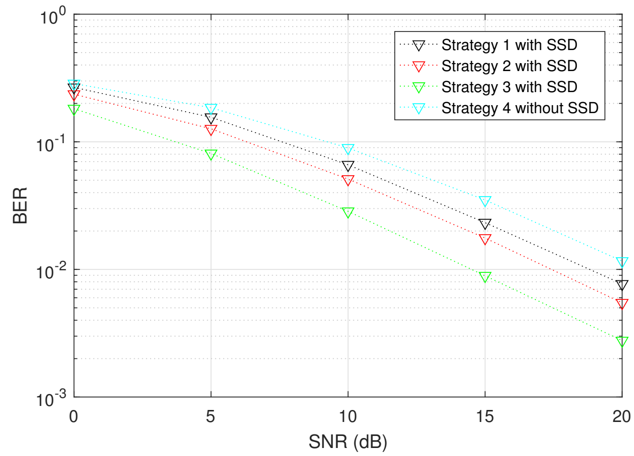

Figure 7. The comparison of the BER performances for the relevant strategies with four transmit and receive antennas under exponential correlation.

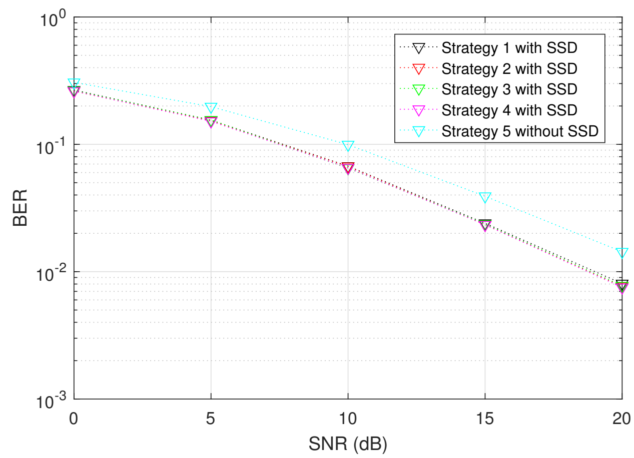

Figure 8. The comparison of the BER performances for the relevant strategies with five transmit and receive antennas under uniform correlation.

$$\mathbf{x} = \begin{bmatrix} s_{1I} + js_{2I} \\ s_{3I} + js_{4I} \\ s_{2Q} + js_{5I} \\ s_{3Q} + js_{4Q} \\ s_{1Q} + js_{5Q} \end{bmatrix}, \mathbf{x} = \begin{bmatrix} s_{1I} + js_{1Q} \\ s_{2I} + js_{2Q} \\ s_{3I} + js_{3Q} \\ s_{4I} + js_{4Q} \\ s_{5I} + js_{5Q} \end{bmatrix}.$$

The correlation coefficient ρ is chosen to be equal to 0.85 in all simulations. It can be seen from Figure 8 that under uniform correlation, all the strategies with SSD performs very similarly. The four strategies with SSD yield about 2.7 dB gain for a BER of 2×10^{-2} as compared to Strategy 5 without SSD. It can be seen from Figure 9 that under dual correlation, Strategy 3 outperforms other strategies. Strategy 3 produces around 2.5 dB gain as compared to Strategy 5 at a BER of 2×10^{-2}. It can be seen from Figure 10 that under exponential correlation, Strategy 1 performs better than other strategies. Strategy 1 produces around 4.3 dB gain as compared to Strategy 5 at a BER of 10^{-2}.

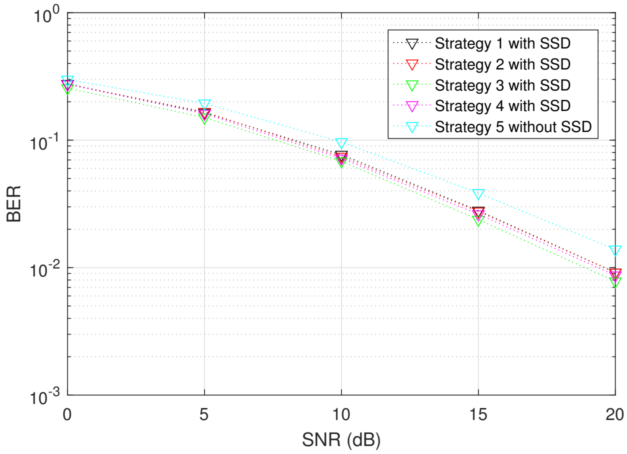

Figure 9. The comparison of the BER performances for the relevant strategies with five transmit and receive antennas under dual correlation.

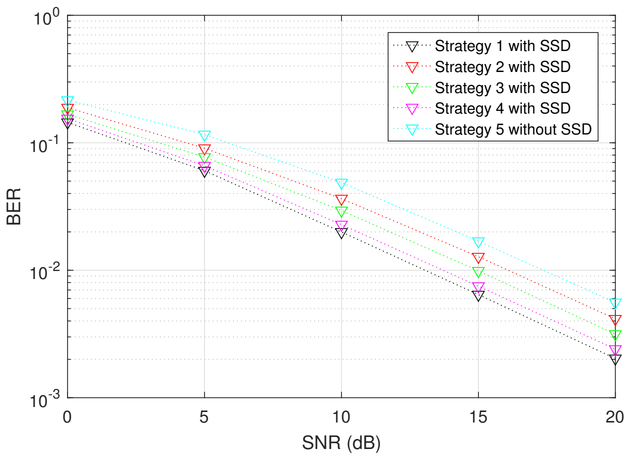

Figure 10. The comparison of the BER performances for the relevant strategies with five transmit and receive antennas under exponential correlation.

6. Conclusion

In this work, the integration of SSD into a ZF MIMO system has been studied over slow flat Rayleigh fading channels with BPSK modulation for correlated transmit antennas and uncorrelated receive antennas. Assuming three models for transmit antenna correlation, bit error rate performances of the different component interleaving strategies have been examined. The proposed scheme has been shown to bring about considerable gains with almost no additional complexity. Also, it is shown that the performance of the system depends on the used component interleaving strategy as different interleaving strategies perform differently under distinct types of correlation models. The presented technique can also be generalized to include other two dimensional modulation schemes with modulation levels higher than two.

7. References

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