Simple and Optimal PI/PID Tuning Formulae for Unstable Time Delay Processes

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Abstract

Many PI and/or PID design methods can be found in the literature for controlling open loop stable processes. However, less PID design methods for controlling unstable processes are considered. Hence, this paper introduces simple and optimal tuning rules for controlling unstable time delay systems. Design method requires a model of the actual process. Here, an unstable first order plus dead time (UFOPDT) model has been used. Based on the UFOPDT model, optimal analytical expressions have been obtained using the ISTE and IST²E criteria, well-known integral performance indexes. Simulation examples are provided to show the use of the obtained tuning rules.

1. Introduction

Proportional-Integral-Derivative (PID) controllers are still commonly used in process industries despite control theory have improved considerably. This is because of the fact that they have only three terms to be tuned for controlling processes and they yield robust and acceptable performances for a wide range of operation conditions.

In the literature, many different methods can be found to obtain tuning parameters of a PID controller. Some PID design methods use information about the open loop step response, such as, the Coon-Cohen reaction curve method [1]. Ziegler-Nichols [2] method use some knowledge of the Nyquist curve of the plant. Åström-Hägglund [3] also suggest a design method that use some knowledge of the Nyquist curve of the plant. An excellent collection of PID controller design methods can be found in [4].

Open loop unstable processes are frequently being encountered in industrial processes. Specific tuning methods have to be developed for these processes. Valentine and Chidambaram [5] suggested a PID controller designed by the dominant pole placement method based on an unstable first-order plus dead-time plant transfer function. Park et al. [6] proposed a PID-P control strategy. Ho and Xu [7] provided PID tuning rules based on gain and phase margin specifications. Visioli [8] used integral performance indexes in order to achieve optimal PID controller parameters for processes with an integrator and an unstable plant transfer function. Majhi and Atherton [9] gave tuning formulas for PI and PI-PD controllers for tuning open loop unstable processes with dead time.

This paper aims to provide simple, optimal and analytical tuning rules for a PI and PID controller to control open loop unstable processes with dead time. Proposed design method needs a model of the actual process. The most widely used model for open loop unstable processes is a first order unstable plant transfer function plus dead time, which is called UFOPDT model. Here, the relay feedback control method, a widely accepted approach to system identification, is employed to describe the UFOPDT model. The relay feedback identification can lead to large errors in the identification if equations derived from the approximate describing function method are used. Therefore, in this paper, equations derived from exact analysis [10] will be used to find unknown parameters of the UFOPDT model. Then, repeated optimizations on the error signal to minimize it are performed to achieve relationships between the PI or PID controller transfer function and the UFOPDT model transfer function parameters. Simulation results have been provided to illustrate the use of the proposed design method.

The paper is organized as follows. The next section gives a very brief review of the relay feedback identification method. Section 3 gives a short review of integral performance criteria. Optimum analytical tuning formulae for PI and PID controllers to control unstable time delay processes are provided Section 4. Simulation examples are supplied in section 5, followed by conclusions in section 6.

2. Relay Feedback Identification

The UFOPDT plant transfer function model given by

$$G(s) = \frac{Ke^{-\theta s}}{(Ts-1)} \tag{1}$$

is needed in order to obtain analytical tuning formulae of PI and PID controllers. Since Åström-Hägglund [3] have successfully applied the relay feedback method to automatic tuning of PID controllers, the technique has gained much attention in recent years. Here, it is used to estimate the unknown parameters of the UFOPDT model using exact limit cycle analysis. The relay feedback control system is shown in Fig. 1.

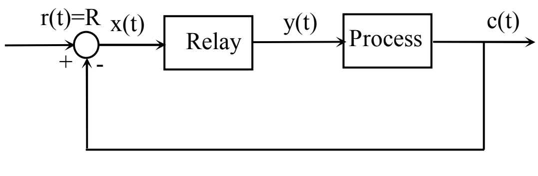

Fig. 1. Relay Feedback Control System

To calculate accurate plant transfer function parameters, the A-Function method [11], which is an exact method for limit cycle investigation, is used. Kaya [12] and Kaya and Atherton [10] used equations derived from the A-Function method to identify unknown parameters of an unstable FOPDT plant transfer function. The detailed analysis of this estimation method ([10], [12]) is not given here, but interested readers may refer to the cited publications.

3. Integral Performance Criteria

Time domain ISE (Integral of Squared Error) criterion is given by

$$J_0 = \int_0^{\infty} e^2(t)dt. \quad (2)$$

The s-domain ISE criterion can be calculated as follows:

$$J_0 = \frac{1}{2\pi j}\int_0^{\infty} E(s)E(-s)ds \quad (3)$$

where *E(s)* is the error signal and assumed to be given by *E(s)=B(s)/A(s)*. *A(s)* and *B(s)* are polynomials with real coefficients given by

$$A(s) = a_0 s^m + a_1 s^{m-1} + \ldots + a_{m-1}s + a_m$$

$$B(s) = b_1 s^{m-1} + \ldots + b_{m-1}s + b_m$$

The integral given in equation (3) can be calculated by using the recursive algorithm of Åström [13]. Time weighted version of the ISE criterion given by

$$J_n = \int_0^{\infty} [t^n e(t)]^2 dt \quad (4)$$

can also be evaluated by using recursive algorithm of Åström [13], since $L\{tf(t)\} = -dF(s)/ds$. Here, L denotes the Laplace transform and *L{f(t)} = F(s)*. Increasing *n* in equation (5) improves the closed loop performance, but usually value of *n* up to 2 or 3 is taken. Taking *n=0* gives the ISE criterion. *n=1*, and *n=2* corresponds to the ISTE and IST²E criteria, which are time moment weighted criteria of the ISE.

4. Design of Optimum PI and PID Controllers

Derivation analytical and optimal PI and PID tuning rules to control open loop unstable time delay processes based on minimization of the error signal in sense of ISTE and IST²E integral performance criteria are provided in this section. Results for the ISE are not given since it yields closed loop performances with large overshoots and long settling times.

4.1 Tuning Rules for PI Controllers

Let us assume that the actual plant transfer function can be modeled by equation (1). The ideal PI controller transfer function given by

$$G_c(s) = K_c(1 + \frac{1}{T_i s}) \quad (5)$$

is considered. Using equations (1) and (5) in the error function of the classical feedback control loop and performing repeated optimizations based on ISTE and IST²E criteria given in the previous section, relations between the normalized dead time, $\theta_n = \theta / T$, and KK_c and T_i / T have been obtained.

Fig. 2 shows the relation between KK_c and θ_n for the ISTE and IST²E criteria. Similarly, Fig. 3 shows the relation between T_i / T and θ_n again for the ISTE and IST²E criteria. In the figures, solid lines correspond to values obtained from the optimizations and asterisks correspond to values achieved from curve fitting formulae for KK_c and T_i / T . Good fitting is obvious from figures.

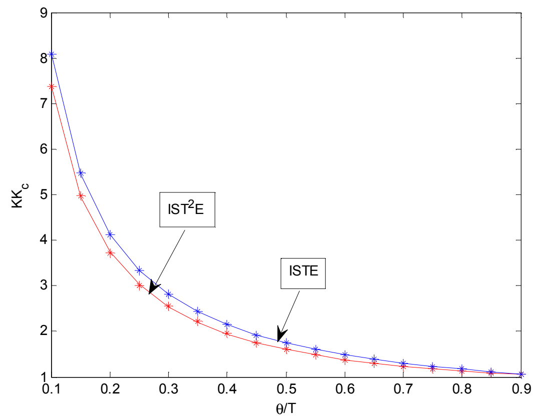

Fig.2. KK_c values for range of $0.1 \le \theta_n \le 0.9$.

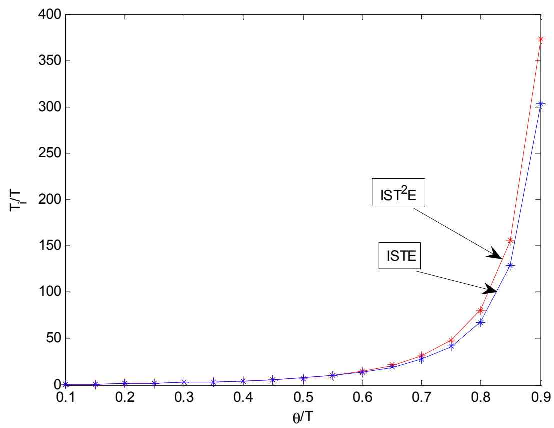

Fig. 3. T_i / T values for range of $0.1 \le \theta_n \le 0.9$.

Following tuning formulae were found from the curve fitting method for KK_c and T_i / T , respectively.

$$KKc = a_1 e^{b_1\theta_n} + c_1 e^{d_1\theta_n} \quad (6)$$

$$\frac{T_i}{T} = a_2 e^{b_2\theta_n} + c_2 e^{d_2\theta_n} \quad (7)$$

Constants in equations (6) and (7) are summarized in Table 1 for two different ranges of θ_n .

Table 1. Optimal Tuning Formulae for PI Controllers

Constants	θ_n ranges			
	$0.1 \le \theta_n \le 0.450$		$0.451 \le \theta_n \le 0.9$	
	ISTE	IST^{2}E	ISTE	IST^{2}E
a_1	19.81	18.26	5.298	5.046
b_1	-16.36	-16.24	-4.148	-4.207
c_1	5.321	4.711	1.279	1.048
d_1	-2.298	-2.233	-0.338	-0.125
a_2	0.592	0.465	0.237	0.178
b_2	5.095	5.615	6.754	7.348
c_2	-0.672	-0.573	1.48*10^{-8}	1.08*10^{-8}
d_2	-8.667	-7.039	25.92	26.48

4.2 Tuning Rules for PID Controllers

Using the procedure given above, tuning formulae for a PID controller will be supplied in this section. The process transfer function is again assumed to be given by equation (1). The ideal PID controller transfer function is assumed to be given by

$$G_c(s) = K_c(1 + \frac{1}{T_i s} + T_d s). \qquad (8)$$

Similar to derivation of PI controller tuning formulae, following formulae were found from the curve fitting method for KK_c , T_i / T and T_d / T respectively.

$$KKc = a + be^{c\theta_n} \qquad (9)$$

$$\frac{T_i}{T} = we^{x\theta_n} + ye^{z\theta_n} \qquad (10)$$

$$\frac{T_d}{T} = ke^{m\theta_n} \qquad (11)$$

Constants in equations (9), (10) and (11) are summarized in Table 2 for two different ranges of θ_n .

5. Simulation Examples

Two simulation examples are given to illustrate the use of the proposed design method. The first example is used to compare the results of the proposed design method with design methods of Visioli [8] and Ho and Xu [7]. Results of the proposed design method is compared with that of Visioli [8] and Majhi and Atherton [9] in the second example. Majhi and Atherton [9] suggest tuning rules for both PI and PI-PD controllers. Comparisons only with PI controller design method of Majhi and Atherton will be provided for a fairer comparison.

Table 2. Optimal Tuning Formulae for PID Controllers

Constants	θ_n ranges			
	$0.1 \le \theta_n \le 0.450$		$0.451 \le \theta_n \le 0.9$	
	ISTE	IST^{2}E	ISTE	IST^{2}E
a	0.208	0.162	0.343	0.395
b	1.166	1.160	1.048	0.969
c	-0.980	-1.001	-1.047	-1.110
w	0.576	0.608	0.504	0.455
x	2.644	2.782	2.806	3.140
y	-0.546	-0.608	-8.78*10^7	-9.50*10^4
z	-2.989	-1.537	-45.030	-30.320
k	0.494	0.436	0.514	0.470
m	-0.00402	-0.00348	-0.0129	-0.0199

Example 1: Consider the following process transfer function:

$$G(s) = \frac{e^{-0.2s}}{(s-1)}.$$

This process transfer function matches exactly with the UFOPDT model. Therefore, using the relay identification method of Kaya and Atherton [10], it has been determined exactly. Consequently, using tuning rules given in the previous section, calculated optimal PI and PID tuning parameters are summarized in Table 3. Responses for designed controllers are shown in Fig. 4. A unit step input and a disturbance with magnitude of -0.3 entering the closed loop system has been assumed. It is seen from the figure that with PID controllers better disturbance rejection have been achieved. Also, PID controllers results in slightly faster settling times when compared to PI controllers.

Table 3. Optimum PI and PID controller parameters for example 1

Optimization	Controller	K_c	T_i	T_d
ISTE	PI	4.1118	1.5207	-
	PID	5.8524	0.6767	0.0949
IST^{2}E	PI	3.7235	1.2890	-
	PID	5.9715	0.6140	0.0836

For comparison, closed loop responses for design methods of Visioli [8] and Ho and Xu [7] will also be provided. PID controller parameters suggested by Visioli [8] and Ho and Xu [7] are, respectively, given by $K_c = 6.228$, $T_i = 0.733$, $T_d = 0.0869$ and $K_c = 2.890$, $T_i = 1.64$. Closed loop performances for all designed methods are illustrated in Fig. 5. For the proposed design methods, the result only for IST^{2}E PID is provided to avoid a complexity in the figure. It is observed that roposed design method and design method of Visioli [8] gives very similar responses. Design method of Ho and Xu [7] yields slower responses for disturbance rejection and set point tracking when compared to both proposed design method and design method of Ho and Xu [7]. Control signals for all design methods are shown in Fig. 6.

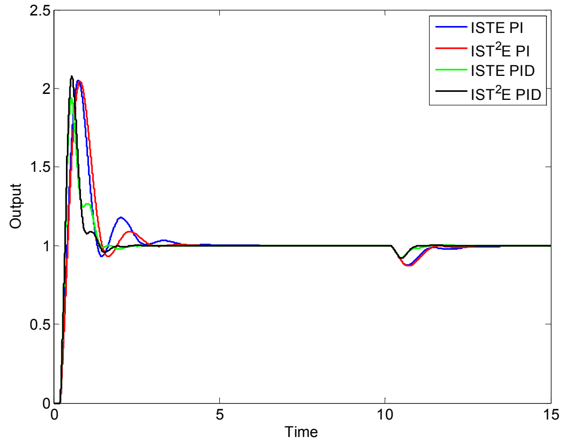

Fig. 4. Set-point and disturbance rejection responses for example 1

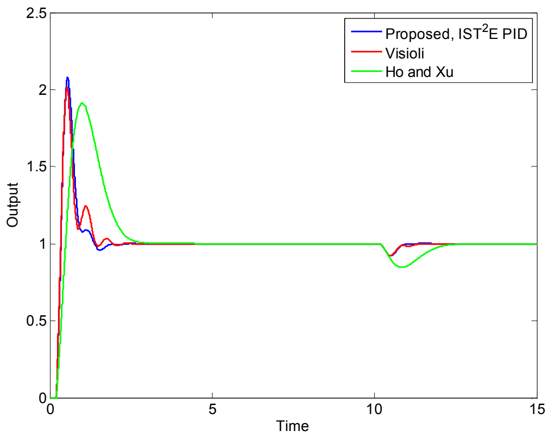

Fig. 5. Comparison of closed loop performances for the proposed and other design methods for example 1

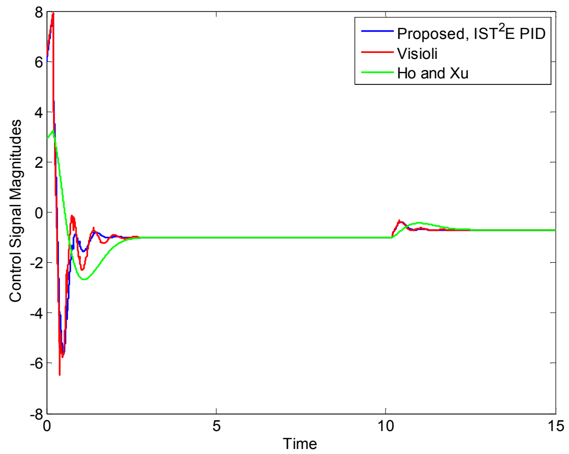

Fig. 6. Control Signals for example 1

Example 2: Here, the following process transfer function has been considered:

$$G(s) = \frac{4e^{-2s}}{(4s-1)}.$$

This process transfer function also matches exactly with the UFOPDT model. As a result, use of the relay identification method suggested by Kaya and Atherton [10] yields exact determination of model parameters. After identification of the UFOPDT model, tuning rules given in section 4 have been used to obtain optimal PI and PID tuning parameters and results are summarized in Table 4. For designed PI and PID controllers, closed loop responses to a unit step input and a disturbance entering the system at time *t=80* s with magnitude of –0.1 are depicted in Fig. 7. For this example, much better set point tracking and disturbance rejection performances have been gained with PID controllers than PI controllers when compared to the first example.

Table 4. Optimum PI and PID controller parameters for example 2

Optimization	Controller	K_c	T_i	T_d
ISTE	PI	0.4364	27.7618	-
	PID	0.6270	8.1408	0.9772
IST^2E	PI	0.4000	28.0047	-
	PID	0.6217	8.6164	0.8608

For this example, closed loop response comparisons will be performed with design methods of Visioli [8] and Majhi and Atherton [9]. Majhi and Atherton [9] provides tuning rules for both a PI and PI-PD controller. Here, comparisons will be done with PI tuning rules for a fairer comparison. PID controller parameters suggested by Visioli [8] and Majhi and Atherton [9] are, respectively, given by $K_c = 0.652$, $T_i = 8.261$, $T_d = 0.967$ and $K_c = 0.421$, $T_i = 31.310$. Closed loop performances for the proposed design method and design methods used for comparison are presented in Fig. 8. Similar to example 1, proposed design method and design method of Visioli [8] results in very similar responses. PI controller design method of Majhi and Atherton [9] gives a set point tracking with larger overshoot and longer settling time and a poorer disturbance rejection. Control signals for all design methods are shown in Fig. 9.

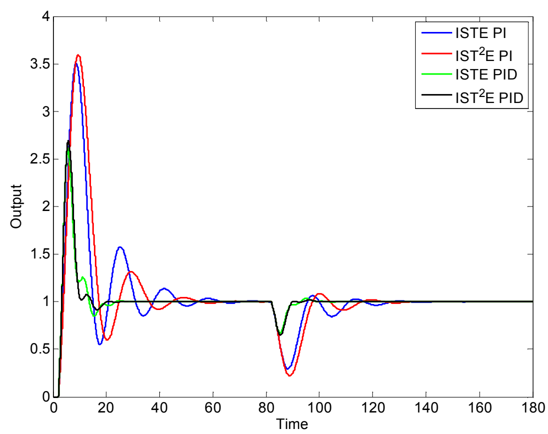

Fig. 7. Set-point and disturbance rejection responses for example 2

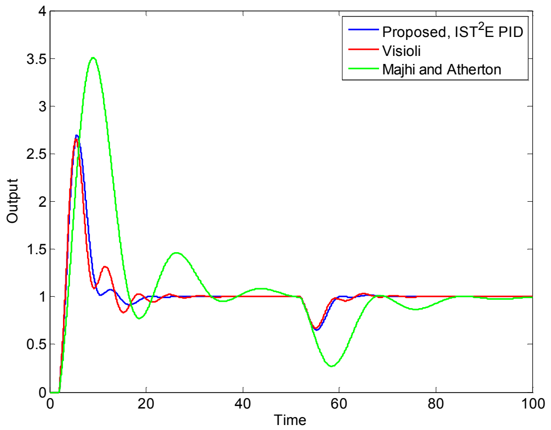

Fig. 8. Comparison of closed loop performances for the proposed and other design methods for example 2

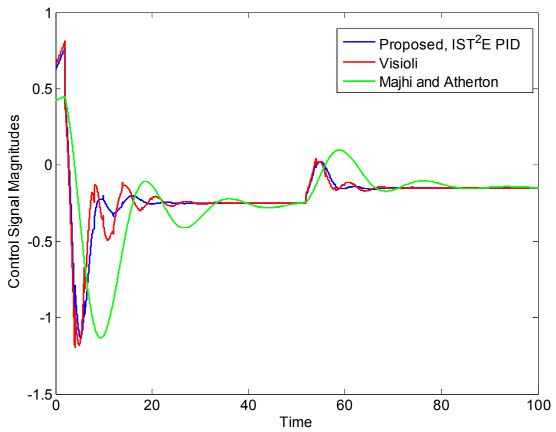

Fig. 9. Control signals for example 2

6. Conclusions

The paper introduced simple optimal tuning rules for open loop unstable processes. The design method requires a unstable first order plus time delay, called UFOPDT, process transfer function. For the identification of the UFOPDT model, widely accepted relay feedback control system is used. Identification method gives exact solutions assuming no measurement errors. Based on the UFOPDT model, optimal analytical expressions have been obtained using the ISTE and IST^{2}E criteria, well-known integral performance indexes. The reason for using ISTE and IST^{2}E criteria for deriving tuning rules is due to fact that they yields pretty good closed loop performances in response to step input changes. Simulation results have been given to illustrate the use of the proposed design method.

7. References

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