

# Gain and Phase Margins Based Delay– Dependent Stability Analysis of Two-Area LFC System with Communication Delays

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## Abstract

**This paper investigates the effect gain and phase margins (GPM) on delay-dependent stability analysis of a two-area load frequency control (LFC) system with communication delays. A frequency-domain based exact method that takes into account both gain and phase margins is utilized to determine stability delay margins in terms of system and controller parameters. A gain-phase margin tester (GPMT) is introduced to the LFC system as to take into gain and phase margins in delay margin computation. For a wide range of proportional – integral (PI) controller gains, time delay values at which LFC system is both stable and have desired stability margin measured by gain and phase margins are computed. The time-domain simulation studies indicate that delay margins must be determined considering gain and phase margins to have a better dynamic performance in term of fast damping of oscillations, less overshoot and settling time.**

## 1. Introduction

Time delays have become an important issue in LFC system since they degrade the performance of controllers and adversely affect system dynamics and stability [1-3]. Time delays are experienced in LFC systems because of the use of an open and distributed communication network used to transmit data from power plant to central controller delays [4]. With the use of open communication infrastructure, large amount of communication delays in the range of 5-15 s are generally observed in LFC systems [2, 6]. Such delays should not be ignored and must be taken into account in the stability analysis and controller design of LFC systems. The existing studies in the stability analysis of time-delayed LFC systems mainly focus on the stability delay margin computation for a given set of controller parameters. Delay margin computation methods could be grouped into two main types, namely frequency-domain direct methods [3, 6, 7-10] and time-domain indirect methods.

The main goal of frequency domain approaches is to compute all critical purely imaginary roots of the characteristic equation and time delay values at which the system will be marginally stable. The indirect time-domain methods that utilize Lyapunov stability theory and linear matrix inequalities (LMIs) techniques have been used to estimate delay margins of the LFC systems [1, 2, 11]. All existing methods aim to compute delay margins just considering stability as the only design consideration and to estimate delay margin values at which the LFC systems will be marginally stable for a given set of PI controller parameters.

However, a practical LFC system cannot operate near such points because of unacceptable oscillations in the frequency response. More importantly, a small increase in the time delay could stabilize the LFC system if the time delay is around the delay margin. Therefore, in addition to the stability consideration, other design specifications such as gain and phase margins that ensure a desired dynamic performance (i.e., damping, steady-state error, settling time, etc.) must be taken into account in delay margins computation. A simple method that includes a gain-phase margin tester (GPMT) in the feedforward part of the control system has been presented in [12] to analyze the gain-phase margins of time-delayed control systems with adjustable parameters.

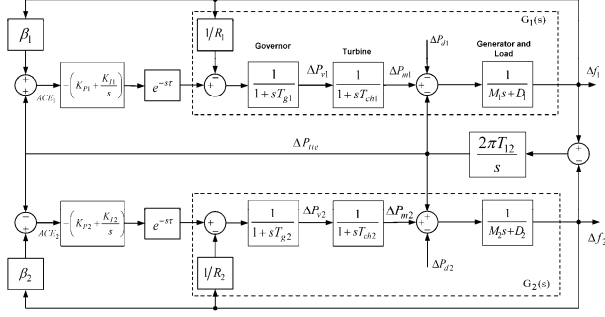
This paper extends our earlier work reported in [3] to compute delay margins based on specified gain and phase margins. For that a purpose, LFC model is modified to include a frequency independent GPMT as a “virtual compensator” in the feedforward part of each control to add desired gain and phase margins to the LFC system with delay. Using the exact method [3, 8], first the delay margins values at which the modified LFC system with a GPMT will be marginally stable are computed for a wide range of PI controller gains. Then, using these delay margins, root crossing frequencies of the imaginary axis and phase margins, time delay values at which the original LFC system without a GPMT will have a desired gain-phase margins are analytically determined. Time-domain simulation results indicate that gain and phase margins as an indication of stability margin should be included in the delay-dependent stability analysis of LFC systems with delay to achieve an improved frequency response in terms of less overshoot, oscillations and settling time.

## 2. Modified LFC model with GPMT

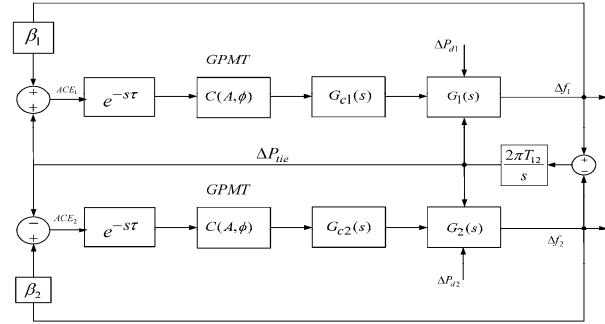
The block diagram of a two-area LFC system with a communication delay into the control loop is shown in Fig. 1. For the stability analysis, the characteristic equation of two-area LFC system with time delay is required [3].

$$\Delta(s, \tau) = P(s) + Q(s)e^{-s\tau} + R(s)e^{-2s\tau} = 0 \quad (1)$$

where  $\tau$  is the total time delay.  $P(s)$ ,  $Q(s)$ ,  $R(s)$  are polynomials in  $s$  with real coefficient and depend on system parameters. Those coefficients are not presented due to insufficient space.



**Fig. 1.** Block diagram of the two-area LFC system with communication delays



**Fig. 2.** Modified block diagram of a single area LFC system with a GPMT

$$\begin{aligned}
 P(s) &= p_9s^9 + p_8s^8 + p_7s^7 + p_6s^6 + \\
 &\quad p_5s^5 + p_4s^4 + p_3s^3 + p_2s^2 \\
 Q(s) &= q_6s^6 + q_5s^5 + q_4s^4 + q_3s^3 + q_2s^2 + q_1s \\
 R(s) &= r_3s^3 + r_2s^2 + r_1s + r_0
 \end{aligned} \quad (2)$$

In Fig. 1,  $\Delta f, \Delta P_m, \Delta P_v, \Delta P_d$  are the deviation in the frequency, the generator mechanical output, the valve position, and the load of each control area, respectively.  $M, D, T_g, T_{ch}$  and  $R$  denote the generator inertia constant, damping coefficient, time constant of the governor and turbine, and speed drop of each control area, respectively.  $ACE$  and  $\int ACE$  represent the area control error and its integral.  $\beta$  is the frequency bias factor. Finally,  $T_{12}$  denotes the tie-line synchronizing coefficient between the control areas.

The user defined Gain and Phase Margins Tester (GPMT) as a “virtual compensator” is added to the feedforward path in each control loop of the LFC system as shown in Fig. 2. The frequency independent GPMT has following form:

$$C(A, \phi) = Ae^{-j\phi} \quad (3)$$

where  $A$  and  $\phi$  represent gain and phase margins, respectively. The characteristic equation of the modified LFC system with the GPMT given in Fig. 2 is then obtained as

$$\begin{aligned}
 \Delta(s, \tau') &= P(s) + Q(s)e^{-s\tau'} Ae^{-j\phi} + R(s)e^{-2s\tau'} A^2 e^{-2j\phi} = 0 \\
 &= P'(s) + Q'(s)e^{-s\tau'} + R'(s)e^{-2s\tau'} = 0
 \end{aligned} \quad (4)$$

where

$$P'(s) = P(s) = p_9s^9 + p_8s^8 + p_7s^7 + p_6s^6 +$$

$$p_5s^5 + p_4s^4 + p_3s^3 + p_2s^2$$

$$Q'(s) = A(q_6s^6 + q_5s^5 + q_4s^4 + q_3s^3 + q_2s^2 + q_1s)$$

$$R'(s) = A^2(r_3s^3 + r_2s^2 + r_1s + r_0)$$

Note that in (4), we have an exponential term  $e^{-s\tau'}$  rather than  $e^{-s\tau}$  as in (1). This is obtained by combining  $e^{-s\tau}$  and  $e^{-j\phi}$  into a single exponential terms for  $s = j\omega_c$  which is the complex root of (4) on the imaginary axis. The relationship between  $\tau'$  and  $\tau$  is given as

$$\tau' = \tau + \frac{\phi}{\omega_c} \quad (5)$$

It must be emphasized that the time delay value  $\tau'$  at which the characteristic polynomial of the modified LFC system with GPMT shown in Fig. 2 has roots on the imaginary axis is the stability delay margin of the modified LFC system, not the original LFC system. Therefore, we first need to compute the purely imaginary roots of the modified LFC system  $s = \pm j\omega_c$  and the corresponding delay margin  $\tau'$ . Then, using these, the time delay value at which the original LFC system will have a desired GPMs,  $A$  and  $\phi$ , could be easily determined using (5).

### 3. Gain and Phase Margins based Delay Margin Computation

The stability of the modified LFC system with GPMT is evaluated by the position of the roots of the characteristic polynomial of the form in (4). For asymptotic stability, all the roots must remain in the left half of the complex plane for a given time delay. Note that the characteristic polynomial of (4) is a transcendental equation due to the term in the form of  $e^{-s\tau'}$ . As a result of this transcendental feature, the characteristic polynomial has infinitely many finite roots and the computation of these roots become quite difficult. However, the main objective here is to compute delay values for which the characteristic polynomial of (4) has roots (if any) on the imaginary axis. Assume that  $\Delta(j\omega_c, \tau') = 0$  has a root on the imaginary axis at  $s = j\omega_c$  for some finite values of  $\tau'_c$ . Since complex root always exist as a conjugate pair, the equation  $\Delta(-j\omega_c, \tau'_c) = 0$  will also have the same root at  $s = j\omega_c$  for the same value of  $\tau'_c$ . As a result, the delay margin problem now becomes a problem of finding values of  $\tau'_c$  such that both  $\Delta(j\omega_c, \tau'_c) = 0$  and  $\Delta(-j\omega_c, \tau'_c) = 0$  have the same root at  $s = j\omega_c$ . This result could be expressed as [8]:

$$\begin{aligned}
\Delta(j\omega_c, \tau'_c) &= P'(j\omega_c) + Q'(j\omega_c)e^{-j\omega_c\tau'_c} + \\
R'(j\omega_c)e^{-2j\omega_c\tau'_c} &= 0 \\
\Delta(-j\omega_c, \tau'_c) &= P'(-j\omega_c) + Q'(-j\omega_c)e^{j\omega_c\tau'_c} + \\
R'(-j\omega_c)e^{2j\omega_c\tau'_c} &= 0
\end{aligned} \tag{6}$$

The exponential terms in (6) should be eliminated to obtain a new characteristic polynomial without transcendentality. This could be easily achieved using a two-step recursive procedure as described below. Let us define new characteristic equations at  $s = \pm j\omega_c$  as [8]

$$\begin{aligned}
\Delta^{(1)}(j\omega_c, \tau'_c) &= P'(-j\omega_c)\Delta(j\omega_c, \tau'_c) - \\
R'(j\omega_c)e^{-2j\omega_c\tau'_c}\Delta(-j\omega_c, \tau'_c) &= 0 \\
\Delta^{(1)}(j\omega_c, \tau'_c) &= [P'(-j\omega_c)P'(j\omega_c) - R'(j\omega_c)R'(-j\omega_c)] + \\
[P'(-j\omega_c)Q'(j\omega_c) - R'(j\omega_c)Q'(-j\omega_c)]e^{-j\omega_c\tau'_c} &= 0
\end{aligned} \tag{7}$$

and

$$\begin{aligned}
\Delta^{(1)}(-j\omega_c, \tau'_c) &= P'(j\omega_c)\Delta(-j\omega_c, \tau'_c) - \\
R'(-j\omega_c)e^{2j\omega_c\tau'_c}\Delta(j\omega_c, \tau'_c) &= 0 \\
\Delta^{(1)}(-j\omega_c, \tau'_c) &= [P'(j\omega_c)P'(-j\omega_c) - R'(-j\omega_c)R'(j\omega_c)] \\
+ [P'(j\omega_c)Q'(-j\omega_c) - R'(-j\omega_c)Q'(j\omega_c)]e^{j\omega_c\tau'_c} &= 0
\end{aligned} \tag{8}$$

It is clear from (7) and (8) that the root  $s = j\omega_c$  of (6) is also a root of the following new characteristic equations. Another way of saying, the purely imaginary root of (6) is preserved in (7) and (8). New characteristic equations in (7) and (8) could be rewritten in a much more compact form as

$$\begin{aligned}
\Delta^{(1)}(j\omega_c, \tau'_c) &= P^{(1)}(j\omega_c) + Q^{(1)}(j\omega_c)e^{-j\omega_c\tau'_c} = 0 \\
\Delta^{(1)}(-j\omega_c, \tau'_c) &= P^{(1)}(-j\omega_c) + Q^{(1)}(-j\omega_c)e^{j\omega_c\tau'_c} = 0
\end{aligned} \tag{9}$$

where

$$\begin{aligned}
P^{(1)}(j\omega_c) &= P'(-j\omega_c)P'(j\omega_c) - R'(j\omega_c)R'(-j\omega_c) \\
Q^{(1)}(j\omega_c) &= P'(-j\omega_c)Q'(j\omega_c) - R'(j\omega_c)Q'(-j\omega_c)
\end{aligned} \tag{10}$$

It should be noted that the new characteristic equations in (9) contain only a single  $e^{j\omega_c\tau'_c}$  and  $e^{-j\omega_c\tau'_c}$ , indicating that the degree of commensurability is reduced from 2 to 1. On the other hand, the degrees of polynomials  $P^{(1)}(j\omega_c)$  and  $Q^{(1)}(j\omega_c)$  now become 18 and 15, respectively after eliminating the term of  $e^{-2j\omega_c\tau'_c}$  in (6). This procedure could be easily repeated to eliminate exponential terms,  $e^{-j\omega_c\tau'_c}$  and  $e^{j\omega_c\tau'_c}$  in (9) and the following augmented characteristic equation not containing any exponential terms could be obtained.

$$\Delta^{(2)}(j\omega_c) = P^{(2)}(j\omega_c) = 0 \tag{11}$$

where

$$P^{(2)}(j\omega_c) = P^{(1)}(j\omega_c)P^{(1)}(-j\omega_c) - Q^{(1)}(j\omega_c)Q^{(1)}(-j\omega_c) \tag{12}$$

It should be noted here that the root (6) for  $\tau'_c$  is also a root of (11) since the elimination procedure preserves the purely imaginary roots of the original characteristic equation of (6). The equation in (11) can be easily rewritten as the following polynomial in  $\omega_c^2$ .

$$W(\omega_c^2) = P^{(1)}(j\omega_c)P^{(1)}(-j\omega_c) - Q^{(1)}(j\omega_c)Q^{(1)}(-j\omega_c) = 0 \tag{13}$$

Note that the new characteristic equation in (13) without any exponential term has the degree of 36 as given below. By substituting  $P'(s = j\omega_c)$ ,  $Q'(s = j\omega_c)$  and  $R'(s = j\omega_c)$  polynomials into (9)-(14) subsequently, we can obtain the augmented characteristic equation as follows

$$W(\omega_c^2) = t_{36}\omega_c^{36} + t_{34}\omega_c^{34} + \dots + t_2\omega_c^2 + t_0 = 0 \tag{14}$$

Due to insufficient space, the coefficients of (14) are not presented.

It is obvious from (14) that the exponential terms in the characteristic equation given in (4) or (6) are now eliminated without using any approximation. For that reason, the positive real roots of (14),  $\omega_c > 0$ , coincide with the imaginary roots of (4),  $s = \pm j\omega_c$  exactly.

For a positive real root  $\omega_c$ , the corresponding value of  $\tau'_c$  of the modified LFC system with the GPMT can be easily computed using (9) as follows [3, 8]:

$$\tau'_c = \frac{1}{\omega_c} \tan^{-1} \left\{ \frac{\text{Im} \left\{ \frac{P^{(1)}(j\omega_c)}{Q^{(1)}(j\omega_c)} \right\}}{\text{Re} \left\{ \frac{P^{(1)}(j\omega_c)}{Q^{(1)}(j\omega_c)} \right\}} \right\} \tag{15}$$

**Table 1.** Delay margin results for  $A=1$  and  $\phi=0^0$

$\tau_{GPM}(s)$	$K_I$					
$K_P$	0.05	0.1	0.15	0.2	<b>0.4</b>	0.6
0	30.81	15.09	9.84	7.21	3.23	1.84
0.05	31.77	15.57	10.16	7.45	3.35	1.92
0.1	32.65	16.01	10.45	7.67	3.45	1.99
<b>0.2</b>	34.12	16.74	10.94	8.04	<b>3.63</b>	2.11
0.4	35.73	17.54	11.47	8.42	3.80	2.19
0.6	34.81	17.07	11.14	8.16	3.59	1.88

**Table 2.** Delay margin results for  $A=2$  and  $\phi=0^0$

$\tau_{GPM}(s)$	$K_I$					
$K_P$	0.05	0.1	0.15	0.2	<b>0.4</b>	0.6
0	15.09	7.21	4.56	3.23	1.09	0.25
0.05	16.01	7.67	4.87	3.45	1.46	0.32
0.1	16.74	8.04	5.11	3.63	1.28	0.36
<b>0.2</b>	17.54	8.42	5.36	3.80	<b>1.28</b>	0.37
0.4	14.25	1.05	0.87	0.75	0.44	0.25
0.6	0.34	0.33	0.31	0.29	0.22	0.15

**Table 3.** Delay margin results for  $A=1$  and  $\phi = 20^0$ 

$\tau_{GPM}(s)$	$K_I$					
	0.05	0.1	0.15	0.2	0.4	0.6
$K_P$	0.05	0.1	0.15	0.2	0.4	0.6
0	23.84	11.61	7.52	5.48	2.37	1.29
0.05	24.80	12.09	7.85	5.72	2.49	1.37
0.1	25.70	12.54	8.15	5.95	2.61	1.45
0.2	27.28	13.33	8.67	6.34	2.79	1.57
0.4	29.33	14.35	9.35	6.84	3.03	1.69
0.6	29.23	14.29	9.29	6.78	2.93	1.49

**Table 4.** Delay margin results for  $A=2$  and  $\phi = 20^0$ 

$\tau_{GPM}(s)$	$K_I$					
	0.05	0.1	0.15	0.2	0.4	0.6
$K_P$	0.05	0.1	0.15	0.2	0.4	0.6
0	11.61	5.48	3.4	2.37	0.70	0.012
0.05	12.54	5.94	3.73	2.61	0.82	0.085
0.1	13.33	6.34	3.99	2.80	0.90	0.135
0.2	14.35	6.84	4.31	3.03	0.94	0.164
0.4	12.18	0.83	0.66	0.55	0.28	0.078
0.6	0.19	0.18	0.16	0.15	0.07	0.009

Once we have found the stability delay margin of the modified LFC system with the GPMT, using (5), we could then easily compute the time delay value for which the original LFC system will have the desired gain and phase margins as follows:

$$\tau_{GPM} = \tau'_c - \frac{\phi}{\omega_c} \quad (16)$$

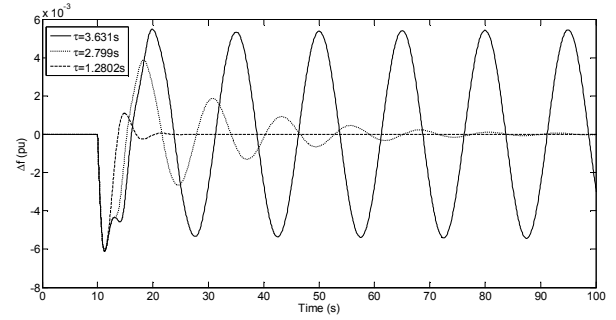
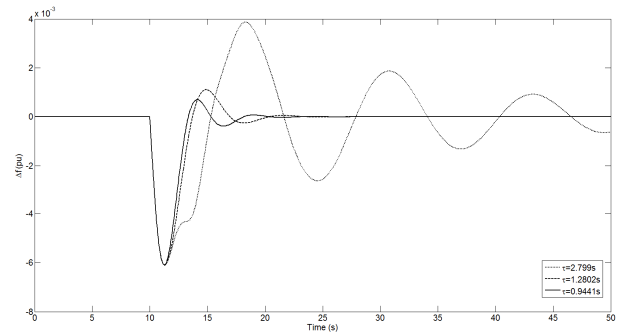
where  $\phi$  is the desired phase margin and  $\omega_c$  represents the root crossing frequency.

#### 4. Results

The section presents gain and phase margins based delay margins results and verification studies using time-domain simulations. The parameters of the two-area LFC system are given in [2, 3].

Delay margins that ensure the desired gain and phase margins are computed using (15) and (16) for a large set of PI controller gains and for various gain and phase margins. Table 1 presents delay margins for  $A=1$  and  $\phi=0^0$ . This case corresponds to the conventional stability delay margin computation which the gain and phase margins are not taken into account. Therefore,  $\tau_{GPM} = \tau'$  since  $\phi=0^0$  as seen from (16). Results show that  $\tau$  decreases as  $K_I$  increases for a fixed  $K_P$ . This indicates that the increase of  $K_I$  causes a less stable LFC system.

Next, the gain and phase margins are selected as ( $A=2, \phi=0^0$ ) to investigate the impact of the gain margin only on the delay margin. The corresponding delay margin results are presented in Table 2. It is clear that the inclusion of the gain margin notably reduces the delay margins for all PI controller gains. The effect on delay margin of phase margin  $\phi$  is also studied. Table 3 give the delay margin results for ( $A=1, \phi=20^0$ ).

**Fig. 3.** Damping effects of the gain and phase margins on the frequency response for ( $A=1, \phi=0$ ), ( $A=1, \phi=20^0$ ), ( $A=2, \phi=0^0$ ) for  $K_P=0.2, K_I=0.4$ **Fig. 4.** Combined damping effect of the gain and phase margins on the frequency response for ( $A=2, \phi=0^0$ ), ( $A=1, \phi=20^0$ ), ( $A=2, \phi=20^0$ )

Similar to the gain margin case, the results clearly indicate that delay margins decreases for all PI controller gains when the phase margin is only considered. However, the decrease in delay margins is less than that of the gain margin case given Table 2.

Finally, both gain and phase margins are included in the delay margin computation. Table 4 illustrates delay margin results for ( $A=2, \phi=20^0$ ). When compared with the gain margin case ( $A=2, \phi=0^0$ ) in Table 2 and the phase margin case ( $A=1, \phi=20^0$ ) in Table 3, it is obvious that the combined effect of the gain and phase margins on delay margins are much remarkable than their individual effects.

Tables 1 to 4 clearly indicate that for all PI controller gains delay margins decrease when gain and phase margins are taken into account. For a selected PI controller gains,  $K_P=0.2, K_I=0.4$  for example, the implication of these delay margins could be explained as follows: As can be seen from Table 1, the LFC system will be marginally stable at  $\tau=3.63 s$ , which means the system will not have any stability margins in terms of the time delay. A small increase in the time delay will destabilize the LFC system. Table 2 indicates that the LFC system will have the gain and phase margins of ( $A=2, \phi=0^0$ ) at  $\tau=1.28 s$ . That means the system is not only stable but also has the desired gain and/or phase margins.

In this case, the system will remain stable if even a small increase is observed in the time delay.

Time-domain simulations are performed to illustrate the reason why the gain and/or phase margins should be considered in delay margin computations. A load disturbance of  $\Delta P_d = 0.1 pu$  at  $t = 10 s$  is considered. Fig. 3 compares the frequency responses of the LFC system for  $(A=1, \phi=0^0)$ ,  $(A=1, \phi=20^0)$ ,  $(A=2, \phi=0^0)$  and  $(K_p=0.2, K_I=0.4)$ . From Table 1, note that delay margin is found to be  $\tau = 3.63 s$  for  $(A=1, \phi=0^0)$ . As seen in Fig. 3, the LFC system exhibits sustained oscillations for this delay value, which indicates that LFC system is marginally stable. However, such oscillations in the frequency deviations are not acceptable from a practical operating point of view.

To eliminate such an undesirable oscillations, we need to consider gain and/or phase margins in delay margin computation. As can be seen in Tables 2 and 3, delay margins based on gain and phase margins are computed as  $\tau = 1.28 s$  for  $(A=2, \phi=0^0)$  and  $\tau = 2.79 s$  for  $(A=1, \phi=20^0)$ . As compared to the case of  $(A=1, \phi=0^0)$ , it is clear from Fig. 3 that oscillations quickly damp out for  $\tau = 1.28 s$  when the gain margin  $(A=2, \phi=0^0)$  is considered. Similar observation could be made for the phase margin case,  $(A=1, \phi=20^0)$  in which oscillations also damp out but in a longer time. Finally, Fig. 4 illustrates the combining effect of gain and phase margins on the damping of LFC frequency response for  $(A=2, \phi=20^0)$ . From Table 4, the corresponding time delay value is computed as  $\tau = 0.94 s$ . Fig. 4 clearly shows that oscillations quickly damp out when both gain and phase margins are considered. This simulation results clearly indicate that gain and/or phase margins must be included in delay margin computation to have an improved dynamic response (fast damping, less overshoot, less settling time, etc.) of the LFC system with time delays.

## 6. Conclusions

This paper has utilized an analytical method to compute delays margins of a time-delayed two-area LFC control system considering not only stability but also gain and phase margins. It has been shown that delay margins obtained considering only the stability result in unacceptable oscillations in the frequency deviation and delay margins based on gain and phase margins improves the dynamic response. Depending on the value of time delay observed in the LFC system, the PI controller parameters could be properly selected such that the two-area LFC system will be not only stable and but also will have a desired dynamic performance in terms of damping, settling time and non-oscillatory behaviors.

## 7. References

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