Memristor-Based Sodium and Potassium Channels Model

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Abstract

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Emulation of known neural behavior is difficult because of the highly complex. The Hodgkin-Huxley model that was developed by Alan Lloyd Hodgkin and Andrew Fielding Huxley explains how to the membrane potentials are conducted from one cell to another cell through the axon. Memristor have attracted great interest for various applications recently. The most interesting application of memristors is neural systems. In this paper, two different special memristor models based Hodgkin-Huxley (HH) circuit are presented. The memristor models are developed using TiO_2 memristor mathematical model to emulate sodium and potassium channels and obtained voltage spikes train applying DC input current. Finally, the conductance characteristics of the developed special memristors are presented to obtain spikes train.

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1. Introduction

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Memristor is a 2-terminal circuit device characterized by a constitutive relation between two variables charge and flux "[1,2]". The new circuit component exhibited specific behavior different from that exhibited by resistors, capacitors or inductors. A simple wire acts as a resistor, two parallel plates separated by a dielectric work as a capacitor and a coiled metallic wire functions as an inductor. Memristor has attracted worldwide interests from scientists ever since it was implemented in 2008 "[3]". Hodgkin and Huxley proposed a circuit for axon membrane "[4]", including two nonlinear resistance, a parallel capacitance and linear leakage resistance. The principal circuit diagram is depicted in Fig.1.a. These two nonlinear resistances are described as sodium and potassium channels. The Hodgkin-Huxley model explains how the membrane potential is conducted from one cell to another cell. Chua rearranged Hodgkin-Huxley model replacing two variable resistors in the potassium and sodium channel with memristors "[5,6]". The rearranged new circuit model is shown in Fig.1.b. Some of the properties of the memristor can actually be outlined here: 1. It works as a passive element (like a resistor); 2.history dependent (depends on state-variable); 3.polarized element (current direction cares); 4.frequency-dependent (time derivative of the state variable). All these properties exist in a synapse! "[7]". Synapses are connections between neurons through which "information" flows from one neuron to another and synapses are history-dependent like memristors. Spike-timing-dependent-plasticity (STDP) rule is known fundamental rule of learning in the neural system.

The first spike timing dependent learning algorithms were reported by Gerstner and coworkers "[8, 9]". STDP has been shown to be more successful than other learning rules "[10-13]". Snider showed how to implement a neuromorphic learning law as STDP in nano devices and proposed STDP learning architecture based on flux driven memristors "[14-15]". Developing an infrastructure for constructing simple memristor-based neural circuit has crucial important. For this reason we focused on the investigation of correlations between memristor and neural systems.

This paper proposes and investigates how to rearranged HP TiO_2 memristor mathematical equations to obtain spikes using Hodgkin-Huxley model. From a theoretic perspective, both sodium and potassium channels are designed developing TiO_2 memristor equations, which provide to us voltage spikes.

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2. New Memristor Model For Sodium and Potassium Channels

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In 2008, the HP research team announced realization of a titanium dioxide memristor, entitled "The Missing Memristor Found" "[3]". The current-voltage relationship is described in Eq. (1).

$$V = \left[R_{ON}\, x + R_{OFF}\,(1-x) \right] I \tag{1}$$

Where, w is width of the doped region, namely, state variable, D is the total length D of the memristor. The speed of the movement of the x value, Joglekar's window function "[16]", and final form of the equation:

$$\frac{dx}{dt} = \eta \frac{\mu_D R_{ON}}{D^2} i(t) \tag{2}$$

$$f(x) = 1-(2x-1)^{2p} \tag{3}$$

$$\frac{dx}{dt} = \eta \frac{\mu_D R_{ON}}{D^2} i(t)\, f(x) \tag{4}$$

The sodium (Na) and potassium (K) ion channels of the Hodgkin-Huxley model are mathematically equivalent to two distinct memristors "[2]". Hodgkin-Huxley circuit is constructed using two new modelled memristors and one parallel capacitor as shown Fig. 1.

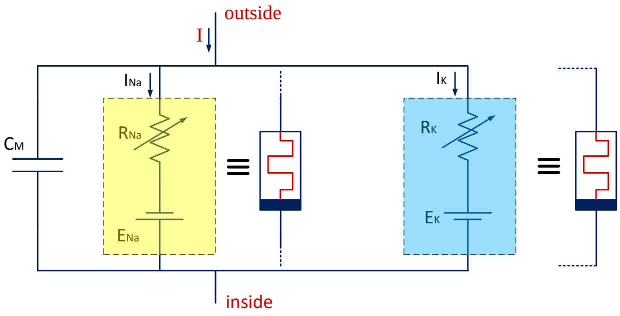

Fig. 1. New Hodgkin-Huxley circuit model.

Membrane can be described as a charge-controlled because of the fact that movement of ions causes voltage change. For this reason, this change gives rise to voltage spikes. Proposed memristor is flux-controlled and voltage change causes current spikes. As a result, current spikes created voltage spikes on capacitor. Two functions are presented to implement sodium and potassium channels using two different memristors. The mathematical function of sodium channel is

$$f_{Na} = \frac{-3}{\left(4e^{-2(\varphi+0.5)}-1\right)^5} + \frac{-24\varphi e^{-2\varphi}}{\left(4e^{-2\varphi}+1\right)^2} \qquad (5)$$

where φ is flux. The resultant new speed of the movement of the x in this case is

$$\frac{dx_{Na}}{dt} = \eta \frac{\mu_D R_{ON}}{D^2} i(t) f(x) f_{Na} \qquad (7)$$

The mathematical function of potassium channel is given by

$$f_K = \frac{12e^{-4\varphi}}{e^{-4\varphi}+1} \qquad (8)$$

As a result, we obtain

$$\frac{dx_K}{dt} = \eta \frac{\mu_D R_{ON}}{D^2} i(t) f(x) f_K \qquad (9)$$

Here, sodium and potassium channels are developed and can be easily obtained by modifying TiO_2 memristor mathematical equations. Each channel was represented by a new modified memristor.

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3. Simulation Results

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Current-voltage characteristic of HP memristor has a pinched hysteresis loop when applied sinusoidal voltage source ”[1]”. The conductivity in a memristor is related both applied voltage and the previous value of the state variable x. If DC supply is applied to the HP memristor, the characteristic of memristor will become linear resistor. The V-I of the memristor driven by periodic source is always a pinched hysteresis loop passing through the origin “[5]”.

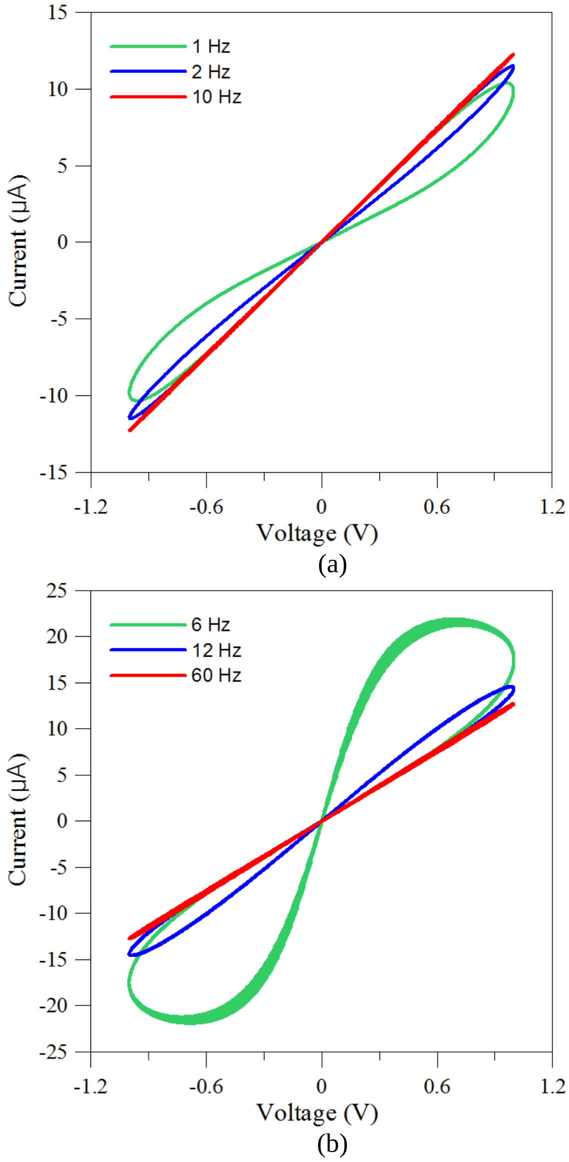


Fig. 2. Current-Voltage characteristic of channels with various frequencies (a) Sodium Channel (b) Potassium Channel.

The current-voltage characteristics of Sodium and Potassium channels with various frequencies are shown in Fig.2. The input signal voltage is 1 V with different frequency ranges. The variations of the pinched hysteresis loop for various frequencies at 1, 2, 10 Hz for sodium channel, and 6, 12, 60 Hz for potassium channel are also shown in Fig. 2. It can see from Fig.2., the operating frequency range of sodium channel is different from potassium channel. Both sodium and potassium channels exhibit hysteresis current-voltage relationship as original sodium and potassium channels.

Finally, HH circuit is rearranged using new designed memristors for sodium and potassium channels.

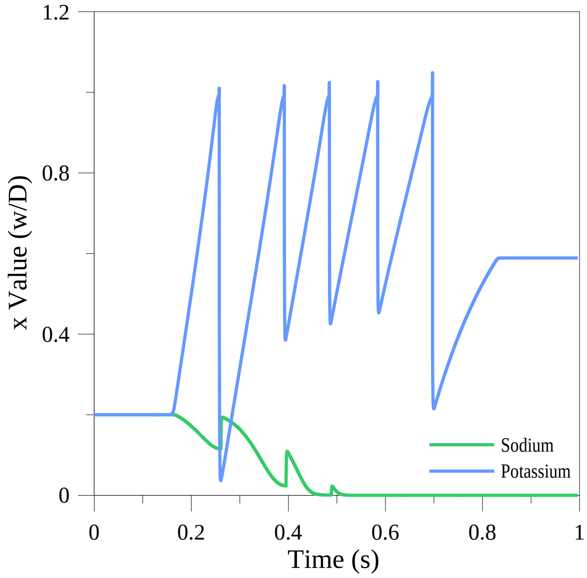
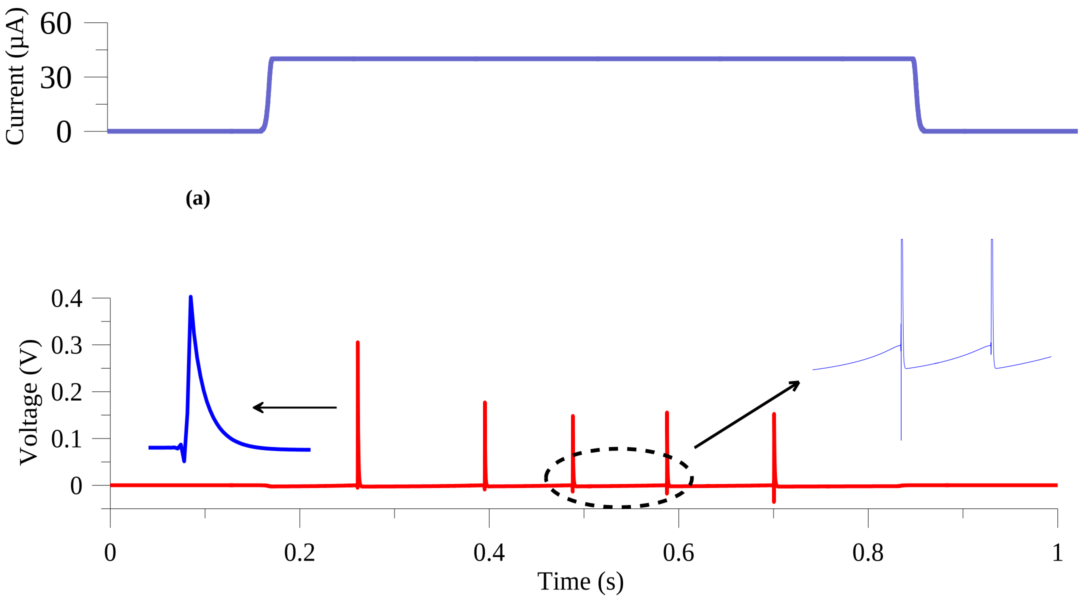


Fig. 3. The simulated x value of channels.

As shown in Fig.3., the sodium channel conductance decreased, simultaneously potassium channel conductance increased when input current is applied to HH circuit. Here initial x value, namely channel conductance is 0.2 (theoretically x variable must be 0<x<1).

After biasing to HH circuit, x value of sodium channel was very small and another x value of potassium channel was higher than before value. In addition variation of conductances moved in opposite direction. Conductance of the channels is consistent with the paper which was published in Nature "[17]".

Lastly, A 40 μA constant current bias is applied to the input terminal of the developed HH circuit as shown in Fig.4.a and shown the regular spiking behavior of the capacitance voltage output (Fig.4.b).



(a)

(b)

Fig. 4. (a) Input DC 20 μA current-source (b) Voltage output of the HH circuit.

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6. Conclusions

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Constructing a compact model of a memristor that is useful for circuit design and simulation is a complex and arduous task. Perhaps after several examples of successful models have been derived and methodology for creating them has become clearer, it will be more straightforward "[18]". The primary goal of this paper is to develop an infrastructure for constructing simple electrical circuit. We showed that sodium and potassium channels can be easily obtained by modifying TiO_2 memristor mathematical equations. Each channel was represented by a new modified memristor. Finally conductance characteristics of sodium and potassium channels have crucial role that is why the changes of the x values for modified memristor are presented.

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