Optimal Signal Reconstruction Based on the Fourier Decomposition Method

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Abstract

The Fourier decomposition method (FDM) is the newest time-frequency-energy analysis tool for signals. Using the FDM, any signal can be represented by the sum of a small number of band-limited orthogonal functions termed Fourier intrinsic band functions (FIBFs). In its present form, however, the FDM is not based on an optimality criterion. The lack of optimality limits the signal recovery capability of the FDM in the presence of disturbances. To find a solution to this limitation and therefore to enhance the capability of the FDM, this paper proposes to adapt two thoughts that are previously applied for the empirical mode decomposition, to the FDM. The other contribution of this paper is that we propose faster procedures than the conventional FDM in finding the FIBFs of a given signal. Simulations show that the proposed approaches result in satisfactory performance in reconstructing the original signal under undesired disturbances.

1. Introduction

The Fourier decomposition method (FDM) derived by Singh *et al.* [1] is the newest signal analysis tool that can applied to any signal. Since it is based on the standard Fourier transform, the FDM is a fully adaptive data-driven method that does not require any prior known basis. On the other hand, unlike the standard Fourier transform, the FDM provides the analytic representation and, thus, the Hilbert transform of a given signal, inherently. Therefore, it is also a suitable tool for time-frequency analysis of any signal [1].

Using the FDM, any signal can be described by a set of a small number of functions named as Fourier intrinsic band functions (FIBFs) that are zero-mean and orthogonal, and have analytic forms with instantaneous frequencies and amplitudes always greater than zero. As compared to another popular signal analysis tool named as the empirical mode decomposition (EMD) method developed by Huang *et al.* [2], the FDM has more formal mathematical foundation and does not suffer from artefacts such as mode mixing, de-trend uncertainty, and end effects during the extraction of the FIBFs of a signal. Furthermore, it is shown in [1] that the FDM provides the better time-frequency-energy estimates and frequency band separation than the EMD through FIBFs. Its ability to effectively separate out the frequency bands of a signal gives an opportunity in using the FDM for classification tasks. Indeed, the recent studies [3, 4] show that the FDM is a resourceful tool for the classification of EEG signals. Despite all these advantages, there is no any implication on the signal reconstruction optimality of the FDM. The lack of such a criterion restricts the signal reconstruction capability of the FDM, especially in the presence of disturbances like interference and noise. In order to find a solution to this limitation and hence enhance the capability of the FDM, in this paper, we propose two approaches that combine the FDM with the mean square error (MSE) criterion. These approaches can be seen as the adaptation of two thoughts that are previously applied for the EMD [5], to the FDM. The first approach, termed optimal signal reconstruction based on FDM (OSR-FDM), tries to reconstruct the given signal through linearly weighted FIBFs. The second approach named as the regularized bidirectional OSR-FDM (RBOSR-FDM) applies a filter bank to the FIBFs. As another contribution of this paper, we propose faster procedures than the conventional FDM in finding the FIBFs of a given signal. To evaluate the signal reconstruction performance of the proposed approaches in the presence of different disturbances, we carry out various simulations on real-life signals and compare the performance of these approaches with existing EMD counterparts.

The paper is organized as follows. A brief description of the EMD and its settings for simulating the signals are given in Section 2. The proposed approaches and procedures for optimal signal reconstruction based on the FDM are presented in Section 3. Section 4 is dedicated to the simulation results. Concluding remarks are mentioned in Section 5.

2. Brief Overview of the EMD-based Optimal Signal Reconstruction

Here, we give a brief description of the EMD method and its optimal versions named as OSR-EMD and RBOSR-EMD. As is seen in the subsequent section, the proposed approaches for optimal signal reconstruction in the framework of the FDM are based on the same mathematical inferences of these two algorithms. Furthermore, we use these EMD-based optimal algorithms as references to evaluate the performance of the proposed signal reconstruction approaches.

The EMD developed by Huang *et al*. [2] is an intrinsically adaptive signal decomposition method suitable for analyzing nonlinear and non-stationary signals as well. Through the EMD, any signal $x(n)$ can be expressed by a finite set of oscillation modes, termed, intrinsic mode functions (IMFs) and a residual function, as in the following:

$$x(n) = \sum_{i=1}^{M} c_i(n) + r(n) = \sum_{i=1}^{M+1} c_i(n) \qquad (1)$$

where $c_i(n)$ denotes the ith IMF and $r(n) = c_{M+1}(n)$ refers the residual function that can be considered as the last IMF.

An IMF is a time series and decomposing the signal into its IMFs as in (1) is carried out by *sifting* process such that the following conditions are met for all the IMFs [2]: (*i*) the absolute difference between the number of extrema and the number of zero crossings is at most one in the complete duration of the time series, (*ii*) the mean of the upper and lower envelopes obtained by fitting of local maxima and local minima is approximately zero at any point of time in the time series.

To represent the signals as in (1), we utilize the EMD algorithm in [2]. The upper and lower envelopes required in the process of EMD are obtained by applying the cubic spline curve fitting to the local maxima and local minima of the time series. To avoid the artifacts arising from the cubic spline fitting at the borders of the signal, we combine the mirror extrapolation approach with the EMD algorithm. In finding the IMFs, the closeness of the mean of the lower and upper envelopes to zero is bounded by 10^{-3}. To meet the abovementioned two conditions for the IMFs, the sifting processes are carried out with a large enough maximum number of iterations and using a small enough value as the stopping criterion. In this connection, we choose the maximum number of iterations and the stopping criterion as 10^4 and 10^{-2}, respectively.

The conventional EMD algorithm [2] does not based on any optimality criterion. In order to make it more flexible for signal reconstruction especially in the presence of disturbances such as interference and noise, two algorithms optimal in the MSE sense are proposed by Weng and Barner [5]. The first of these algorithms is named as the optimal signal reconstruction based on EMD (OSR-EMD), which reconstructs the original signal via the sum of the linearly weighted IMFs:

$$\hat{s}(n) = \sum_{i=1}^{M+1} a_i c_i(n) \tag{2}$$

where $\hat{s}(n)$ is the reconstructed version of the original signal $s(n)$ from its noise-corrupted version $x(n)$ and a_i denotes the weight assigned to the time series $c_i(n)$ corresponding to the ith IMF of the noisy signal $x(n)$. The weighting coefficients $\{a_i\}$ of the IMFs are determined such that the MSE defined by $E\{[s(n) - \hat{s}(n)]^2\}$ is minimized (for details see equations (6)-(13) in [5]).

The second algorithm proposed by Weng and Barner [5] is named as the regularized bidirectional OSR-EMD (RBOSR-EMD) that reconstructs the original signal by applying a filter bank to the IMFs:

$$\hat{s}(n) = \sum_{i=1}^{M+1} \sum_{j=-W}^{W} b_{ij} c_i(n-j) \tag{3}$$

where W stands for the half window length of a FIR filter and b_{ij} denotes the coefficients of the FIR filter applied to the ith IMF. The coefficients $\{b_{ij} \mid i = 1, 2, \ldots, M+1, j = -W, -W+1, \ldots, W\}$ for $M + 1$ FIR filters are determined by solving a linear system of equations formed on the basis of the MSE criterion and the quadratic programming is used so as to obtain the solution (for details see equations (18)-(24), (32), (33) in [5]).

Different from the OSR-EMD, the RBOSR-EMD algorithm uses knowledge of the relationship between the IMFs. Thus, it can capture the signal dynamics better than the OSR-EMD [5].

3. Optimal Signal Reconstruction in the Framework of the Fourier Decomposition Method

Although the EMD is a general method for analyzing nonlinear and non-stationary signals, it is an empirical method and thus has not a formal mathematical description yet. Also, the EMD method requires user-defined parameters such as stopping criterion and a type of interpolation, which affect its signal analysis performance. As an alternative method for the analysis of signals, the FDM proposed by Singh *et al.* [1] has a more concrete mathematical foundation compared to the EMD. As a new signal decomposition method, the FDM is also free from some limitations like stopping criterion, a type of interpolation, mode mixing and end effect artifacts that the EMD suffers. On the other hand, similar to the conventional EMD algorithm before the proposals of Weng and Barner [5], at present, there is no any implication on the reconstruction optimality of the FDM. It is expected that the lack of optimality will limit the signal recovery capability of the FDM, particularly in the presence of disturbances like interference and noise. As a suggestion to overcome this limitation and its effects, in this paper, we propose adapting mathematical inferences developed in [5] to FDM for optimal signal reconstruction.

The FDM is based on the representation of a real-valued N-sample signal $x(n)$ according to the inverse discrete Fourier transform (IDFT) as follows [1]:

$$x(n) = X(0) + \mathrm{Re}\{z(n)\} + (-1)^n X(N/2) \tag{4}$$

where N is an even number and $X(k)$ expresses the N-point DFT coefficients for $k = 0, 1, \ldots, N-1$. The N DFT coefficients $X(k)$ obtained by applying the well-known fast Fourier transform (FFT) algorithm to the N-sample signal $x(n)$ give the frequency components ranging from 0 to f_S Hz, where f_S denotes the sampling frequency. Each component has a frequency of $f = kf_S/N$ corresponding to the frequency index k. In (4), $\mathrm{Re}\{z(n)\}$ stands for the real part of the analytic signal $z(n)$ defined by

$$z(n) = 2\sum_{k=1}^{N/2-1} X(k) e^{j2\pi kn/N} = \sum_{i=1}^{T} w_i(n) e^{j\phi_i(n)}. \tag{5}$$

Considering (4), the representation defined by (5) is the first form of obtaining the FIBFs by evaluating the instantaneous frequency (IF) [1]

$$f_i(n) = (1/2\pi)\left[\phi_i(n+1) - \phi_i(n)\right], \forall n \tag{6}$$

according to

$$w_i(n) e^{j\phi_i(n)} = 2\sum_{k=N_{i-1}+1}^{N_i} X(k) e^{j2\pi kn/N};\; i = 1, 2, \ldots, T \tag{7}$$

with $N_0 = 0$, $N_T = N/2 - 1$, and $n = 0, 1, \ldots, N-1$.

The key point in obtaining the FIBFs is the determination of the frequency indices $N_1, N_2, \ldots, N_{T-1}$. These indices are determined as follows [1]. Each index N_i is searched in the interval of $[N_{i-1} + 1, N_T]$ for $i = 1, 2, \ldots, T-1$. Equation (7) is evaluated for each N_i value in this interval, and then instantaneous phase (IP) $\phi_i(n)$ is calculated. Using the IPs corresponding to N_i values ranging from $N_{i-1} + 1$ to N_T in (6), the corresponding IFs $f_i(n)$ are calculated. Thus, the maximum value of N_i satisfying the condition of $f_i(n) \geq 0$ is determined as N_i. This procedure is named as the *low to high frequency scan* (LTH-FS) in [1]. Hence, the FIBFs $\Omega_i(n)$ is determined as

$$\Omega_i(n) = \mathrm{Re}\left\{2\sum_{k=N_{i-1}+1}^{N_i} X(k) e^{j2\pi kn/N}\right\} \tag{8}$$

for $i = 1, 2, \ldots, T$ and $n = 0, 1, \ldots, N-1$.

It is noted that the LTH-FS procedure requires unnecessary operations increasing the processing time and the memory in the course of determining the frequency indices $N_1, N_2, \ldots, N_{T-1}$ and thus the FIBFs. To make the existing LTH-FS procedure more practical, the following procedure, termed, *modified LTH-FS* (M-LTH-FS) is proposed. The proposed procedure for obtaining the frequency indices N_1, N_2, …, N_{T-1} and therefore the FIBFs are summarized in the following steps:

Step 1. Set $p = 0$, $i = 1$ and for $n = 0, 1, \ldots, N-1$ obtain

$$sum(n) = 2\sum_{k=1}^{N/2-1} X(k)e^{j2\pi kn/N} = w(n)e^{j\phi(n)}$$

Step 2. Assign $sum(n)$ to $temp_i(n)$ signal, $temp_i(n) = sum(n)$.

Step 3. Obtain the IPs $\phi_i(n)$ of the analytical signal $temp_i(n)$ and find the corresponding IFs $f_i(n)$ via (6).

Step 4. Control all the IFs, $f_i(n)$ for $n = 0, 1, \ldots, N-1$. If the condition of $f_i(n) \geq 0$ is not satisfied for all n, set $p = p + 1$ and update the signal $temp_i(n)$ as follows:

$$temp_i(n) = temp_i(n) - 2X(N/2-p)e^{j2\pi n(N/2-p)/N}$$

Step 5. Repeat Step 3 and Step 4 until the condition of $f_i(n) \geq 0$ is satisfied for all n.

Step 6. If the condition of $f_i(n) \geq 0$ is satisfied for all n, the frequency index N_i is obtained and then it will be $p = N/2 - 1 - N_i$. Thus, the ith FIBF will be determined as

$$\Omega_i(n) = \mathrm{Re}\{temp_i(n)\}$$

Step 7. Set $i = i + 1$, $p = 0$ and obtain

$$temp_i(n) = sum(n) - \sum_{l=1}^{i-1} temp_l(n)$$

Step 8. Repeat Steps 3-7 until it is not found a $temp_i(n)$ signal that satisfy the condition of $f_i(n) \geq 0$ or until it is $i < T + 1$, where $N_{T-1} < N_T$.

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Based on the representation (5), the second form of obtaining the FIBFs through the evaluation of the IF defined by (6) is

$$w_i(n)e^{j\phi_i(n)} = 2\sum_{k=N_i}^{N_{i-1}-1} X(k)e^{j2\pi kn/N};\ i = 1, 2, \ldots, T \qquad (9)$$

with $N_0 = N/2$, $N_T = 1$, and $n = 0, 1, \ldots, N-1$.

As is in the LTH-FS procedure, the key point in obtaining the FIBFs based on (9) is the determination of the frequency indices N_1, N_2, …, N_{T-1}. These indices are determined as follows [1]. Each index N_i is searched in the interval of $[N_{i-1} - 1, N_T]$ for $i = 1, 2, \ldots, T-1$. Equation (9) is evaluated for each N_i value in this interval, and then the IP $\phi_i(n)$ is calculated. Using the IPs corresponding to N_i values ranging from $N_{i-1} - 1$ to N_T in (6), the corresponding IFs $f_i(n)$ are found. Thus, the minimum value of N_i satisfying the condition of $f_i(n) \geq 0$ is determined as N_i. This procedure is termed as the *high to low frequency scan* (HTL-FS) in [1]. Hence, the FIBFs $\Omega_i(n)$ is determined as

$$\Omega_i(n) = \mathrm{Re}\left\{2\sum_{k=N_i}^{N_{i-1}-1} X(k)e^{j2\pi kn/N}\right\} \qquad (10)$$

for $i = 1, 2, \ldots, T$ and $n = 0, 1, \ldots, N-1$.

As is the LTH-FS procedure, the HTL-FS procedure requires unnecessary operations that increase the processing time and memory during the determination of the frequency indices N_1, N_2, …, N_{T-1} and thus the FIBFs. To make the HTL-FS procedure more practical, we propose a *modified HTL-FS* (M-HTL-FS) procedure. The proposed procedure for obtaining the frequency indices N_1, N_2, …, N_{T-1} and therefore the FIBFs are defined as in the following steps:

Step 1. Set $p = 0$, $i = 1$ and for $n = 0, 1, \ldots, N-1$ obtain

$$sum(n) = 2\sum_{k=1}^{N/2-1} X(k)e^{j2\pi kn/N} = w(n)e^{j\phi(n)}$$

Step 2. Assign $sum(n)$ to $temp_i(n)$ signal, $temp_i(n) = sum(n)$.

Step 3. Obtain the IPs $\phi_i(n)$ of the signal $temp_i(n)$ and then find the corresponding IFs $f_i(n)$ by using (6).

Step 4. Control all the IFs, $f_i(n)$ for $n = 0, 1, \ldots, N-1$. If the condition of $f_i(n) > 0$ is not satisfied for all n, set $p = p + 1$ and update the signal $temp_i(n)$ as follows:

$$temp_i(n) = temp_i(n) - 2X(p)e^{j2\pi np/N}$$

Step 5. Repeat Step 3 and Step 4 until the condition of $f(n) \geq 0$ is satisfied for all n.

Step 6. If the condition of $f_i(n) \geq 0$ is satisfied for all n, the frequency index N_i is obtained and then it will be $p = N_i - 1$. Thus, the ith FIBF is to be

$$\Omega_i(n) = \mathrm{Re}\{temp_i(n)\}$$

Step 7. Set $i = i + 1$, $p = 0$ and obtain

$$temp_i(n) = sum(n) - \sum_{l=1}^{i-1} temp_l(n)$$

Step 8. Repeat Steps 3-7 until it is not found a $temp_i(n)$ signal that satisfy the condition of $f_i(n) \geq 0$ or until it is $i < T + 1$, where $N_{T-1} > N_T$.

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As a result, using the mathematical inferences defined by (5)-(8) in (4) turns out

$$x(n) = X(0) + \sum_{i=1}^{T} \Omega_i(n) + (-1)^n X(N/2) \qquad (11)$$

which corresponds to the representation of the real-valued signal $x(n)$ by using the FDM performed with the LHT-FS procedure. Similarly, the real-valued signal $x(n)$ can also be obtained in the form of (11) by using (5), (6), (9), and (10) in (4). In this case, (11) corresponds to the representation of the real-valued signal $x(n)$ by using the FDM performed with the HTL-FS procedure.

As mentioned before, the existing FDM does not based on any optimality criterion. The lack of optimality limits the reconstruction performance the FDM in the presence of disturbances like interference and noise. Inspired by the OSR-EMD and RBOSR-EMD algorithms in [5], we propose two optimal versions of the FDM so as to reconstruct the original signal $s(n)$ from its noisy counterpart $x(n)$. The proposed optimal FDMs are introduced in the subsequent subsections.

3.1. OSR-FDM

The proposed first optimal version of the FDM (OSR-FDM) is the linearly weighted version of (11), which is defined by

$$\hat{s}(n) = \sum_{i=1}^{T+2} a_i \Omega_i(n). \qquad (12)$$

For the sake of simplicity, the first and the last terms in (11) are included as $\Omega_1(n)$ and $\Omega_{T+2}(n)$ in (12). The remaining components are the T FIBFs generated by the LHT-FS (or proposed M-LHT-FS) and HTL-FS (or proposed M-HTL-FS) procedures separately. Note that, in (12), $\hat{s}(n)$ denotes the reconstructed version of the original signal $s(n)$ from its noisy version $x(n)$ and a_i refers the ith weight assigned to $\Omega_i(n)$ that corresponds to the ith time series of the noisy signal $x(n)$. The weighting coefficients $\{a_i\}$ of the related time series are determined such that the MSE defined by $E\{[s(n) - \hat{s}(n)]^2\}$ is minimized. To do this, in the equations (6)-(13) of [5], we use $\Omega_i(n)$ instead of $c_i(n)$.

3.2. RBOSR-FDM

Inspired from the RBOSR-EMD algorithm in [5], the proposed second optimal version of the FDM (RBOSR-FDM) tries to reconstruct the original signal through a linearly weighted version of (11) along both in vertical and horizontal directions:

$$\hat{s}(n) = \sum_{i=1}^{T+2} \sum_{l=-W}^{W} b_{il} \Omega_i (n-l). \tag{13}$$

Equation (13) also corresponds to the application a FIR filter bank to the time series $\Omega_i(n)$ involving T FIBFs generated by using LHT-FS (or proposed M-LHT-FS) and HTL-FS (or proposed M-HTL-FS) procedures separately. W indicates the half window length of the FIR filter applied on each time series $\Omega_i(n)$. $\{b_{il}\}$ states a set of FIR filter coefficients that are determined by solving a system of equations formed on the basis of the MSE criterion. The solution is performed by the quadratic programming of the equations (18)-(24), (32), (33) in [5]. Note that we use $\Omega_i(n)$ instead of $c_i(n)$ in these equations.

4. Simulation Results

The performance of the proposed OSR-FDM and RBOSR-FDM both with M-LTH-FS and M-HTL-FS procedures is evaluated on reconstructing ECG signal from its disturbed counterparts with white Gaussian noise (WGN) and baseline wander noise (BWN) separately. Because of the page limitation, we perform the simulations only on the ECG signal and only with signal-to-noise ratio (SNR) of 0 dB. While the WGN signal is generated synthetically from the Matlab, the ECG and BWN signals are provided from the PhysioBank database [6]. Under this database, the clean ECG used as the original signal $s(n)$ is the 'MLII' signal placing in the record of '100', which is provided from the MIT-BIH arrhythmia database. Also, the BWN signal 'noise1' placing in the record of 'bw' is extracted from the MIT-BIH noise stress test database. In both databases, the ECG and BWN recordings have a total duration of about 30 minutes and the sampling frequency is 360 Hz. In simulations, we use their first 10 second part and thus the total number of samples for the ECG and BWN signals is $N = 3600$ samples. It is noteworthy that, before generating the noisy ECG signals at SNR of 0 dB, the original and noise signals are processed to eliminate base and gain. To do this, normalization and standardization are performed on these signals. The processed signals involving clean ECG, noisy ECG with BWN and noisy ECG with WGN are shown in Fig. 1.

To gain insight into the performance of the proposed algorithms, comparisons are made with the OSR-EMD and RBOSR-EMD algorithms [5] by evaluating the signal-to-error ratio (SER) defined in decibel by

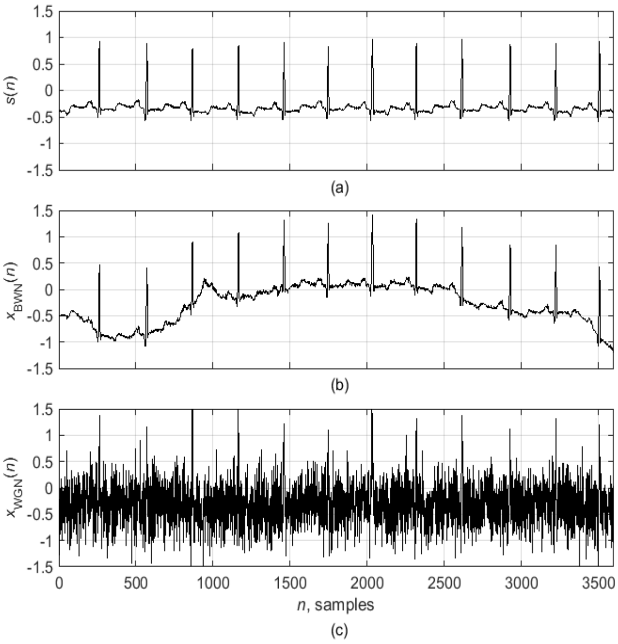

Fig. 1. Signals used in simulations: (a) Original ECG signal, (b) ECG signal disturbed by BWN with SNR of 0 dB, (c) ECG signal disturbed by WGN with SNR of 0 dB,

$$\mathrm{SER(dB)} = 10\log_{10}\left((1/(N \times MSE))\sum_{n=0}^{N-1} s^2(n)\right)$$

where N denotes the number of samples for the original signal $s(n)$, MSE is defined by

$$MSE = (1/N)\sum_{n=0}^{N-1}[s(n) - \hat{s}(n)]^2,$$

and $\hat{s}(n)$ indicates the reconstructed version of $s(n)$. Note that the SER(dB) should be as high as possible for a good signal reconstruction performance.

During the course of simulations, in the RBOSR-based methods, the FIR filter width is taken as $W = 5$ and the regularization bound required in the quadratic programming is chosen to be 1.

In order to estimate the weighting coefficients of the IMFs $\{c_i(n)\}$ for the OSR-EMD and RBOSR-EMD methods and to estimate the weighting coefficients of the time series $\{\Omega_i(n)\}$ for the OSR-FDM and RBOSR-FDM, we use the first S_k samples of the original ECG signal $s(n)$ as the *training signal*. Once these coefficients are estimated, they are used in (2) and (3) for reconstructing the original signal based on the OSR-EMD and RBOSR-EMD methods, respectively. Similarly, using the estimated weighting coefficients in the reconstruction formulae (12) and (13), four reconstructed counterparts of the original signal are obtained based on the four optimal versions of the FDM named as OSR-FDM(M-LTH-FS), OSR-FDM(M-HTL-FS), RBOSR-FDM(M-LTH-FS) and RBOSR-FDM(M-HTL-FS), respectively. We perform simulations by taking the length of training signal $\{s(n)|n = 0, 1, \ldots, S_k - 1\}$ as $S_k = 30 \times k$ for $k = 1, 2, \ldots, 120$. Thus, we have 120 signal reconstruction results for each compared algorithm. The SER results related to the

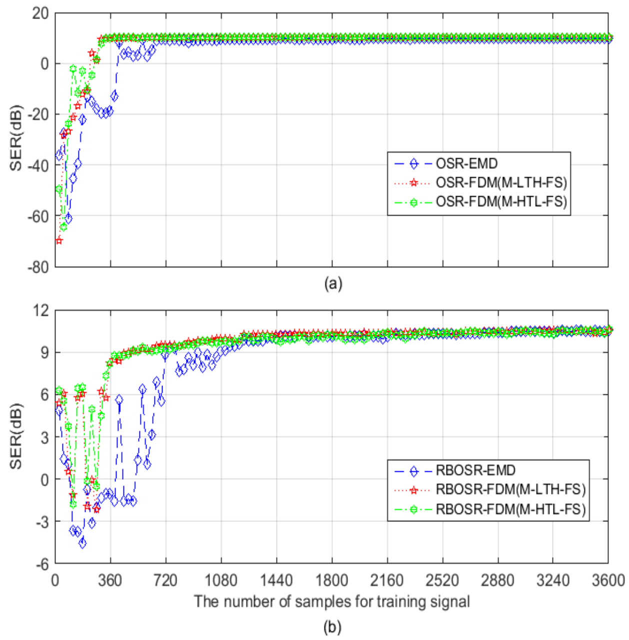

Fig. 2. SER results arising from the reconstruction of the ECG signal from its WGN disturbed counterparts by using 120 different training signal: (a) Results obtained by the OSR-based methods, (b) Results obtained by the RBOSR-based methods.

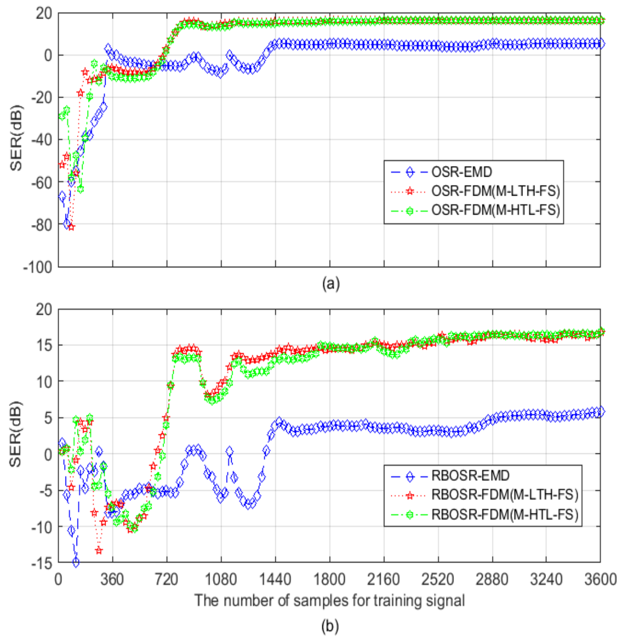

Fig. 3. SER results arising from the reconstruction of the ECG signal from its BWN disturbed counterparts by using 120 different training signals: (a) Results obtained by the OSR-based methods, (b) Results obtained by the RBOSR-based methods.

reconstruction of the ECG signal from its WGN and BWN disturbed versions are depicted in Fig. 2 and Fig 3, respectively.

It is clear from Fig. 2 that the proposed OSR-FDM and RBOSR-FDM approaches outperform their EMD counterparts, in the case of reconstructing original signal from its WGN distorted version with training signals that have a small number of samples. On the other hand, all methods exhibit almost the same performance, in the case of reconstructing the original signal from its WGN contaminated version with training signals that have a big number of samples. Unlike the Fig. 2, in reconstructing the original signal from its BWN contaminated version, it is clearly seen from Fig. 3 that the proposed approaches provide the best results almost for all cases of the training signal.

5. Conclusions

Inspired by the OSR-EMD and RBOSR-EMD algorithms, we propose two approaches to perform optimal signal reconstruction in the framework of the recently developed signal decomposition method known as FDM. It is supported by simulations that the proposed approaches result in superior performance than those of their existing counterparts even with training signals that have a small number of samples. This is an important superiority of the proposed approaches because they require a small knowledge of the original signal for reconstructing the original signal from its disturbed versions.

In addition to these proposed approaches, as another contribution of this paper is that we propose two effective procedures named M-LTH-FS and M-HTL-FS for obtaining the FIBFs of a given signal. Since these procedures are computationally and memory efficient, they are more practical than the existing ones termed LTH-FS and HTL-FS.

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