Calculation of Self- and Mutual Inductances of Power Transformers Using a 3D Modified Magnetostatic Integral Equation Approach

Bojan Trkulja[1], Željko Štih[1], and Tomislav Župan[2]
[1]Faculty of Electrical Engineering and Computing, University of Zagreb, 10000 Zagreb, Croatia
bojan.trkulja@fer.hr, zeljko.stih@fer.hr
[2] KONČAR Electrical Engineering Institute, Inc.
tzupan@koncar-institut.hr

Abstract

This work presents a novel three-dimensional calculation of self- and mutual inductances of a power transformer, based on a 3D magnetostatic integral equations approach and point matching. Influence of a ferromagnetic core is modeled with a constant magnetization over an element. In order to avoid the well-known cancellation error in calculation of the magnetic field inside the closed ferromagnetic core of a transformer, an additional line integral equation of magnetic field in a closed path, which is explicitly enforced to match magnetomotive force, is appended to the procedure. Overdetermined system of linear equations is assembled by point matching and solved by the least squares method. Results of the calculation for high voltage windings of a power transformer are in good agreement with results obtained by commercial FEM package Ansys Maxwell.

1. Introduction

The calculation of self- and mutual inductances of power transformers is a subject of many studies throughout the years. Classical approach to inductance calculation is to use commercial packages based on the well-established FEM procedures [1], [2]. Application of FEM demands discretization of the entire domain. In transformer design phase, it is necessary to calculate transformer inductances fast and with sufficient accuracy. In order to solve a problem using FEM, remeshing is required for each change in geometry, which is time consuming [2].

Therefore, another approach used is to employ in-house developed codes especially designed for transformers [3], [4]. Magnetostatic moment method is a promising approach, which can be used in calculating inductances, since it only requires meshing of a ferromagnetic core of a transformer. Ferromagnetic core is meshed into uniformly magnetized prisms. Application of the magnetostatic moment method coupled with the reluctance network method to such problems in current transformer is described in [4].

In this paper, a novel method based on the magnetostatic moment method is used to calculate inductances in a power transformer. Since the transformer core forms a closed magnetic circuit, for highly permeable ferromagnetic materials cancellation error occurs in calculation of the magnetic field strength inside the core [2], [5]. In order to resolve this problem, an additional line integral equation of the magnetic field over a closed path through the core, which is explicitly enforced to match the magnetomotive force, is appended to the procedure.

In the following sections, the method is described and applied to calculation of solving self- and mutual inductances of a power transformer.

2. Basic theory

A conducting ring with a rectangular cross-section with uniform current density J is wound around a ferromagnetic core according to Fig. 1.

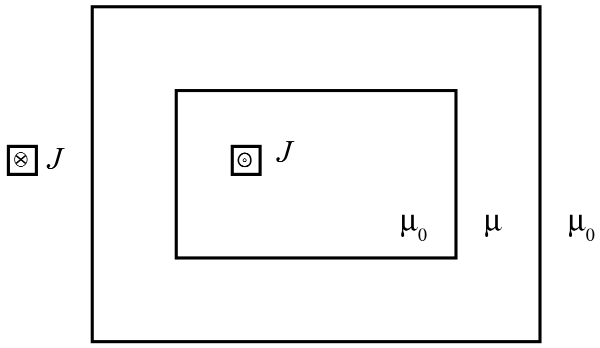

Fig. 1. Conducting ring and ferromagnetic core

The magnetic field at an arbitrary observation point $\vec{r}$ is

$$H(\vec{r}) = \vec{H}^e(\vec{r}) + \vec{H}^c(\vec{r}), \quad (1)$$

where $\vec{H}^e(\vec{r})$ is the magnetic field due to a circular conductor with current density J, calculated directly from the Biot-Savart law

$$\vec{H}^e(\vec{r}) = \frac{1}{4\pi}\iiint_{V'} \frac{\vec{J}\times(\vec{r}-\vec{r}\,')}{\left|\vec{r}-\vec{r}\,'\right|^3} dV', \quad (2)$$

and $\vec{H}^c(\vec{r})$ is the contribution of the ferromagnetic core.

The core is subdivided into n_c uniformly magnetized elements and the total contribution of the core is calculated by superposition of contributions of each element of the core.

In order to find the unknown magnetization of each element of the iron core, it is necessary to solve a magnetostatic problem governed by the following equations:

$$\nabla \cdot \vec{B} = 0, \quad (3)$$

$$\vec{B} = \mu_0 \left(\vec{H} + \vec{M} \right). \tag{4}$$

Solution in the form of volume and surface integrals is

$$\vec{H}(\vec{r}) = \frac{1}{4\pi} \iiint_{V'} \frac{(\vec{r}\,' - \vec{r}) \nabla \cdot \vec{M}}{\left|\vec{r}\,' - \vec{r}\right|^3} dV' - \frac{1}{4\pi} \oiint_{S'} \frac{(\vec{r}\,' - \vec{r}) \vec{M} \cdot \vec{n}_{S'}}{\left|\vec{r}\,' - \vec{r}\right|^3} dS'. \tag{5}$$

For a uniformly magnetized prism, (5) is reduced to

$$\vec{H}(\vec{r}) = \left(\frac{1}{4\pi} \oiint_{S'} \frac{(\vec{r} - \vec{r}\,') \otimes \vec{n}_{S'}}{\left|\vec{r}\,' - \vec{r}\right|^3} dS' \right) \cdot \vec{M} = \mathbf{T}(\vec{r}) \cdot \vec{M}. \tag{6}$$

Surface integral in (6) can be analytically evaluated for an arbitrary polyhedron [6]. In this paper, the iron core is discretized into rectangular parallelepipeds, where point C is at the center of element and O is an observation point (Fig. 2)

In (6) $\mathbf{T}(\vec{r})$ is a [3x3] matrix

$$\mathbf{T}(\vec{r}) = \begin{bmatrix} T_{xx} & T_{xy} & T_{xz} \\ T_{yx} & T_{yy} & T_{yz} \\ T_{zx} & T_{zy} & T_{zz} \end{bmatrix} \tag{7}$$

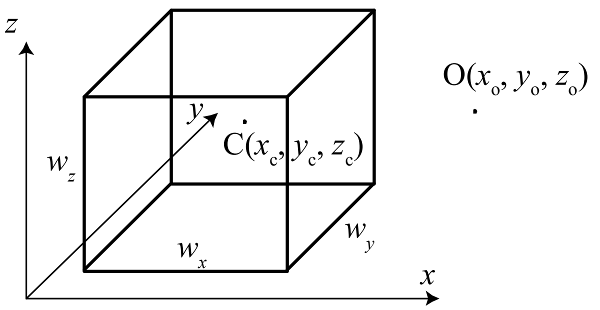

Fig. 2. Rectangular parallelepiped element

Diagonal element T_{xx} in $\mathbf{T}(\vec{r})$ matrix is [7]

$$\begin{aligned} T_{xx} = \frac{1}{4\pi} (&\tan^{-1}(\frac{y_1 z_1}{x_1\sqrt{x_1^2 + y_1^2 + z_1^2}}) - \tan^{-1}(\frac{y_1 z_1}{x_2\sqrt{x_2^2 + y_1^2 + z_1^2}}) \\ &- \tan^{-1}(\frac{y_2 z_1}{x_1\sqrt{x_1^2 + y_2^2 + z_1^2}}) + \tan^{-1}(\frac{y_2 z_1}{x_2\sqrt{x_2^2 + y_2^2 + z_1^2}}) \\ &- \tan^{-1}(\frac{y_1 z_2}{x_1\sqrt{x_1^2 + y_1^2 + z_2^2}}) + \tan^{-1}(\frac{y_1 z_2}{x_2\sqrt{x_2^2 + y_1^2 + z_2^2}}) \\ &+ \tan^{-1}(\frac{y_2 z_2}{x_1\sqrt{x_1^2 + y_2^2 + z_2^2}}) - \tan^{-1}(\frac{y_2 z_2}{x_2\sqrt{x_2^2 + y_2^2 + z_2^2}})), \end{aligned} \tag{8}$$

where

$$\begin{aligned} x_1 &= x_c - x_o - 0.5 w_x, \\ x_2 &= x_c - x_o + 0.5 w_x, \\ y_1 &= y_c - y_o - 0.5 w_y, \\ y_2 &= y_c - y_o + 0.5 w_y, \\ z_1 &= z_c - z_o - 0.5 w_z, \\ z_2 &= z_c - z_o + 0.5 w_z. \end{aligned} \tag{9}$$

Parameters x_c, y_c, z_c, x_o, y_o, z_o, w_x, w_y, w_z are set according to Fig.2.

Non-diagonal element T_{xy} in (5) is [7]

$$\begin{aligned} T_{xy} = \frac{1}{4\pi} (&\log(z_1 + \sqrt{x_2^2 + y_1^2 + z_1^2}) + \log(z_1 + \sqrt{x_1^2 + y_2^2 + z_1^2}) \\ &+ \log(z_2 + \sqrt{x_1^2 + y_1^2 + z_2^2}) + \log(z_2 + \sqrt{x_2^2 + y_2^2 + z_2^2}) \\ &-(\log(z_1 + \sqrt{x_1^2 + y_1^2 + z_1^2}) + \log(z_1 + \sqrt{x_2^2 + y_2^2 + z_1^2}) \\ &+ \log(z_2 + \sqrt{x_2^2 + y_1^2 + z_2^2}) + \log(z_2 + \sqrt{x_1^2 + y_2^2 + z_2^2}))). \end{aligned} \tag{10}$$

Other elements in the matrix $\mathbf{T}(\vec{r})$ are determined from (8) and (10) by using circular permutations of indices.

In order to solve (1) for the unknown magnetization $\vec{M}$ on n_c elements of the ferromagnetic core, a system of equations is formed for the unknown magnetic field $\vec{H}_k$ at the center of each element k of the prisms in a ferromagnetic core

$$\vec{H}_k = \vec{H}_k^e + \sum_{i=1}^{n_c} \mathbf{T} \cdot \vec{M}_i. \tag{11}$$

Taking into account that at the center of k-th element the magnetic field

$$\vec{M}_k = [\chi] \vec{H}_k, \tag{12}$$

the system of n_c equations in (11) is then rearranged in the form

$$-\vec{H}_k^e = \sum_{i=1}^{n_c} \mathbf{T} \cdot \vec{M}_i - \vec{M}_k \cdot [\chi]^{-1}. \tag{13}$$

where $[\chi]$ is a local susceptibility tensor.

If the magnetization $\vec{M}$ is obtained directly from (12), for a highly permeable closed core a strong cancellation error will occur due to a subtraction of large, nearly equal numbers [5].

Therefore, a solution is obtained by adding a one line-integral equation with the integration path through the core

$$\oint_c \vec{H} \cdot d\vec{l} = NI, \tag{14}$$

and enforcing it to match the magnetomotive force. An overdetermined system of equations (12) and (13) is than solved by the least squares method for the unknown magnetizations $\vec{M}$.

The self- and the mutual inductances are then determined, taking into account the contribution of a ferromagnetic core, with an integral of the vector magnetic potential $\vec{A}_c(\vec{r})$

$$\vec{A}_c(\vec{r}) = \sum_{k=1}^{n_c} \iiint_{V_i'} \frac{\vec{M}_k(\vec{r}\,')\times(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3} dV_k' \quad (15)$$

over the path of a conductor

$$L_c = \frac{\psi}{I} = \frac{\oint_{l'} \vec{A}_c(r)\cdot d\vec{l}\,'}{I}, \quad (16)$$

while the contribution of the conducting ring is more convenient to be calculated from the energy expression [8]

$$L_{s,i,j} = \frac{\mu_0 N_i N_j}{(r_{2,i}-r_{1,i})(r_{2,j}-r_{1,j}) h_{w,i} h_{w,j}} \int_0^{\pi} \cos(\varphi)\, d\varphi \int_{r_{1,i}}^{r_{2,i}} \rho_1 d\rho_1 \int_{r_{1,j}}^{r_{2,j}} \rho_2 d\rho_2$$
$$\int_{h_{w1,i}}^{h_{w2,i}} dz_1 \int_{h_{w1,j}}^{h_{w2,j}} \frac{dz_2}{\sqrt{(z_2-z_1)^2+\rho_1^2+\rho_2^2-2\rho_1\rho_2\cos(\varphi)}}. \quad (17)$$

Inductances are then

$$L_{i,j} = L_{s,i,j} + L_{c,i,j} \quad (17)$$

3. Numerical example

The proposed formulation has been tested on a power transformer geometry (Fig. 3). The results for the self- and the mutual inductances are benchmarked against *Ansys Maxwell 16.0*. One of the most important goals of this study is to find a sufficiently accurate solution that is fast enough to be applicable in the design phase.

This method is particularly suitable for that purpose because the interaction element-to-element matrices **T** in (12) have to be calculated only once. Therefore, for a change of geometry of windings, only $-\vec{H}_k^e$ on the left side of (12) has to be calculated again, which is very efficient in the optimization problems.

The power transformer HV windings layout is presented in Fig. 4. Windings are numerated (1) – (50).

The magnetization in the core (Fig. 5) is calculated using (12) and (13). In order to retain clarity, only half of the core and the central leg windings are shown.

The self-inductances and the mutual inductances are calculated and the inductance matrix is formed. Results of the calculation for the self-inductances benchmarked against FEM are presented in Fig. 6. The core is discretized in only 192 elements, and the maximum error in the self-inductance

The results of the calculation of mutual inductances of winding (1) to windings (2)–(50) are shown in Fig. 7. The maximum error in the calculation of a mutual inductance of a winding compared to FEM is 2.1% (Table I).

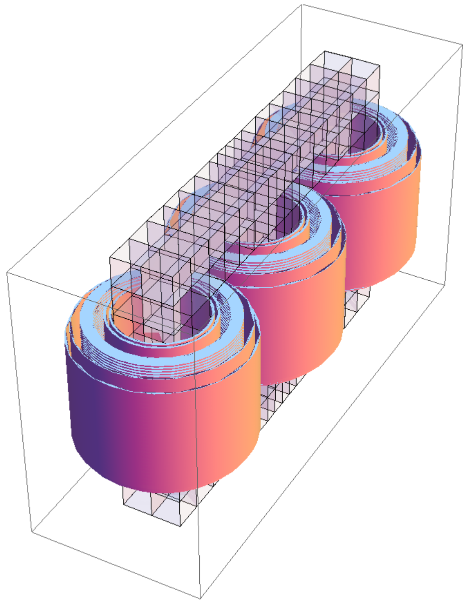

Fig. 3. Power transformer core and windings

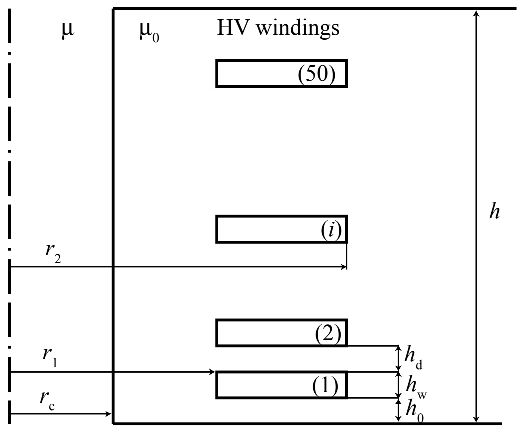

Fig. 4. Transformer HV windings layout (r_1=669 mm, r_2=796.5 mm, r_c=340 mm, h_0=89 mm, h_w=12.6 mm, h_d=19.8 mm, h=1800 mm, μ=4000μ_0)

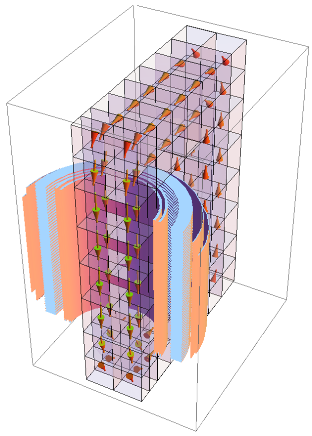

Fig. 5. Magnetization in core section

TABLE I
MAXIMUM ERROR IN THE SELF-INDUCTANCES AND THE MUTUAL INDUCTANCES OF THE HV WINDINGS COMPARED TO 3D FEM *ANSYS MAXWELL 16.0*

	L_1	$M_{1,13}$
FEM	161.27 mH	160.51mH
MMM	158.20 mH	157.22mH
Error[%]	1.9	2.1

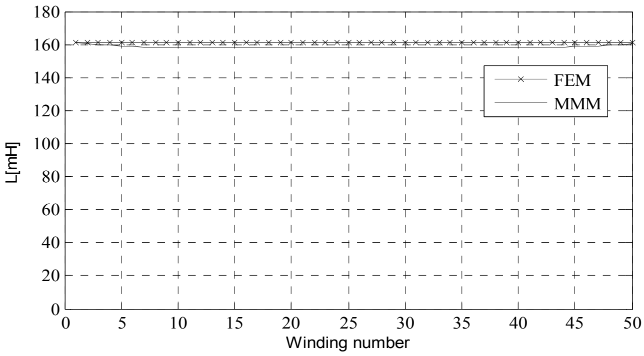

Fig. 6. Self-inductances of the HV windings

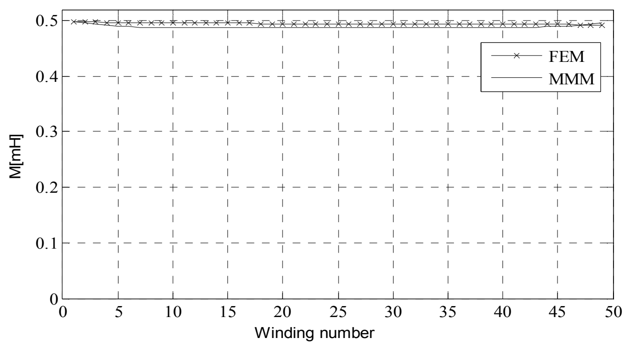

Fig. 7. Mutual inductances of winding (1) to windings (2)–(50)

4. Conclusion

Power transformers are crucial devices in efficient and reliable transmission and distribution of electricity. Fundamental design principles and basic elements of transformers (tank, core, windings and insulation system) are known for over a hundred years, yet designing a modern transformer is a challenging task. The focus in power transformer design is on higher efficiency and reliability during service and more economic solutions. Power transformer self- and mutual inductances are important parameters in the design phase and optimization of a transformer. In this paper a modified magnetostatic moment method developed for the calculation of self- and mutual inductances of coils on closed highly permeable core is presented. In order to handle the cancellation error in the iron core, an additional line integral equation of the magnetic field on a closed path through the core is enforced to equal the magnetomotive force. Finally, a comparison shows that the results of the calculation of inductances using a modified magnetostatic method are in good agreement with FEM.

5. Acknowledgement

This work has been fully supported by Croatian Science Foundation under the project IP-2013-1118.

6. References

[1] Y. Li, J. Du, X. Li, and D. Li, "Calculation of capacitance and inductance parameters based on FEM in high-voltage transformer winding," in *Electrical Machines and Systems (ICEMS), 2011 International Conference on*, Beijing, 2011, pp. 1-4.

[2] A. T. Phung, O. Chadebec, P. Labie, Y. Le Floch, and G. Meunier, "Automatic cuts for magnetic scalar potential formulations," *IEEE Trans. Magn.*, vol. 41, no. 5, pp. 1668-1671, May 2005.

[3] F. Janet, J.-L. Coulomb, C. Chillet, and P. Mas, "Magnetic moment and reluctance network mixed method applied to transformer modeling," *IEEE Tran. Magn.*, vol. 41, no. 5, pp. 1428-1431, May 2005.

[4] P. Gomez and F. de Leon, "Accurate and efficient computation of the inductance matrix of transformer windings for the simulation of very fast transients," *IEEE Trans. Power Delivery*, vol. 26, no. 3, pp. 1423-1431, Jul. 2011.

[5] W. Hafla, F. Groh, A. Buchau, and W. M. Rucker, "Magnetic field computation with integral equation method and energy-controlled relaxation," *IEEE Trans. Magn.*, vol. 42, no. 4, pp. 719-722, Apr. 2006.

[6] O. Chubar et al., "Application of finite volume integral approach to computing of 3d magnetic fields created by distributed iron-dominated electromagnet structures," in *Proceedings of EPAC 2004*, Lucerne, 2004, pp. 1675-1677.

[7] P. Elleaume, O. Chubar, and J. Chavanne, "Computing 3D magnetic fields from insertion devices", *Particle Accelerator Conference, 1997. Proceedings of the 1997*, vol. 3, pp. 3509-3511, 12-16 May 1997.

[8] D. Yu, and K. S. Han, "Self-inductance of air-core circular coils with rectangular cross section," *IEEE Trans. Magn.*, vol.23, no.6, pp.3916-3921, Nov. 1987.

[9] T. Župan, B. Trkulja, R. Obrist, T. Franz, B. Cranganu-Cretu, and J. Smajić, "Transformer Windings' RLC Parameters Calculation and Lightning Impulse Voltage Distribution Simulation", *IEEE Trans. On Magn..*, vol.52,no.3, March 2016

[10] B. Trkulja, A. Drandić, V. Milardić, T. Župan, I. Žiger, D. Filipović-Grčić: "Lightning impulse voltage distribution over voltage transformer windings — Simulation and measurement", Electric power systems research. vol. 147, June 2017, pp 185-191