A Lyapunov Based Model Reference Adaptive Control of A Quadrotor

Onur Akgün[1], Evren Subaşı[2], Türker Türker[2]

[1] Faculty of Engineering, Turkish-German University, Istanbul, Turkey
akgun@tau.edu.tr
[2] Dept. of Control & Automation Engineering, Yildiz Technical University, Istanbul, Turkey
evren1subasi@gmail.com, turker@yildiz.edu.tr

Abstract

Model reference adaptive controller is proposed to control the position of a quadrotor with uncertain model parameters. The controller design procedure is provided considering position and attitude dynamics as outer and inner loops respectively. Reference signals for roll and pitch angles are obtained from the outer loop controller while the inner loop controller drives the tracking error of roll and pitch angle dynamics to zero. Stability analysis for both inner and outer loop error dynamics are performed and the adaptation laws are derived based on Lyapunov stability theory. Since the reference models can also be assigned separately for inner and outer loops, the speed of the responses of position and attitude dynamics are adjusted independently improving the control performance. The viability and the success of the proposed controller are tested through simulations studies performed for different tracking scenarios.

1. Introduction

Unmanned aerial vehicles (UAVs) have captured increasing attention in last few decades due to their wide area of usage and advantageous features. One of the most popular UAV is the Quadrotor having preferable capabilities such as vertical take-off and landing (VTOL), agile motion and simple manufacturing process [1]. VTOL property brings quadrotors forward since it provides the ability of vertical, stationary and low speed flight options [2]. In addition, quadrotors are able to move in any direction both horizontal, vertical and their combination [3]. Quadrotors have a large variety of application fields such as mapping, search and rescue, surveillance, traffic monitoring, patrolling for forest fires [1], military and security purposes [2, 3].

Due to high nonlinearity and complexness of the dynamical model, the control problem of quadrotors have been considered extensively. Moreover, the underactuation –defined as having fewer actuators than the degree of freedom– in the dynamical model makes the control problem more challenging. This control problem can be taken into account as the combination of position control on the parallel plane to the ground, attitude and altitude control. Linear and nonlinear control methods are proposed in literature to achieve the control of these vehicles. Once the system is linearized around a certain equilibrium point, it can be stabilized via linear controllers such as full state feedback [4] and linear quadratic regulator [5]. A linear matrix inequality based method is proposed for the gain calculation in PID controllers for the linearized model [6]. In addition to linear controllers, a model reference adaptive controller (MRAC) is also proposed for the linearized model of a quadrotor in hovering operation [7]. On the other hand, the dynamic model of the quadrotor helicopter contains a combination of high-order nonlinear structure and system states. Therefore, the domain of attraction of the controllers designed after the linearization process is very limited. These controllers are generally effective for hovering [5] or working with low velocities and small attitude values [4].

Due to the limited motion capability of linear controllers, various nonlinear methods are proposed for the control of quadrotors such as backstepping controller [8], adaptive sliding mode controllers [1, 9], direct adaptive controller [10], adaptive PID controller [11], controllers based on neural networks [12]. In particular, different approaches of adaptive controllers are proposed for the attitude control of quadrotors with uncertain parameters [13, 14]. In addition to attitude control, altitude control is also provided in [15] via a model reference adaptive controller. An improved performance is obtained with these adaptive controllers applied to quadrotors with the uncertain terms existing in the dynamical model. Furthermore, in order to control also the position of the uncertain model of quadrotors, various different adaptive controllers are proposed as well [16–18]. A backstepping based adaptive controller is designed to deal with the uncertainty only on the mass of the vehicle [16]. In [17], an adaptive controller is designed for the changes in center of gravity of the quadrotor. Besides, based on Cerebellar Model Arithmetic Computer (CMAC), an approximate adaptive controller is applied aiming to provide the robustness against both uncertainty on mass and external disturbances.

A model reference adaptive controller scheme is proposed in this paper to control the position of the quadrotor in 3-dimensional space considering mass, gravitational acceleration and inertia values uncertain. After designing the altitude controller, the position control in xy plane is provided with roll and pitch angles being virtual control inputs, and these virtual inputs are utilized as desired values for the roll and pitch angles. Then, a controller is designed for the attitude motion of the vehicle. Reference models are derived considering position and attitude dynamics separately allowing to adjust the speed of the dynamical responses. Numerical simulations are performed to show the effectiveness of the proposed control structure and satisfying results are obtained.

2. Dynamical Model of a Quadrotor

The quadrotor examined in this paper has four fixed propellers placed perpendecularly. The motion control of a quadrotor helicopter is performed by controlling the speed of propellers that generate necessary thrusts. Dynamic model ignoring gyroscopic effects and friction forces is given as follows [2]:

$$\ddot{x} = \frac{1}{m}(\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi)u_1 \tag{1}$$

$$\ddot{y} = \frac{1}{m}(\cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi)u_1 \tag{2}$$

$$\ddot{z} = \frac{1}{m}(\cos\phi\cos\theta)u_1 - g \tag{3}$$

$$\ddot{\phi} = \frac{l}{I_x}u_2 \tag{4}$$

$$\ddot{\theta} = \frac{l}{I_y}u_3 \tag{5}$$

$$\ddot{\psi} = \frac{1}{I_z}u_4 \tag{6}$$

where $\zeta = [x, y, z]^T$ represents the position of the quadrotor and $\eta = [\phi, \theta, \psi]^T$ denotes the attitude states that are roll, pitch and yaw angles respectively. m stands for the mass of the helicopter and g is the gravitational acceleration. l stands for the length of the arm, $I = \mathrm{diag}\{I_x, I_y, I_z\}$ denotes the inertia moments of the axes. u_1, u_2, u_3, u_4 are the control inputs being composed of the linear combination of propeller propulsive forces.

A method to change the position of a quadrotor is described as adjusting pitch and roll angles properly. The problem of motion of a quadrotor from one point to another on the 3-dimensional space can be considered independent of the yaw angle. This assumption does not only facilitates the solution but also provides the possibility of generation plain control rules for the problem. Assuming that the yaw angle is zero, the dynamics given by (1)-(3) can be modified as

$$m\ddot{\zeta} = m\begin{bmatrix}\ddot{x}\\ \ddot{y}\\ \ddot{z}\end{bmatrix} = \begin{bmatrix}0\\ 0\\ -mg\end{bmatrix} + v \tag{7}$$

where

$$v = \begin{bmatrix}v_1\\ v_2\\ v_3\end{bmatrix} = \begin{bmatrix}\cos\phi\cos\theta\tan\theta u_1\\ -\cos\phi\tan\phi u_1\\ \cos\phi\cos\theta u_1\end{bmatrix} \tag{8}$$

It should be noted that there is only one control input (u_1) in (7) which is used for the control of the altitude. There is no direct control input for the motion on x and y axes. Thus, ϕ and θ angles are utilized to provide motion on x and y axes. In other words, roll and pitch angles of the quadrotor are adjusted conveniently in order to eliminate x and y axes position errors. Following this idea, the controller design procedure is given in detail in the next section.

3. Controller Design

In this section, a model reference adaptive controller is designed for the position control of the quadrotor helicopter. The purpose of the controller is to ensure that the system states follow the states of the reference model when the physical parameters of the dynamical model are uncertain. Since the motion on x and y axes is not directly controlled, roll and pitch angles are considered as virtual control inputs firstly. Then, the desired motion for these dynamics is obtained and tracking controllers for roll and pitch angles are designed. Position and attitude errors are defined as follows

$$e_\zeta = \zeta - \zeta_m \tag{9}$$

$$e_\eta = \eta - \eta_m \tag{10}$$

where $\zeta_m = [x_m, y_m, z_m]^T$ and $\eta_m = [\phi_m, \theta_m, \psi_m]^T$ are position and attitude vectors of the reference model. The dynamics of reference models are defined as

$$\ddot{\zeta}_m = \ddot{\zeta}_d + K_{\zeta_2}(\dot{\zeta}_d - \dot{\zeta}_m) + K_{\zeta_1}(\zeta_d - \zeta_m) \tag{11}$$

$$\ddot{\eta}_m = \ddot{\eta}_d + K_{\eta_2}(\dot{\eta}_d - \dot{\eta}_m) + K_{\eta_1}(\eta_d - \eta_m) \tag{12}$$

with ζ_d and η_d being desired position and attitude vectors. Here K_{ζ_1}, K_{ζ_2}, K_{η_1} and K_{η_2} are 3×3 real and positive definite gain matrices. Note that, the solutions of both models track the desired trajectories asymptotically.

The dynamics of the position error defined in (9) can be obtained utilizing (7) as follows

$$m\ddot{e}_\zeta = \begin{bmatrix}-m\ddot{x}_m\\ -m\ddot{y}_m\\ -m\ddot{z}_m - mg\end{bmatrix} + v. \tag{13}$$

Note that there are two physical quantities in position error dynamics that are m and g. These constants will be considered uncertain in the design procedure and the vector containing these quantities is defined as follows

$$\gamma_\zeta = \begin{bmatrix}m & mg\end{bmatrix}^T. \tag{14}$$

In addition, since γ is considered uncertain, the adaptation error is given as

$$\tilde{\gamma}_\zeta = \gamma_\zeta - \hat{\gamma}_\zeta \tag{15}$$

where $\hat{\gamma}_\zeta$ is the estimation of γ_ζ.

In order to obtain the stabilizing control signals, consider the following candidate Lyapunov function

$$V_\zeta = \frac{1}{2}e_\zeta^T e_\zeta + \frac{1}{2}m(\dot{e}_\zeta + K_{\zeta_1}e_\zeta)^T(\dot{e}_\zeta + K_{\zeta_1}e_\zeta) + \frac{1}{2}\tilde{\gamma}_\zeta^T\kappa_\zeta^{-1}\tilde{\gamma}_\zeta \tag{16}$$

where $K_{\zeta_1} \in \mathbb{R}^{3\times3}$ and $\kappa_\zeta \in \mathbb{R}^{2\times2}$ are positive definite gain matrices. Taking the time derivative of (16) and adding $\pm e_\zeta^T K_{\zeta_1} e_\zeta$ to the right hand side yield

$$\dot{V}_\zeta = -e_\zeta^T K_{\zeta_1} e_\zeta + (\dot{e}_\zeta + K_{\zeta_1}e_\zeta)^T(e_\zeta + m\ddot{e}_\zeta + mK_{\zeta_1}\dot{e}_\zeta) - \tilde{\gamma}_\zeta^T\kappa_\zeta^{-1}\dot{\hat{\gamma}}_\zeta \tag{17}$$

Utilizing (13), one can obtain the following relation

$$m\ddot{e}_\zeta + mK_{\zeta_1}\dot{e}_\zeta = \left[\begin{array}{c|c} K_{\zeta_1}\dot{e}_\zeta - \ddot{\zeta}_m & -z_b \end{array}\right]\gamma_\zeta + v \tag{18}$$

with $z_b = \begin{bmatrix}0 & 0 & 1\end{bmatrix}^T$. Defining

$$\varphi_\zeta = \left[\begin{array}{c|c} K_{\zeta_1}\dot{e}_\zeta - \ddot{\zeta}_m & -z_b \end{array}\right]^T, \tag{19}$$

(17) turns out to be

$$\dot{V}_\zeta = -e_\zeta^T K_{\zeta_1} e_\zeta + (\dot{e}_\zeta + K_{\zeta_1}e_\zeta)^T(e_\zeta + \varphi_\zeta^T\gamma_\zeta + v) - \tilde{\gamma}_\zeta^T\kappa_\zeta^{-1}\dot{\hat{\gamma}}_\zeta. \tag{20}$$

Assigning the control input for position dynamics as

$$v = -e_\zeta - \varphi_\zeta^T\gamma_\zeta - K_{\zeta_2}(\dot{e}_\zeta + K_{\zeta_1}e_\zeta) \tag{21}$$

where $K_{\zeta_2} \in \mathbb{R}^{3\times3}$, and the adaptation rule for γ_ζ as

$$\dot{\hat{\gamma}}_\zeta = \kappa_\zeta\varphi_\zeta(\dot{e}_\zeta + K_{\zeta_1}e_\zeta), \tag{22}$$

the time derivative of the Lyapunov function defined in (16) can be obtained as

$$\dot{V}_\zeta = -e_\zeta^T K_{\zeta_1} e_\zeta - (\dot{e}_\zeta + K_{\zeta_1}e_\zeta)^T K_{\zeta_2}(\dot{e}_\zeta + K_{\zeta_1}e_\zeta)^T. \tag{23}$$

Considering (16) and (23), the function in (23) is negative semi-definite ensuring that the estimated variables are bounded and $[e_\zeta, \dot{e}_\zeta]^T \to 0$ as $t \to \infty$ according to LaSalle-Yoshizawa theorem [19]. Even though the convergence result is obtained, it is only valid when v can be applied directly. However, this is not the case because of the absence of the direct control input to roll and pitch angle dynamics. In order to control the position of the quadrotor on x and y axes, it is considered that the designed controller provides the desired angles for roll and pitch and u_1 is used to control the position of z axis. Therefore, utilizing (8),

$$u_1 = \frac{v_3}{\cos\phi\cos\theta} \tag{24}$$

$$\phi_d = \tan^{-1}\left(\frac{\cos\theta v_2}{v_3}\right) \tag{25}$$

$$\theta_d = \tan^{-1}\left(\frac{v_1}{v_3}\right) \tag{26}$$

can be obtained. In this work, it is assumed that ϕ and θ angles take values in $\pm\pi/2$, so that the control signal defined in (24) can be applied. This assumption can be considered as a weakness but quadrotor loses the controllability property on the z axis when such $\cos\phi = 0$ or $\cos\theta = 0$ and it is a common assumption in the literature [9, 10, 16]. Once the desired signals for roll and pitch angles are obtained, the attitude controller design procedure can be presented. The attitude dynamics of the quadrotor given in (4)-(7) can be rewritten as

$$\frac{1}{l} I \ddot{\eta} = u \tag{27}$$

where $u = \begin{bmatrix} u_2 & u_3 & u_4 \end{bmatrix}^T$. Consider the similar candidate Lyapunov function to the one given in (16) as

$$V_\eta = \frac{1}{2} e_\eta^T e_\eta + \frac{1}{2}(\dot{e}_\eta + K_{\eta_1} e_\eta)^T I (\dot{e}_\eta + K_{\eta_1} e_\eta) + \frac{1}{2}\tilde{\gamma}_\eta^T \kappa_\eta^{-1} \tilde{\gamma}_\eta \tag{28}$$

with

$$\tilde{\gamma}_\eta = \gamma_\eta - \hat{\gamma}_\eta \tag{29}$$

where $K_{\eta_1} \in \mathbb{R}^{3\times3}$ and $\kappa_\eta \in \mathbb{R}^{3\times3}$ are positive definite gain matrices,

$$\gamma_\eta = \begin{bmatrix} \frac{I_x}{l} & \frac{I_y}{l} & \frac{I_z}{l} \end{bmatrix}^T \tag{30}$$

and $\hat{\gamma}_\eta$ is its estimation. Following the same procedure performed for position dynamics, the time derivative of V_η can be obtained as

$$\dot{V}_\eta = -e_\eta^T K_{\eta_1} e_\eta - (\dot{e}_\eta + K_{\eta_1} e_\eta)^T K_{\eta_2} (\dot{e}_\eta + K_{\eta_1} e_\eta)^T \tag{31}$$

when the control vector is assigned as

$$u = -e_\eta - \varphi_\eta^T \hat{\gamma}_\eta - K_{\eta_2}(\dot{e}_\eta + K_{\eta_1} e_\eta) \tag{32}$$

and the adaptation law is set as

$$\dot{\hat{\gamma}}_\eta = \kappa_\eta \varphi_\eta (\dot{e}_\eta + K_{\eta_1} e_\eta), \tag{33}$$

where $K_{\eta_2} \in \mathbb{R}^{3\times3}$ is a positive definite matrix and

$$\varphi_\eta^T = \mathrm{diag}\{K_{\eta_1}\dot{e}_\eta - \ddot{\eta}_m\}. \tag{34}$$

The boundedness of adapted variables and convergence of the error signals to zero can be concluded with (28) and (31) by means of previous similar analysis.

4. Simulation Results

Various simulation studies have been implemented using Matlab in order to test the performance of the proposed controller. The model parameters of the quadrotor utilized in the simulations are given in Table 1. The step size of the solver is set as 1μs and the controller sampling time is adjusted as 1ms in simulations. Two simulations are run for two different trajectories. The reference signals for x, y, and z are taken as a square wave in the first simulation while the reference signals are sinusoidal for x and y, and a ramp for z in the second simulation. All the states are set to zero initially except the yaw angle which is equal to $\pi/4$ in both simulations, and Table 2 gives the gain values for each simulation.

Table 1. Model parameters

Parameter	Value	Unit
m	3	kg
g	9.81	m/s^2
l	1	m
I_x	10^{-3}	$kg\, m^2$
I_y	10^{-3}	$kg\, m^2$
I_z	10^{-3}	$kg\, m^2$

The responses of the first simulation are given in Figures 1 to 3. In Figure 1 the changes of position and attitude are shown for the first reference signals. The position of the quadrotor converges to the reference position while the attitude values

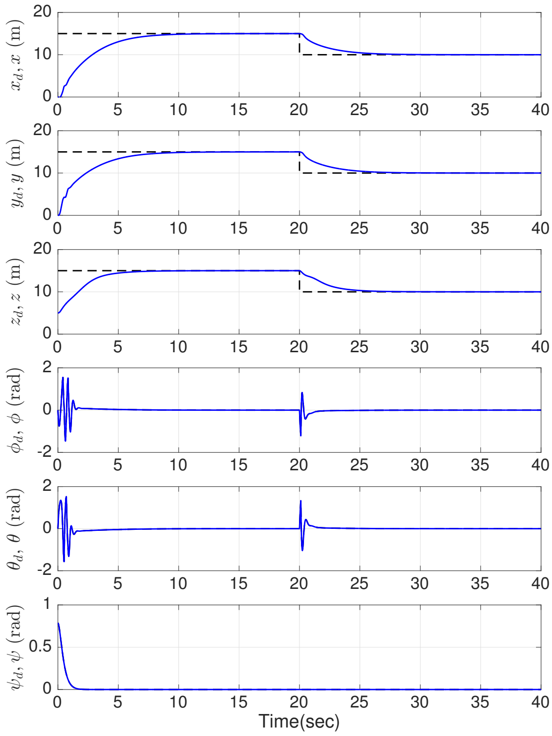

Figure 1. Results for simulation #1: Change of position and attitude signals (Reference signals are given in dashed (black)).

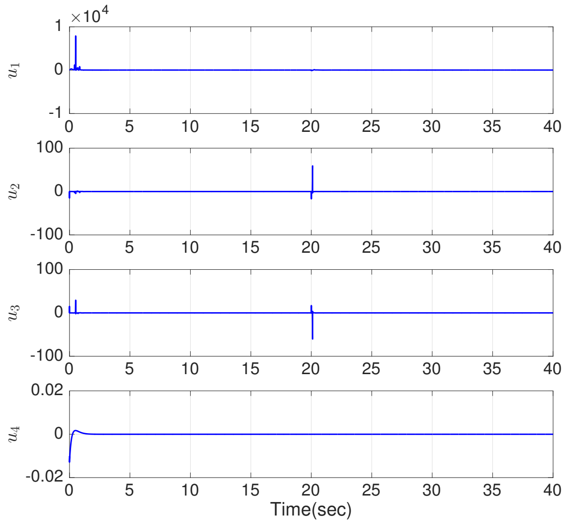

Figure 2. Results for simulation #1: Change of control signals.

Table 2. Gain values used in simulations

Parameter	Simulation #1	Simulation #2
$K_{m_{\zeta_1}}$	2	4
$K_{m_{\zeta_2}}$	4	4
$K_{m_{\eta_1}}$	16	16
$K_{m_{\eta_2}}$	8	8
K_{ζ_1}	1	2
K_{ζ_2}	10	20
K_{η_1}	0.1	0.1
K_{η_2}	1	1

converge to zero as commanded. The change of control inputs for uncertain parameters are demonstrated in In Figure 2. There is no saturation in control signals in the simulations and these signals take relatively large values when a step reference is applied. This issue does not effect the stability of the quadrotor, however the response would be more sluggish when control signals are saturated. In Figure 3, adapted signals for uncertain parameters are depicted. The adapted signals change during the transient of the error signals and they converge to a bounded values. Although the convergence of the adapted signals to their actual values is not guaranteed theoretically, the trajectories of the system track the reference signals successfully. Hence, it can be concluded from the results that the position and attitude values converges to the desired values following the reference models without steady-state error when the proposed controller is implemented.

Figures 4 and 5 present the response for the second simulation with the proposed controller. The trajectory in this simulation corresponds to a circular path in 3D space with the altitude increasing by time. The results for position and attitude values are given in Figure 4 in which the success on trajectory tracking can be observed. Figure 5 depicts the motion of the quadrotor in 3D space. Note that the trajectory is followed with a steady state error caused by the reference model being type-0.

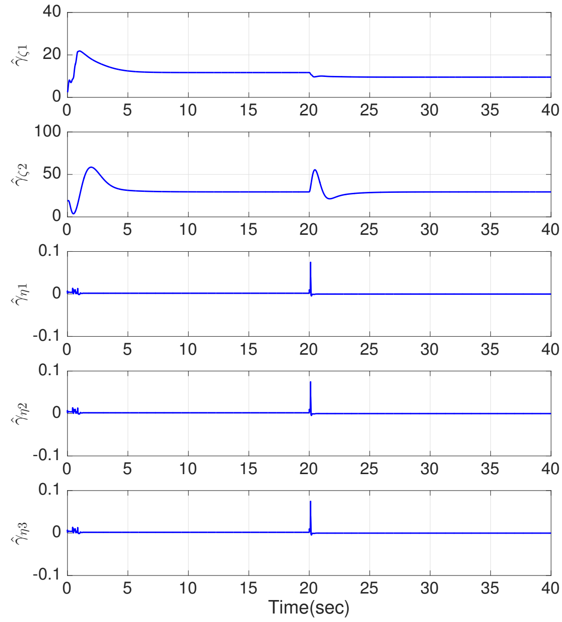

Figure 3. Results for simulation #1: Change of adapted signals.

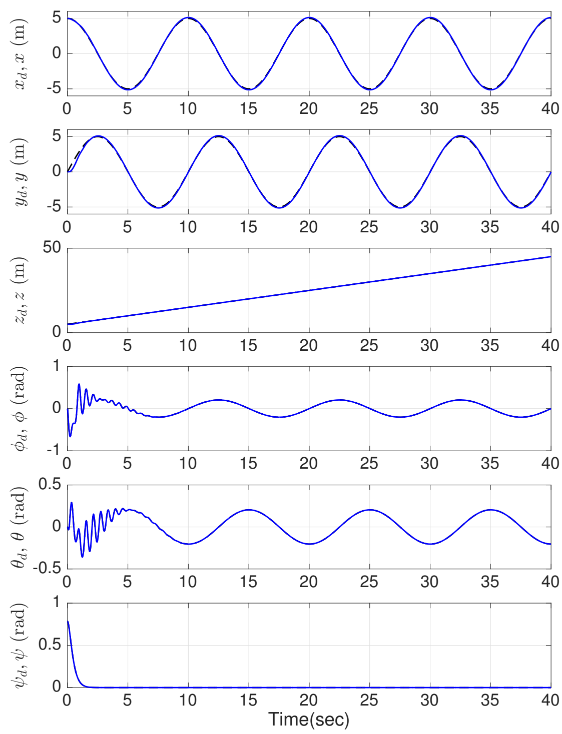

Figure 4. Results for simulation #2: Change of position and attitude signals (Reference signals are given in dashed (black)).

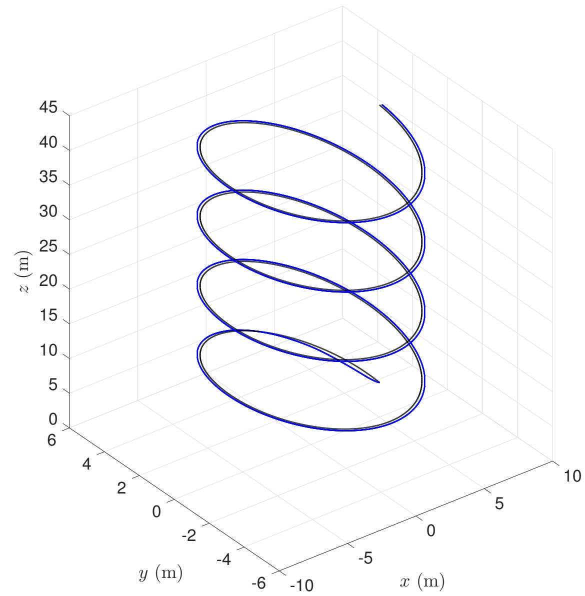

Figure 5. Results for simulation #2: Motion of the quadrotor in 3D space.

5. Conclusion

A Lyapunov based model reference adaptive controller design has been presented for the position control of uncertain dynamical model of the quadrotor. Position and attitude dynamics have been taken into account separately and tracking adaptive controllers have been designed for both dynamics. Lyapunov stability analysis has been performed for the proposed controllers. Simulation studies implemented considering the model uncertainties have shown the successive performance of the proposed adaptive control structure.

6. References

[1] Daewon Lee, H. Jin Kim, Shankar Sastry, ”Feedback Linearization vs. Adaptive Sliding Mode Control for a Quadrotor Helicopter”, *International Journal of Control, Automation, and Systems*, vol. 7, no. 3, pp. 419-428, 2009.

[2] S. Bouabdallah, P. Murrieri, R. Siegwart, ”Design and control of an indoor micro quadrotor”, *IEEE International Conference on Robotics and Automation*, vol. 5, pp. 4393-4398, 2004.

[3] M. A. Ma’sum, M. K. Arrofi, G. Jati, F. Arifin, M. N. Kurniawan, P. Mursanto, and W. Jatmiko, ”Simulation of intelligent unmanned aerial vehicle (uav) for military surveillance”, *International Conference on Advanced Computer Science and Information Systems*, pp. 161-166, 2013.

[4] F. Solc, ”Modelling and control of a quadrocopter”, *Advanced in Military Technology*, vol. 5, pp. 29-38, 2010.

[5] F. Sabatino, ”Quadrotor control: modeling, nonlinear control design, and simulation”, M.S. thesis, *KTH Royal Institute of Technology*, 2015.

[6] T. Ryan and H. J. Kim, ”LMI-based gain synthesis for simple robust quadrotor control”, *IEEE Transactions on Automation Science and Engineering*, vol. 10, no. 4, pp. 1173-1178, 2013.

[7] Zachary T. Dydek, Anuradha M. Annaswamy, Eugene Lavretsky, ”Adaptive Control of Quadrotor UAVs: A Design Trade Study With Flight Evaluations”, *IEEE Transactions on Control Systems Technology*, vol. 21, no. 4, pp. 1400-1406, 2013.

[8] Abhijit Das, Frank Lewis, Kamesh Subbarao, ”Backstepping Approach for Controlling a Quadrotor Using Lagrange Form Dynamics”, *Journal of Intelligent and Robotic Systems*, vol. 56, no. 1-2 pp. 127-151, 2009.

[9] Zhi Li, Xin Ma, Zhigang Xu, Yafang Wang, Yibin Li, ”Chattering Free Sliding Adaptive Attitude Control for Quadrotor”, *IEEE Chinese Guidance, Navigation and Control Conference*, pp. 707-712, 2016.

[10] Gianluca Antonelli, Elisabetta Cataldi, Filippo Arrichiello, Paolo Robuffo Giordano, Stefano Chiaverini, Antonio Franchi, ”Adaptive Trajectory Tracking for Quadrotor MAVs in Presence of Parameter Uncertainties and External Disturbances”, *IEEE Transactions on Control System Technology*, vol. 99, pp. 1-7, 2017.

[11] Pisan Moonumca, Yoshio Yamamoto, Nattawoot Depaiwa, ”Adaptive PID for Controlling a Quadrotor in a Virtual Outdoor Scenario: Simulation Study”, *IEEE International Conference on Mechatronics and Automation*, pp. 1080-1086, 2013.

[12] M. Imran Rashid, Suhail Akhtar, ”Adaptive Control of a Quadrotor with Unknown Model Parameters”, *Mechatronics*, pp. 8-14, 2012.

[13] Matthias Schreier, ”Modeling and Adaptive Control of a Quadrotor”, *IEEE International Conference on Mechatronics and Automation*, pp. 383-390, 2012.

[14] Stanislav I. Tomashevich, Alexander L. Fradkov, Boris Andrievsky, Anderey O. Belyavskyi, Konstantin Amelin, ”Simple adaptive control of quadrotor attitude. Algorithms and experimental results”, *Mediterranean Conference on Control and Automation* , pp. 933-938, 2017.

[15] Mostafa Mohammadi, Alireza Mohammad Shahri, ”Adaptive Nonlinear Stabilization Control for a Quadrotor UAV: Theory, Simulation and Experimentation”, *Journal of Intelligent and Robotic Systems*, vol. 72, no. 1, pp. 105-122, 2013.

[16] Mu Huang, Bin Xian, Chen Diao, Kaiyan Yang, and Yu Feng, ”Adaptive Tracking Control of Underactuated Quadrotor Unmanned Aerial Vehicles via Backstepping”, *American Control Conference*, pp. 2076-2081, 2010.

[17] Ivana Palunko, Rafael Fierro, ”Adaptive Control of a Quadrotor with Dynamic Changes in the Center of Gravity”, *The International Federation of Automatic Control*, vol. 44 no. 1 pp. 2626-2631, 2011.

[18] C. Nicol, C.J.B. Macnab, A. Ramirez-Serrano, ”Robust adaptive control of a quadrotor helicopter”, *Mechatronics*, vol. 21 no. 6 pp. 927-938, 2011.

[19] Miroslav Krstic, Ioannis Kanellakopoulos, and Peter V. Kokotovic. Nonlinear and adaptive control design. Wiley, 1995.