Transmission and Stopband Characteristics of One-Dimensional Photonic Crystals

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Abstract

In this work, an analytical investigation and numerical simulation of one-dimensional photonic crystals (1D-PCs) are studied. In the analysis, two different analytical methods are used to determine bandgaps of periodic structure. Namely, plane wave expansion method (PWE) is used to solve eigenvalue equation and generalized scattering matrix method (GSM) is used to calculate reflection and transmission coefficients of the unit cell. In both techniques Bloch theorem is applied to determine dispersion relation of 1D-PC structure for both transverse electric (TE) and transverse magnetic (TM) modes. Simulations are carried out to demonstrate omnidirectional reflection of 1D-PCs. Results show that characteristics of the transmission and stopband can be modified by changing the physical and electrical parameters of 1D-PC as well as the bandwidth of omnidirectional reflector.

1. Introduction

Photonic crystals have been investigated in large number of application areas such as telecommunications, electronic circuits and medical applications. One-dimensional photonic crystals (1D-PCs) yield number of properties used for optical devices employed as optical waveguide, power dividers or couplers, low loss dielectric reflectors known as Bragg mirrors particularly used for omnidirectional reflection. A certain frequency band in which the photonic crystals exhibit full reflectivity independent from polarization and incidence angle is called the omnidirectional band. This feature makes photonic crystals heavily used in many important applications such as distributed feed-back lasers, dielectric Fabry-Perot filters, adjustable polarizers, narrow-band filters and dielectric reflectors **[1]**.

The propagation characteristics, reflection and transmission properties, forbidden bands are generally obtained by solving the eigenvalue equation for a single unit cell and computed via construction of band diagrams (sometimes called dispersion diagram). Techniques that used to calculate transmission and stop band characteristics of 1D-PCs have been well studied in the literature. Finite-difference time-domain (FDTD), transfer matrix method (TMM) and plane-wave expansion (PWE) are among the most widely used techniques to calculate bandgaps of PCs **[2]**. In this paper we used generalized scattering matrix (GSM) method to find band characteristics of cascaded 1D-PCs. Furthermore solution of eigenvalue equation is given for the comparison which is a conventional method to determine bandgaps of 1D-PCs. Unlike existing methods in the literature, band-edge frequency values can be determined without solving the eigenvalue equation by obtaining analytical functions based on the proposed method in **[3]**. Several numerical examples are given to demonstrate verified results on 1D-PCs.

2. Theory

A periodic 1D-PC along the *z*-direction consisting of alternating multilayers depicted in Figure 1. The structure has N cascaded (isotropic and nonmagnetic) dielectric layers with two different refractive indices of n_1 and n_2. It's assumed that a plane wave is incident to the structure with an angle of incidence. The refractive index profile of considered structure assumed as

$$n(z) = \begin{cases} n_1 & 0 < z < p_1 \\ n_2 & p_1 < z < p \end{cases} \tag{1}$$

where $p=p_1+p_2$ is the period (or lattice constant) of the unit cell. In that case, dielectric constant is described with a function having a periodicity in the direction of propagation as

$$\varepsilon(z) = \varepsilon(z+p) \tag{2}$$

In order to use periodicity, fields can be expressed with periodic functions and such that;

$$\begin{aligned} \vec{E} &= E_K(z)e^{-jKz} \\ \vec{H} &= H_K(z)e^{-jKz} \end{aligned} \tag{3}$$

where periodic field functions are

$$\begin{aligned} E_K(z) &= E_K(z+p) \\ H_K(z) &= H_K(z+p) \end{aligned} \tag{4}$$

Starting with the Maxwell's equations electric field amplitudes in different layers are related by using the boundary conditions.

It is assumed that solutions of wave equations are in the form of plane waves and the term K indicates Bloch wave vector. The wave propagation in periodic media can be described by using the Floquet (or Bloch) theorem. Electric field in each layer can be expressed in terms of the travelling waves propagating in the forward and backward directions. Continuity conditions are satisfied at the interfaces to find the incident and reflected field coefficients a_n, b_n, c_n, d_n as shown in Figure 1.

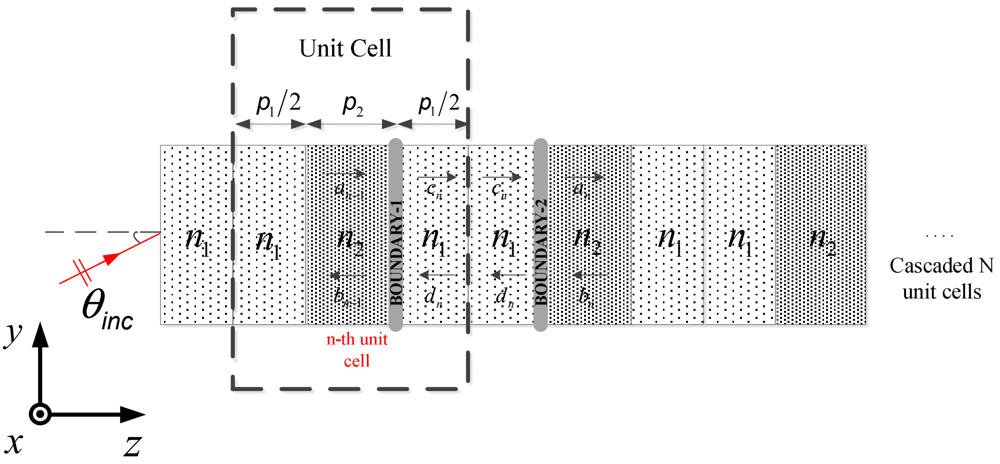

Fig. 1. Geometry of cascaded 1D-PC

The resulted values expressed in unit-cell (UC) translation matrix given by

$$\begin{bmatrix} a_{n-1} \\ b_{n-1} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} a_n \\ b_n \end{bmatrix} \tag{5}$$

which relates the amplitudes of the plane waves in second layer of *n-th* unit cell and second layer of *(n+1)-th* unit cell.

According to Bloch theorem unit-cell translation matrix satisfy the Bloch condition by the relation

$$\begin{bmatrix} a_n \\ b_n \end{bmatrix} = e^{-iKp} \begin{bmatrix} a_{n-1} \\ b_{n-1} \end{bmatrix} \tag{6}$$

After some algebraic steps eigensystem representation is represented in the form of $Ax = \lambda x$. In eigenvalue equation e^{iKp} is the eigenvalue of the matrix system. Solution of the eigenvalue equation provides the information of band structures which is obtained to be

$$\cos(Kp) = \frac{A+D}{2} \tag{7}$$

The term *cos(Kp)* states the band structure of 1D-PC as follows: In the region where $|\cos(Kp)|<1$, *cos(Kp)* is real and gives propagating Bloch waves. In the region where $|\cos(Kp)|>1$, *cos(Kp)* has real and imaginary parts corresponding to evanescent and propagating Bloch waves. These are called Photonic Bandgaps (PBG) of periodic medium. The photonic band edge regions appear at $|\cos(Kp)|=1$ **[4]** .
Therefore eigenvalue equation of the periodic structure for TE and TM cases can be expressed by

$$\cos(Kp) = \cos\left(k_1^z p_1\right)\cos\left(k_2^z p_2\right) - M \sin\left(k_1^z p_1\right)\sin\left(k_2^z p_2\right) \tag{8}$$

$$M = \begin{cases} \dfrac{1}{2}\left(\dfrac{k_2^z}{k_1^z} + \dfrac{k_1^z}{k_2^z}\right) & TE \\ \dfrac{1}{2}\left(\dfrac{n_1^2 k_2^z}{n_2^2 k_1^z} + \dfrac{n_2^2 k_1^z}{n_1^2 k_2^z}\right) & TM \end{cases} \tag{9}$$

$$k_i^z = \sqrt{\left(\frac{\omega n_i}{c}\right)^2 - \left(k^y\right)^2} \quad i = 1,2 \tag{10}$$

Photonic bandgap characteristics of UC are determined by scattering matrix parameters. Assuming the unit cell as a two-port network, scattering parameters are expressed by corresponding normalized wave amplitudes at input/output ports of the UC of PC.

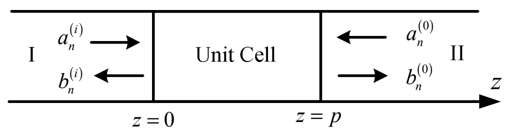

Fig. 2. Unit cell of periodic structure (2-port network)

Scattering parameters of the UC given in Fig.2 are represented by

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \tag{11}$$

After applying the boundary conditions and Floquet periodicity condition eigenvalue equation of the unit cell is obtained which has the passband information of the system.

$$\begin{bmatrix} I & -S_{11} \\ 0 & -S_{21} \end{bmatrix} \begin{bmatrix} b^{(i)} \\ a^{(i)} \end{bmatrix} + \lambda \begin{bmatrix} -S_{12} & 0 \\ -S_{22} & I \end{bmatrix} \begin{bmatrix} b^{(i)} \\ a^{(i)} \end{bmatrix} = 0 \tag{12}$$

An alternative analytical expression for the eigenvalue equation can be derived in terms of the scattering matrix parameters for the symmetric UC as follows **[3]**

$$\cos(Kp) = \frac{1 - S_{11}^2 + S_{21}^2}{2S_{21}} \tag{13}$$

Generalized scattering matrix (GSM) of cascaded networks is well analyzed in the literature. Straightforward cascading of two scattering matrices solves the electromagnetic problem of junction scattering to avoid the often occurring large transmission matrix elements that lead to computational difficulties. After calculating the final scattering parameters of N cascaded unit cells, auxiliary functions X_+ and X_- are used to determine photonic edge frequencies by observing the zero transitions of the imaginary part of $S_{11} \pm S_{21}$ **[3]**.

$$X_{\pm} = 2\,\mathrm{Im}\{S_{11} \pm S_{21}\} \qquad (14)$$

3. Numerical Examples

In this subsection, we consider two different 1D-PCs (in the form of depicted in Fig.1) consist of dielectric layers with refractive indices n_1=4.6 and n_2=1.6 (polystyrene and tellurium) and thicknesses p_1 and p_2 respectively. In the first part bandgap (or stopband) characteristics of cascaded 1D-PC are analyzed to investigate the effect of different filling ratio values for only normal incidence. In the second part, we have kept the dielectric permittivity and thickness of the considered layers fixed and taking into account change of incidence angle which allows us to obtain omnidirectional mirror.

3.1. Bandgap Analysis

Two different values of layer thicknesses in the case of normal incidence are considered to observe transmission spectra. In order to verify numerical results calculated with Matlab compared with those of obtained from commercial tool Lumerical (see Fig.3).

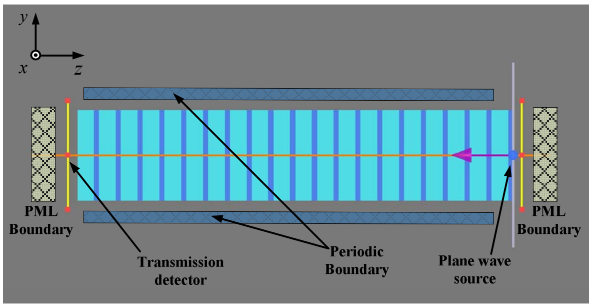

Fig. 3. Schematic diagram modeled in Lumerical for a finite multilayer 1D-PC (20 cascaded slabs)

It may be seen that various bandgaps that the propagation is forbidden plotted in Fig. 4 and Fig. 5. The GSM method is executed with Matlab. Computed bandgap edge frequencies are given in Table 1.

Table 1. Parameters and bandgap spectrum of 1D-PCs

Thicknesses of layers	Polarization	Photonic Bandgap
p_1=48 nm p_2=155 nm	TE/TM 0^0	$217.45 < f(THz) < 421.62$
p_1=65 nm p_2=137 nm	TE/TM 0^0	$197.76 < f(THz) < 377.22$

PBG of the 1D-PC1s is PBG_1 = 204 THz, center frequency is f_{c1}=319.53 THz plotted in Fig. 4 and PBG of the 1D-PC2s is PBG_2=179 THz, center frequency is f_{c2} = 287.49 THz plotted in Fig. 5.

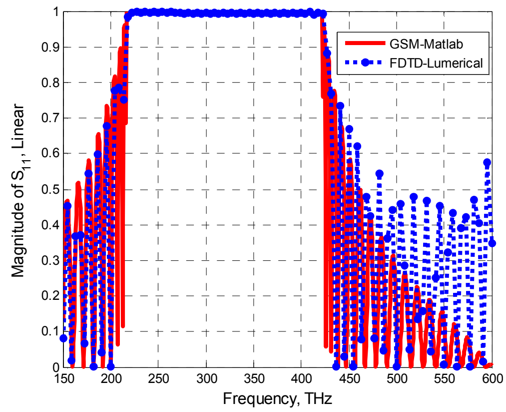

Fig. 4. Reflectivity spectrum of 20 period 1D-PC (p_1=48 nm, p_2=155 nm)

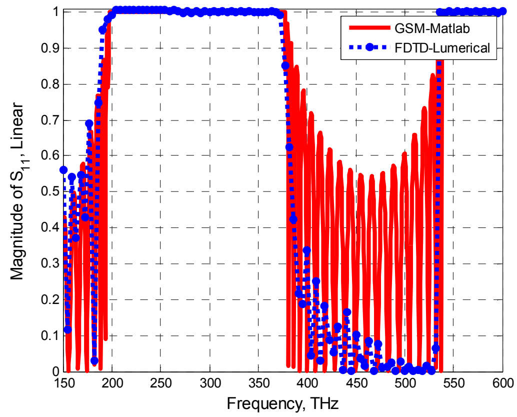

Fig. 5. Reflectivity spectrum of 20 period 1D-PC (p_1=65 nm, p_2=137 nm)

We can observe that PC_1 has a wider PBG than the PC_2 in addition to that center frequency is shifted to a higher frequency region. Here, we have demonstrated that it is possible to modify PBG of 1D-PC by varying the thicknesses of layers (filling ratio). It is found that there is a very good agreement between our results and Lumerical FDTD results.

3.2. Omnidirectional Mirror Example

Cascading of unit cells as periodic or quasi-periodic is used to employ broad omnidirectional bandgap (OBG) characteristics. In this subsection we demonstrate the OBG using 1D-PC of PC1 with given parameters in section 3.1 (n_1=4.6 and n_2=1.6 (polystyrene and tellurium) and thicknesses p_1 and p_2 respectively). In Figures 6-9 dispersion diagrams and variation of X_+ and X_- functions illustrated for TE/TM polarizations and for the incidence angles of θ_{inc}=0^0 and θ_{inc}=90^0 . It's shown that when the incidence angle is increased, gap of TE polarized wave increases; on the other hand gap of TM polarized wave decreases. This means TM polarized wave solutions are more narrow and sufficient to calculate OBG of 1D-PCs.

It can be noted that omnidirectional bandgap of a structure must be satisfied in every value of incidence angle. Therefore, omnidirectional bandgap region is obtained as 272.4THz-421.35THz as seen from figures 6 to 9. It should be noted that there is a limitation to obtain broad omnidirectional bandgap when only one type of PC is used in the design as stressed in **[5]**.

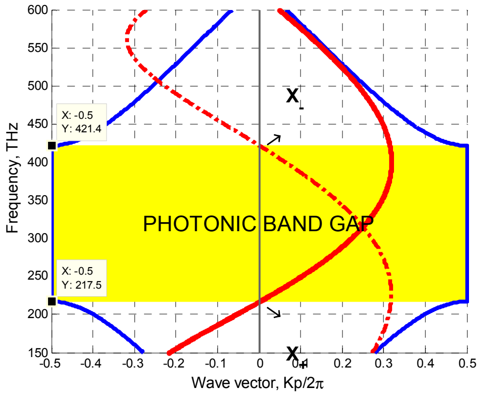

Fig. 6. Dispersion diagram (blue-line) and variation of X_+ and X_- (red-line) of 20 period 1D-PC, TM polarization, $\theta_{inc}=0^0$ (p_1=48 nm, p_2=155 nm)

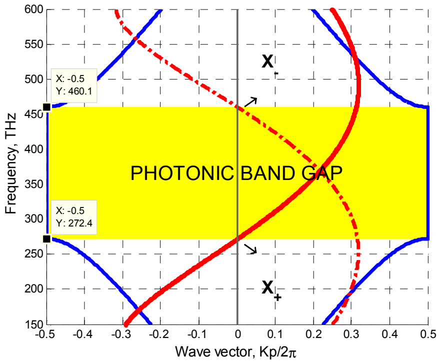

Fig. 7. Dispersion diagram (blue-line) and variation of X_+ and X_- (red-line) of 20 period 1D-PC, TM polarization, $\theta_{inc}=90^0$ (p_1=48 nm, p_2=155 nm)

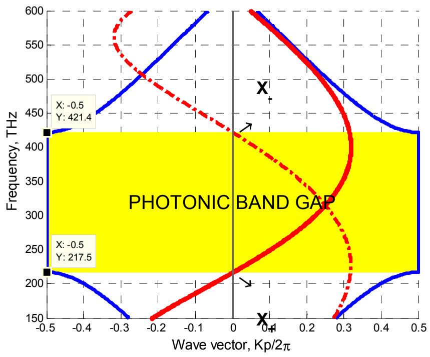

Fig. 8. Dispersion diagram (blue-line) and variation of X_+ and X_- (red-line) of 20 period 1D-PC, TE polarization, $\theta_{inc}=0^0$ (p_1=48 nm, p_2=155 nm)

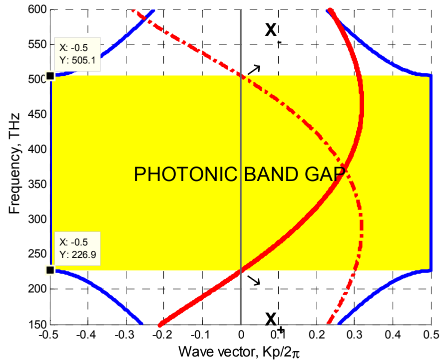

Fig. 9. Dispersion diagram (blue-line) and variation of X_+ and X_- (red-line) of 20 period 1D-PC, TE polarization, $\theta_{inc}=90^0$ (p_1=48 nm, p_2=155 nm)

By considering the off-axis propagation structure does not have a complete bandgap, but it's possible to create a mirror that reflects light from any incidence angle for any polarization for a specified range of frequency. In Figure 10, the omnidirectional mirror of a multilayer structure is illustrated. Refractive indices of structure are 4.6 and 1.6 respectively. In the figure the right side indicates TM (s-polarized) modes and left side indicates TE (p-polarized) modes. Straight red line is called the "light line". Above the light line all propagating and non-propagating wave vectors wave vectors exist. The yellow area of the band structure is the band gap for the light that is allowed to propagate in the air which means that the crystal will reflect all incoming light with frequencies in the band gap without a dependency to angle or polarization of incoming wave. This region indicates omnidirectional bandgap.

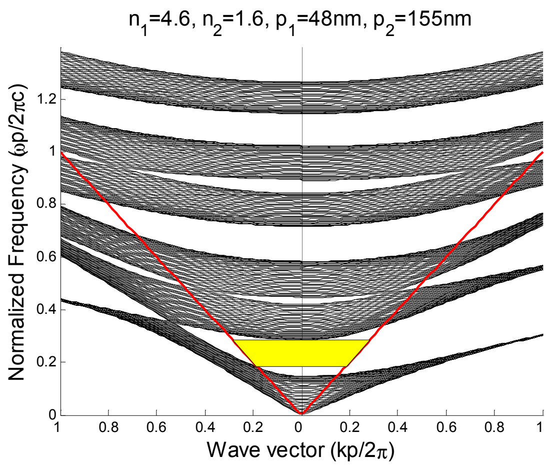

Fig. 10. Dispersion diagram (blue-line) and variation of X_+ and X_- (red-line) of 20 period 1D-PC, TE polarization, $\theta_{inc}=0^0$ (p_1=48 nm, p_2=155 nm)

From the Fig. 10, it is clearly seen that top edge and bottom edge of yellow area are the band edge frequencies of OBG, i.e. 0.285 and 0.184 respectively. These values of band edge frequencies correspond to ~421 THz and ~272 THz which are high edge frequency of TM-wave 0^0 and low edge frequency of TM-wave 90^0 respectively.

4. Conclusions

The research presented in this paper focuses on transmission and stopband characteristics of 1D-PCs in terms of alternating thicknesses of layers and dielectric permittivity. It has been shown that varying the filling ratio allows to design desired structure for a specified center frequency and allows to adjust bandwidth of PBG. The angle of incidence is another factor that PBG sensitivity depends on. With these results omnidirectional dielectric mirrors can be constructed with desired optical properties.

5. References

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