Discrete Time Sliding Mode Control of Magnetic Levitation System with Enhanced Exponential Reaching Law

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Abstract

In this study, the control of magnetic levitation system in discrete time domain is considered. Magnetic ball levitation system model is identified, linearized near the equilibrium point and discretized in convenient sampling period. Discrete time sliding mode controller with enhanced exponential reaching law is designed and compared to traditional discrete time constant proportional rate reaching law for the magnetic ball levitation system which is subject to many control problems since it has unstable structure and it has nonlinear dynamics. The asymptotically stability of the system is analyzed by using Lyapunov stability condition with discrete time approach. In order to evaluate the performance of the considered control technique, simulations are conducted and the results show that discrete time enhanced exponential reaching law provides better performance in terms of both reference tracking and disturbance noise rejection as compared to conventional constant proportional rate reaching law techniques.

1. Introduction

Magnetic levitation systems provide a non-contact mobility, free of friction, heating, noise, and unwanted vibrations from general engineering problems. Hence, these systems are used in contactless high velocity rail systems in Germany and Japan, spacecraft simulators designed for gravity-free environments, biomechanical implant emplacement, and even satellite launchers due to they have specific advantages [1]. However, magnetic ball suspension systems are highly nonlinear and unstable due to their electromechanical structure [2]. The sensitivity requirement which is derived from the system dynamics is a necessity for designing a robust, efficient and practical controller. Many control algorithms have been proposed in the literature for the control of magnetic levitation system. In [3], Prasanta et al. suggested fractional order SMC (sliding mode control) for magnetic ball levitation system. They have compared with traditional SMC on the laboratory experimental setup and shown that fractional order SMC outperform than traditional SMC. An observer-based control mechanism aimed at moving the non-contact train structure based on the magnetic suspension system along with a guide line was proposed in [4] and the response of the system to disturbance effects was examined. In another study [5], a fuzzy sliding mode controller with evolutionary programming based on a magnetic suspension system was designed and compared with other sliding-mode control approaches.

System stability can be ensured when the control methods such as PID or LQR are used. But in such cases the controller's operating performance is limited due to the fact that the controller parameters are fixed. To remedy this matter, variable structure control systems are proposed as a solution for better performance. The sliding-mode control, which is a variable-structure control technique, was first introduced in the soviet union in the late 1950's and the first work was done by Emel'yanov in early 1960's [6]. Sliding mode control is a highly robust control technique that can provide the desired dynamic behavior despite the uncertainties in the system, parameter changes and disturbance effects when appropriate conditions are ensured [7]. Presently, SMC is applied in the area of power system control, satellite attitude control, robotic manipulator control, war plane route control etc. The sliding mode control consists of a set of subsystems supported by appropriate switching continuous control functions. It is assumed that this control technique is subject to discontinuity on a particular surface in the system state space [8]. Theoretically the SMC technique is based on the fact that the error vector of a system is forced into a desired dynamic and kept in this dynamic. Linear and nonlinear systems are drawn on the surface defined in the state space and held on the surface using the infinite switching feedback control. This surface comprises of state variables and is called the sliding surface or sliding manifold. Another advantage of sliding mode control is that it can transform a nth-order control problem into a first-order control problem [9]. The sliding mode is realized in three stages: reaching mode, sliding mode and steady-state mode. Once the system with controller reaches the sliding surface, the system becomes independent of parameter changes and disturbance effects [10]. This property has known in the literature as the condition of uniformity.

In this study, discrete time sliding mode control of the magnetic levitation system is considered. In order to obtain satisfactory performance results, different from each other SMC techniques which are discrete time constant proportional rate reaching law (CPRL), and enhanced exponential reaching law (EERL) are designed. Various simulations are conducted for the performance comparison of these control techniques.

2. Magnetic Levitation System

The structure of a magnetic levitation system is shown in Fig. 1. The purpose of the magnetic suspension system is to keep or change the distance x between the electromagnetic coil and the steel ball with mass m. The distance x mentioned here is positioned relative to the reference value or changes. The ball position is sensed by a sensor and the electromagnetic force is changed by increasing or decreasing the amount of current supplied from the current source as seen in Fig. 1. Thus, the aim is to determine the position of the ball. In this study, a simulation study was carried out based on the magnetic suspension system produced by Googol Company. The physical parameters of the system are given in Table 1 [11].

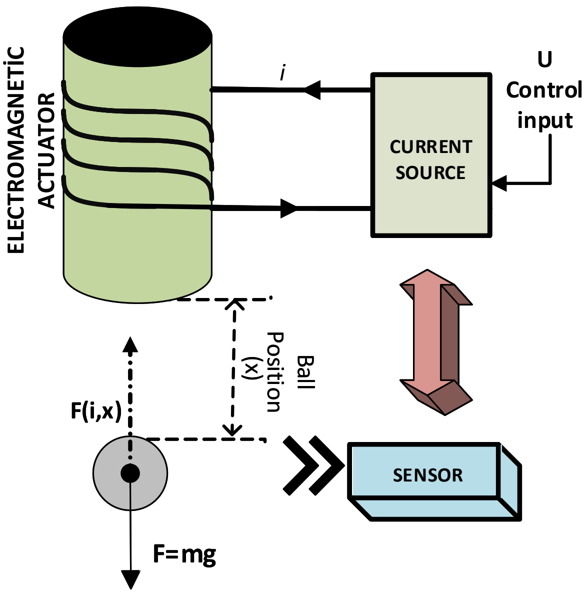

Fig. 1. General scheme of magnetic levitation system

In the most general case, the nonlinear equation relationship between current i, which exist in the electromagnetic coil and ball position x, is as follows.

$$m\ddot{} \quad - \quad k\frac{i^2}{x^2} \tag{1}$$

In this equation, g is the gravitational acceleration and k is the general gain coefficient for the coil parameters.

Since the system model given in (1) is in nonlinear form, it is required to linearize the system in the vicinity of an equilibrium point in order to design an appropriate controller.

Table 1. Realizable parameters of magnetic levitation system

Parameters	*Values*
m- Mass of Ball	22 gr
g- Gravitational Acceleration	9.81 m/s^2
i_0- Equilibrium Point of Coil Current	1.2441 A
x_0- Equilibrium Point of Ball Position	35 mm
k_1- The Gain Coefficient of Coil Current	5.8829 A/V
k_2- The Gain Coefficient of Sensor Current	458.715 V/m

If it is desired to obtain the transfer function by taking the points represented by x_0 and i_0 as equilibrium points, the equation $\ddot{}$ is obtained as follows.

$$\ddot{} \quad - \quad (x,i), f(x,i) = k\frac{i^2}{mx^2} \tag{2}$$

By equating the derivative as $\ddot{}$, we get

$$g = f(x,i) \rightarrow i_0, x_0 \tag{3}$$

where i_0, x_0 values can be obtained. Then *k* is defined as

$$k = \frac{mgx_0^2}{i_0^2} \tag{4}$$

(4) provides the presence of the k coefficient In order to obtain the transfer function of the system, the $f(i,x)$ in the (1) and (2) is implemented by the Tylor series expansion and high order terms are neglected and then Laplace transform is performed. The result is given as

$$\frac{X(s)}{I(s)} = \frac{-1}{(i_0/2g)s^2 - i_0/x_0} \tag{5}$$

In the actual transfer function of the system, the relationship between the controlled voltage value of the input signal u and value of the position sensor output x_v, can be expressed as follow.

$$G(s) = \frac{X_V(s)}{U(s)} = \frac{-(k_1/k_2)}{(i_0/2g)s^2 - i_0/x_0} = \frac{b}{s^2 - a} \tag{6}$$

where $b = \frac{2g}{x_0}$, $a = \frac{-2gk_1}{i_0k_2}$ represent the variables of the system.

As it is seen in (6), the magnetic levitation system is a second order unstable system since it has a real root on the right half s plane. Therefore, it is necessary to combine the closed loop control with the convenient controller to ensure that the steel ball can be suspended at the desired position.

3. Design and Formulation of Discrete Time Sliding Mode Controller

The SMC design consists of two phases. First phase is designing of sliding surface or sliding manifold and the latter is designing of the controller. The sliding surface is designed in the state space by the root locus method according to the desired closed loop system response [12]. The expression of the sliding surface according to the error value and the derivative of the error is given as follows

$$S = \dot{} \tag{7}$$

In the second stage, the control input signal is obtained by equivalent controller, reaching law techniques and Lyapunov stability theory. The Lyapunov sliding condition is forces the system to reach the surface and keep it on the surface. Then the formulation is obtained as

$$\frac{1}{2}\frac{d}{dt}S^2 \leq -\eta \, | \, S \, | \qquad (8)$$

where η is a strictly positive constant coefficient. When the condition of (8) is satisfied it keeps the system trajectories remaining on the sliding surface [12]. Lyapunov based sliding mode reaching condition can be expressed as below

$$S\dot{S} \ldots \qquad (9)$$

Up to this point, continuous expressions about sliding mode control are given. But in today's applications, instead of analog elements, digital signal processors, microcontrollers are widely used due to their flexibility and their ability to perform complex control algorithms. The controllers and systems must be expressed in discrete time form for a suitable sampling period to be able to perform sliding mode control with these equipment's. In this regard, Lyapunov stability condition in (9) should be examined in discrete time domain. Dote and Hoft first investigated discrete-time SMC systems in literature [10] . They have derived discrete-time reaching and stability condition from continuous-time approach in (10). In this study, this formulation has been used while Lyapunov stability condition is analyzed.

$$\left[s(k+1) - s(k)\right]s(k) < 0 \qquad (10)$$

The state variables of the system modeled in section 2 are determined and transformed into the state space matrix form as follows

$$\begin{bmatrix} 0 & 1 \\ b & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ a \end{bmatrix} u(t) \qquad (11)$$

When a discrete-time sliding-mode controller is devised, the system must be transformed into a discrete form as in (12) where we have

$$x(k+1) = Gx(k) + Hu(k) \qquad (12)$$

Then the switching function $S(x)$ is determined as in the sliding surface. That is, $S(x)=0$ and $S(k)$ is obtained at an appropriate sampling period. The last stage is obtaining the control input signal $U(x)$ in discrete-time form as $U(k)$.

The displacement of the system at the reference input signal and its output are represented by r and x1, respectively. The steel ball position error $e(t)$ in the system and the derivative [illegible] of this error are defined as follows

$$z_1 = e(t) = r(t) - x_1$$
$$z_2 = \text{[illegible]} \qquad (13)$$

In this study, EERL, which is the compound of the reaching laws in the different concepts in [13] and [14], was proposed in order to facilitate the intervention of the reaching and sliding phases. The property of the EERL method provides faster reaching speed to the sliding surface than the constant rate reaching law method, which has similar or the same K value. The EERL is newer in the literature than the other reaching laws [12].

The discrete-time model of the magnetic levitation system required for discrete-time controller design is derived from the continuous-time model given in Eq. (6) and (11). In order to discretize the system, the A and B matrices are transformed into G and H matrices using the ZOH (Zero Order Hold) method in T_S=1ms. sampling period. Then, discrete time form of the switching surface is obtained by using continuous time form in the same sampling period for designing the discrete time constant proportional rate reaching law [15].

$$S(k+1) = (1 - KT_s)S(k) - \varepsilon sign(S(k)) \qquad \varepsilon > 0, \quad K > 0 \qquad (14)$$

The following result is obtained when stability condition given in (10) is investigated.

$$\left[s(k+1) - s(k)\right]s(k) \;=\; -KT_sS(k)^2 - \varepsilon\left|S(s)\right| < 0 \qquad (15)$$

So, the system can meet the condition in (10).

The following equation could be written if the existence of a linear sliding surface is considered.

$$S(k) = C^* z(k) \qquad (16)$$

when the state variables reach the sliding surface,

$$S(k+1) = S(k) = 0 \qquad (17)$$

From the Eq. (15)

$$C^* x(k+1) = (1 - KT)S(k) - \varepsilon sign(S(k)) \qquad (18)$$

Using the Constant Proportional-rate reaching law, the discrete-time control input signal is obtained as follows

$$u(k) = \left(C^* H\right)^{-1}\begin{bmatrix} -C^* G.x(k) + (1-KT)S(K)... \\ -\varepsilon.sign(S(k)) \end{bmatrix} \qquad (19)$$

where $C^* = [c \quad 1]$, G=$\begin{bmatrix} 1.003 & 0.001 \\ 0.5606 & 1.0003 \end{bmatrix}$ and

H=$\begin{bmatrix} 0.0006 \\ 1.2298 \end{bmatrix}$, Since the discrete-time controller design is considered with the EERL approach proposed in this study the switching function is discretized by the same sampling period.

Discrete time switching function has the form of

$$S(k+1)=(1-\lambda T)S(k)-\frac{KT}{D(S(k))}|S(\text{k})|^{\gamma}\,sign(S(k)) \tag{20}$$

where

$$D(S(k))=\alpha+(1-\alpha)e^{-\beta_x|S(k)|} \tag{21}$$

For analyzing the Lyapunov condition in (10), the result is obtained as follow

$$-\lambda T_s S(k)^2-\frac{KT}{D(S(k))}|S(\text{k})|^{\gamma}|S(k)|<0 \tag{22}$$

From which we observe that this system can also meet the condition.

From (16), (17) and (20), one can get

$$C^* x(k+1)=(1-\lambda T_s)S(k)-\frac{KT_s}{D(S(k))}|S(\text{k})|^{\gamma}\,sign(S(k)) \tag{23}$$

Finally, using the Enhanced exponential reaching law, the discrete-time control input signal is obtained as follows

$$u(k)=\left(C^*H\right)^{-1}\begin{bmatrix}-C^*G.x(k)+(1-\lambda T)S(k)... \\ -\dfrac{KT}{D(S(k))}|S(k)|^{\gamma}\,sign(S(k))\end{bmatrix} \tag{24}$$

4. The Simulation Study

In this paper, the discrete-time EERL sliding mode control and CPRL sliding mode control have been adapted on the magnetic ball levitation system model. System and two different sliding mode controllers are discretized in same sampling time, then closed loop controller scheme has formed. The model of magnetic ball levitation system with sliding mode controllers is built in MATLAB/SIMULINK environment. The physical parameters of this system are presented in Table 1. Gain coefficients of the controller system in CPRL are chosen as follow, K=5, $\varepsilon=2.25$ and c=15 respectively. For EERL, the coefficients are determined as α=0.1, β=0.9 and γ=0.1.

When the system is simulated, a square wave signal with amplitude of 0.2 unit and a period of 4 sec. is used as a input signal. The simulation results of the system are shown in Fig. 2.

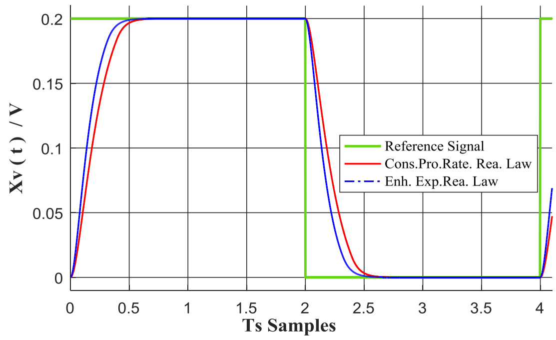

Fig. 2. Position tracking curves

As it is seen from Fig. 2, both controllers have good tracking performance and no overshoot is observed in both reaching laws. But it is obvious that the enhanced exponential reaching law method converges more quickly to the reference input signal. The sampling period depended sliding manifolds of constant-proportional rate reaching law and enhanced exponential reaching law are shown in Fig. 3.

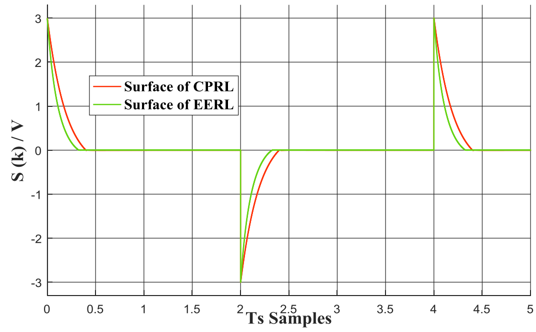

Fig. 3. Sliding surfaces of CPRL& EERL.

A systematic disturbance effect was applied to the system output in order to examine the disturbance noise rejection behavior of the system. As can be seen in Fig. 4 enhanced exponential reaching law method provides better results than constant proportional rate reaching law technique.

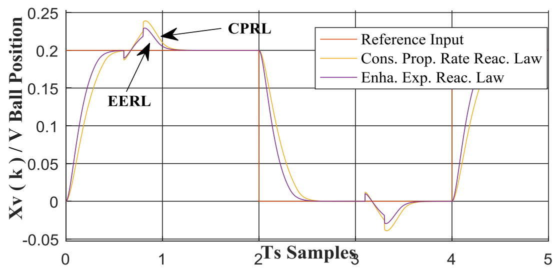

Fig.4. Disturbance noise rejection behavior of controllers.

The control input signals of both controllers in the discrete time form are shown in Fig.5

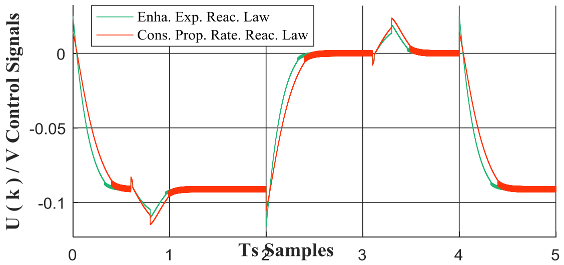

Fig.5. Control input signal of both controllers

5. Conclusions

In this paper, the control of magnetic levitation ball system is considered in discrete time. In order to obtain an effective control performance, enhanced exponential reaching law control technique (EERL) is implemented and its performance is compared with the conventional technique that is constant proportional rate reaching law (CPRL). The results show that EERL shows superior performance in terms of reference point tracking and disturbance noise rejection behavior.

6. References

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