# Some Quadrature Waveform Generators

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## Abstract

**The oscillators with different type of output signal e.g. sinusoidal, triangular, square etc. are often produced as function generators. Moreover quadrature oscillators are used in measurements and communication applications, e.g. mixers and single sideband modulators. In measurement applications this types of oscillators are used in selective voltmeters and vector generators. In this paper some problems with amplitude of generated waveforms of quadrature oscillators are described and also simulation results and techniques of oscillator's construction are presented.**

## 1. Introduction

The electronic oscillators can be divided into two main groups according to the type of signal. Oscillators generating sinusoidal signals are termed "linear" oscillators. All other oscillators are termed relaxation oscillators [1-4]. Linear oscillators (without control) are normally considered as second order systems. The block diagram of ideal conservative system - linear quadrature oscillator with sinusoidal outputs $x_1$, $x_2$ is displayed in Fig.1. Such system can be described in state space by equations

$$\begin{aligned} \dot{x}_1(t) &= \omega x_2(t) \\ x_2(t) &= -\omega x_1(t) \end{aligned} \quad (1)$$

where $\omega$ is angular frequency. The ideal nonlinear quadrature oscillator with nonlinear function *f(.)* is shown in Fig. 2.

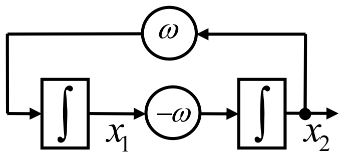


**Fig. 1.** Linear quadrature oscillator with sinusoidal output

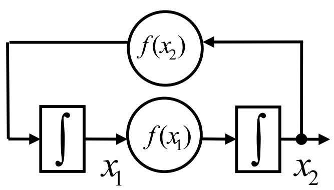


**Fig. 2.** Ideal nonlinear quadrature oscillator

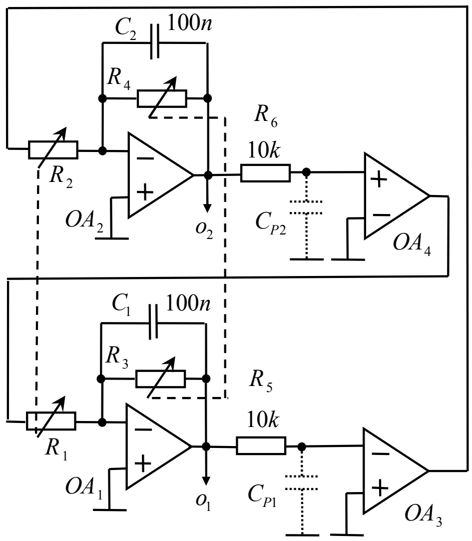


**Fig. 3.** Example of triangular wave quadrature generator with $OA_1$ – $OA_4$ (TL074), parasitic capacitors $C_{P1} = C_{P2} \approx 30$ pF connected to comparators inputs. $R_1$ and $R_2$ are used for frequency control, $R_3$ and $R_4$ for amplitude control

If the linear or nonlinear oscillator is constructed according Fig. 1 or Fig. 2 (with operational amplifiers - *OA*), amplitude of output signal is increasing up to limitation, because of parasitic capacitors. If the roots of closed loop characteristic equation of the oscillator (linear or linearized system) are located in right half of *s*-plane, the oscillations would be increasing up exponentially. They would be limited by some inherent non linearity of the active device, such as the saturation type nonlinear characteristic of the *OA* [5-8]. Such limitation may also distort the output waveforms. Good example is triangular wave quadrature generator shown in Fig. 3, consists of 2 integrators and 2 comparators where nonlinear function is *f(.)=sign(.)*. Parasitic capacitors are $C_{P1}$ and $C_{P2}$ (dashed). The resistors $R_5$, $R_6$ with parasitic capacitors causes instability – amplitude increasing of output signals (system is 4-th order). One possible amplitude control (stabilization) is based on integrators dissipation by means of resistors $R_3$ and $R_4$. In following parts, some methods of amplitude control of oscillations are described.

## 2. Principles of Stability Control and Results

There are several possibilities of the stability (amplitude) control. The 3 possible feedback principles are shown in Fig. 4. Feedback value is $\beta$.

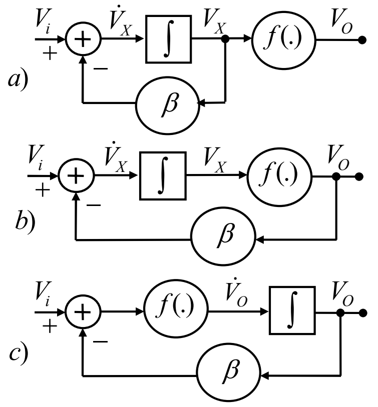

**Fig. 4.** Principles of the feedback amplitude control containing integrator and nonlinear function: a) Feedback around the integrator, b) Feedback from the output of nonlinear function, c) Feedback from the integrator output

The feedback's described in Fig. 4 can be described by equations

$$
\begin{aligned}
a)\quad & \frac{dV_x}{dt} = \left(V_i - \beta V_x\right); \quad V_O = f\left(V_x\right) \\
b)\quad & \frac{dV_x}{dt} = \left(V_i - \beta f\left(V_x\right)\right); \quad V_O = f\left(V_x\right) \\
c)\quad & \frac{dV_O}{dt} = f\left(V_i - \beta V_O\right)
\end{aligned}
\qquad (2)
$$

Amplitude control according (2) is described in next part.

## 3. Simulations and Constructions

The first block diagram of the system (according Fig. 4 a) is displayed in Fig. 5, circuit diagram is shown in Fig. 3.

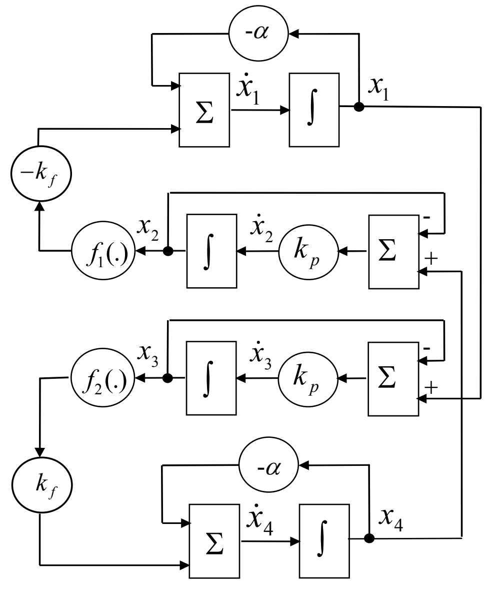

**Fig. 5.** Structure of the system with feedback $\alpha$ around the integrators

In Fig. 5 (and also in Fig. 7 and 9) the $k_f$ is constants for frequency control, $k_P = 1/\tau_P$ where $\tau_P = R \cdot C_P$ (time constants of resistor $R_5$ and $R_6$ and parasitic capacitors $C_P$), $\alpha$ is dissipation parameter. The structure shown in Fig. 5 can be described by state space equations

$$
\begin{aligned}
\dot{x}_1(t) &= -\alpha\, x_1(t) - k_f \cdot f_1\left(x_2(t)\right) \\
\dot{x}_2(t) &= k_P\left(-x_2(t) + x_4(t)\right) \\
\dot{x}_3(t) &= k_P\left(-x_3(t) + x_1(t)\right) \\
\dot{x}_4(t) &= -\alpha\, x_4(t) + k_f \cdot f_2\left(x_3(t)\right)
\end{aligned}
\qquad (3)
$$

Singular point is (0,0,0,0) and linearized equations are

$$
\begin{aligned}
\dot{x}_1(t) &= -\alpha\, x_1(t) - k_f \cdot x_2(t) \\
\dot{x}_2(t) &= k_P\left(-x_2(t) + x_4(t)\right) \\
\dot{x}_3(t) &= k_P\left(-x_3(t) + x_1(t)\right) \\
\dot{x}_4(t) &= -\alpha\, x_4(t) + k_f \cdot x_3(t)
\end{aligned}
\qquad (4)
$$

For the numerical values $k_f = 1$ and $k_P = 100$ linearized state space system with dissipation parameter $\alpha$ is

$$
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix}
=
\begin{bmatrix} -\alpha & -1 & 0 & 0 \\ 0 & -100 & 0 & 100 \\ 100 & 0 & -100 & 0 \\ 0 & 0 & 1 & -\alpha \end{bmatrix}
\cdot
\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}
\qquad (5)
$$

System described by eq. (5) was solved for local stability, see Fig. 6, where $\alpha$ is on X- axis and maximal value of real part of eigenvalues on Y- axis. From this figure can be seen that for $\alpha > 0.01$ the system is stable (all real parts of eigenvalues are negative).

The second structure of the system (according Fig. 4 b) is shown in Fig. 7. System is controlled by changing the value of $c_O$. State space equations are

$$
\begin{aligned}
\dot{x}_1(t) &= -c_O x_3(t) - k_f \cdot f_1\left(x_2(t)\right) \\
\dot{x}_2(t) &= k_P\left(-x_2(t) + x_4(t)\right) \\
\dot{x}_3(t) &= k_P\left(-x_3(t) + x_1(t)\right) \\
\dot{x}_4(t) &= -c_O x_2(t) + k_f \cdot f_2\left(x_3(t)\right)
\end{aligned}
\qquad (6)
$$

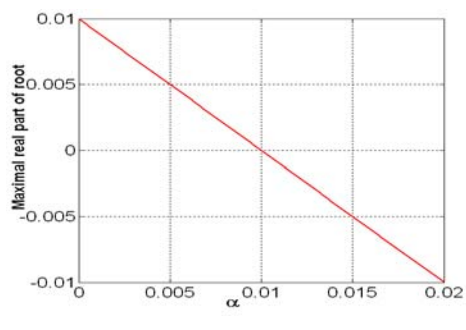


**Fig. 6.** Maximal value of real part of eigenvalues as a function of dissipation parameter $\alpha$

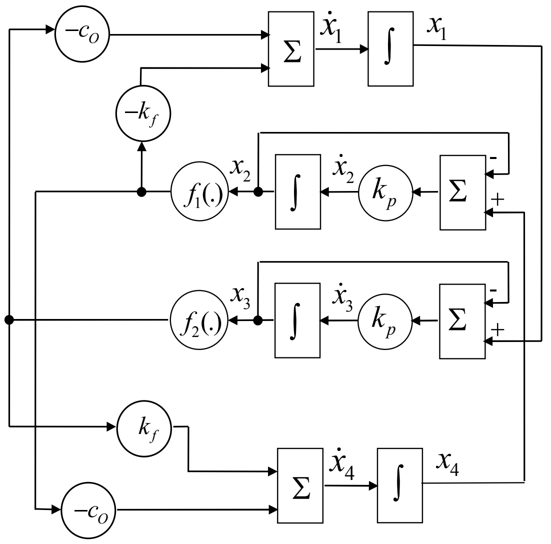

**Fig. 7.** Structure of the system with feedback taken from the output of nonlinear function

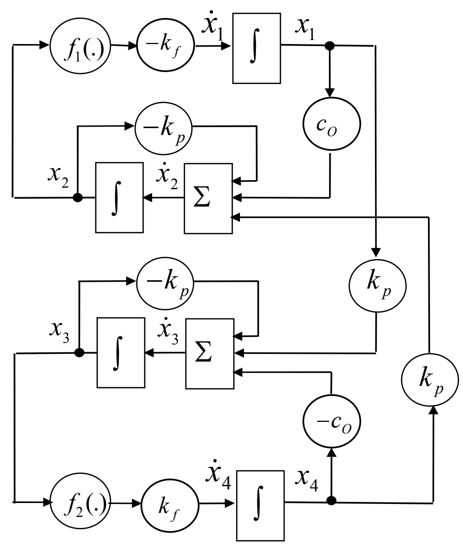

**Fig. 9.** Structure of the system with feedback from the output of integrator

By the same way as in previous system, for numerical values $k_f$ =1 and $k_P$=100 linearized state space system controlled by parameter $c_O$ is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & -1 & -c_o & 0 \\ 0 & -100 & 0 & 100 \\ 100 & 0 & -100 & 0 \\ 0 & -c_o & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \tag{7}$$

The result is exactly the same as graph in Fig. 6. System is stable for $c_O$>0.01. Circuit diagram is presented in Fig. 8.

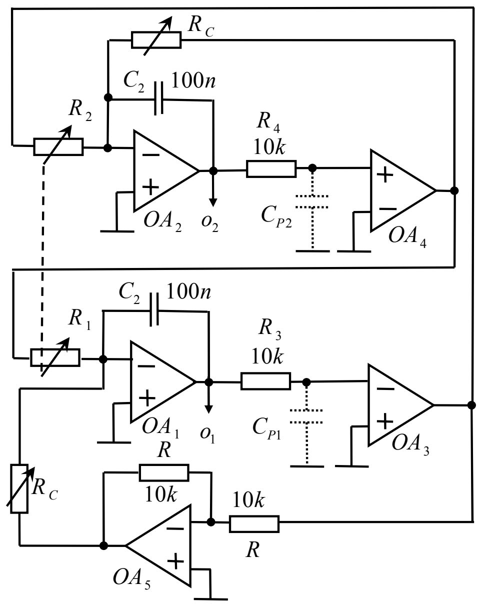

**Fig. 8.** Circuit diagram of the system where feedback is derived from the output of nonlinear function *sign(.)*

The third structure of the system (according Fig. 4 c) is shown in Fig. 9. System is controlled by changing the value of $c_O$. State space equations are

$$\begin{aligned} \dot{x}_1(t) &= -k_f \cdot f_1\left(x_2(t)\right) \\ \dot{x}_2(t) &= k_P\left(-x_2(t) + x_4(t)\right) + c_O x_1(t) \\ \dot{x}_3(t) &= k_P\left(-x_3(t) + x_1(t)\right) - c_O x_4(t) \\ \dot{x}_4(t) &= k_f \cdot f_2\left(x_3(t)\right) \end{aligned} \tag{8}$$

By the same way as in previous examples, the numerical values for $k_f$ =1 and $k_P$=100, linearized state space system controlled by parameter $c_O$ is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ c_o & -100 & 0 & 100 \\ 100 & 0 & -100 & -c_o \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \tag{9}$$

Result is presented in Fig. 10. System is stable for $c_O$>1. Circuit diagram is shown in Fig. 11.

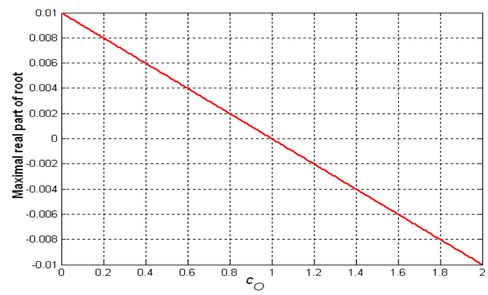


**Fig. 10.** Maximal value of real part of eigenvalues as a function of dissipation parameter $c_O$

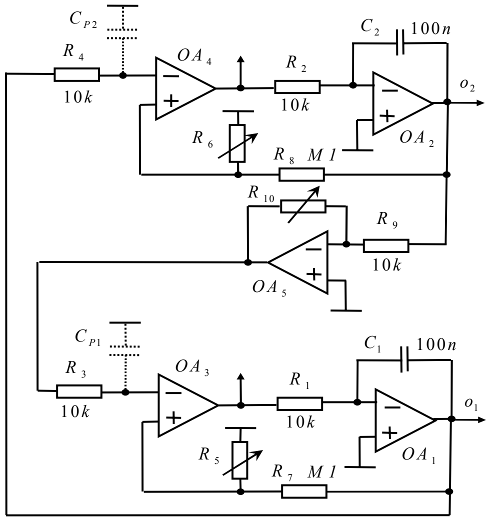


**Fig. 11.** Circuit diagram of the system where feedback is derived from the output of integrator and *f(.)*=sign*(.)*

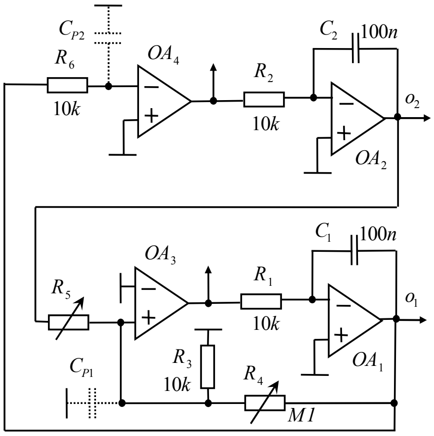


**Fig. 12.** Simplest version of quadrature oscillator derived from Fig. 11 with nonlinear function *f(.)*=sign*(.)*

Circuit diagram from Fig. 11 can be simplified by means of removing $OA_5$ and one resistor divider. The simplest version of triangular wave quadrature oscillator is shown in Fig. 12. Simulation results of this oscillator are displayed in Fig. 13, 14.

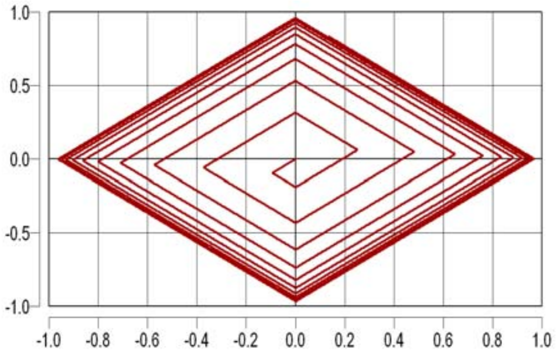


**Fig. 13.** Phase plane trajectory. Simulation of the start-up of the oscillator from Fig. 12

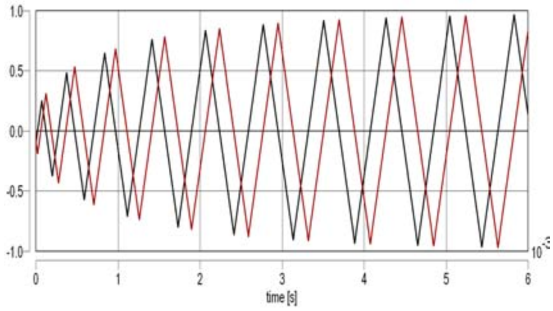


**Fig. 14.** Time evolution. Simulation of the start-up of the oscillator from Fig. 12, *f*=1250 Hz

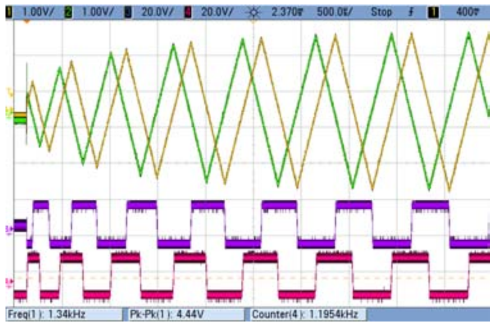


**Fig. 15.** Scope of measured signals, start-up of oscillator according Fig. 12. Signals at the outputs of integrators (top) and outputs of comparators (bottom), *f*=1195 Hz

**Table 1.** Steady state output frequency and peak to peak voltage PP as function of $R_5$ (oscillator according Fig. 12)

| $R_5$ [Ω] | *f* [kHz] | $V_O$ PP [V] |
|---|---|---|
| 2k7 | 0.35 | 15.6 |
| 3k9 | 0.5 | 11.3 |
| 5k6 | 0.65 | 8.8 |
| 8k2 | 0.88 | 6.44 |
| 10k | 1.02 | 5.57 |
| 15k | 1.42 | 3.91 |
| 22k | 1.87 | 3 |
| 33k | 2.38 | 2.35 |

The oscillators from Fig. 3, 8, 11 and 12 was constructed and tested. Measuring results of the simplest oscillator according Fig. 12 are presented in Fig. 15 and Table 1. The frequency is affected by amplitude control which can be compensated (but not presented in this work). Example of amplitude control is illustrated in Fig. 16.

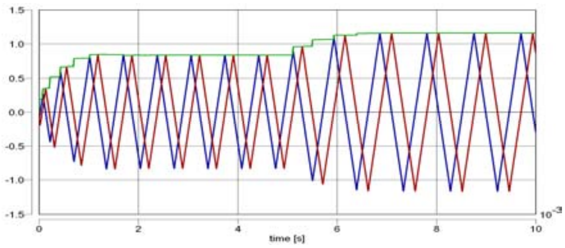


**Fig. 16.** Simulation of amplitude control. For *t*<0.005, prescribed amplitude is 0.8 V, for *t*>=0.005 is 1.2 V

## 4. Another Types of Quadrature Oscillators

In previous simulation and oscillators constructions nonlinear functions were $f_1(.)=f_2(.)=sign(.)$. For different type's shapes of waves the simulation with other linear-nonlinear function was done. List of the functions $f_1(.)=f_2(.)$ is in (10)

$$\begin{aligned} f(x) &= k \cdot x; & f(x) &= x^3 \\ f(x) &= tanh(k \cdot x); & f(x) &= sign(x) \cdot x^2 \\ f(x) &= sin(k \cdot x); & f(x) &= cos(k \cdot x) \end{aligned} \qquad (10)$$

There are possible construct the oscillator also with mixed version of function $f_1(.)\neq f_2(.)$ eg. according eq. (11)

$$f_1(x) = k \cdot sign(x); \qquad f_2(x) = k \cdot x \qquad (11)$$

Simulation of phase portrait of state variables for mixed version functions is shown in Fig. 17, scope of steady state signals is illustrated in Fig. 18. The measured results are with good agreement with theory and simulations.

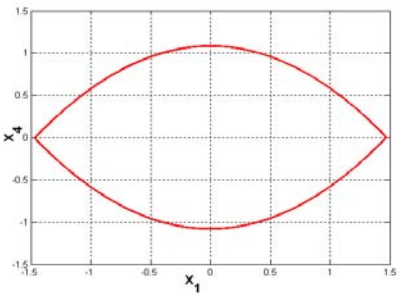


**Fig. 17.** Phase portrait of steady states variables $x_1$ and $x_4$ for mixed version of function $f_1(.)\neq f_2(.)$ according (10). Such system generate triangular and sine wave outputs

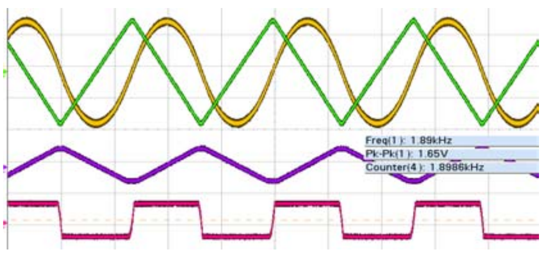


**Fig. 18.** Scope of steady state signals of mixed version system. Function $f_1(.)=k.sign(.)$; $f_2(.)=k$ according eq. (10). Signals at the outputs of integrators (top), outputs of amplifier (middle) and comparator (bottom).

## 5. Conclusions

In this work the nonlinear dynamical system used as quadrature oscillator was described. The unstable behavior of system was stabilized by means several types of feedback. The mathematical description and structure of the system was used as the first, after the results of simulation were shown and finally also construction of generator and measuring confirmed previous results. Generator can be of course constructed from modern building block current and voltage conveyors, e.g. CCII [9, 10], digital potentiometers used for control [11], etc., but important is universal principle of stabilization solution approach, which can be used also for other types of dynamical systems and different shapes of signals.

## 6. Acknowledgment

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