Spectral Analysis of Chirp and Sinusoidal Signals in Complex Domain

Dur-E-Jabeen, Kundan Kumar, A. Haseeb, and M. Faizan

[1]Elcetronics Engineering, Faculty of Engineering Science and Technology, Hamdard University, Karachi, Pakistan
dure.jabeen@hamdard.edu.pk, durejabeen@hotmail.com

Abstract

This paper comprises of the comparative analysis of modulation schemes Amplitude and Frequency. Sinusoids and chirp signals are studied with their spectral energy in the sequency domain using Complex Hadamard transform by varying the size of data vector. A Conjugate Symmetric variant of Complex Hadamard transform has similar main lobes to Discrete Fourier transform in the frequency domain with number of side bands. Whereas, its sequency ordered complex Hadamard transform has some higher energy side lobes at the particular side with the shifted main bands in the spectra. Signal to noise ratio, the peak to average power ratio and energy spectral density are observed at the transmitter end. Two sided chirp has higher peak to average power in the experimental analysis. Distribution of spectral coefficients shows that maximum energy is presented in the main lobes in both sequency and frequency domains. Whereas the total energy spectral density of the various data sets is similar.

1. Introduction

Signal processing is the widespread domain of analog and digital signals. It plays very important role in signal processing [1], wireless communication [2], image processing [3], multimedia applications [4] etc. Therefore, signals are represented with their mathematical formation, statistical models and with their computational parameters. For particular application signals are synthesized and analyzed according to a specific domain.

Chirp signals are linear and exponential (angle modulated) signals and classified as up and down chirp signals. The chirp signal provides an ideal rectangular spectrum; it has better small spectral power density among the existing transmission signals. In frequency domain side lobes are translated at positive and negative frequencies. Energy and power spectral density can be analyzed and calculated easily using transform domain. Study of spectra in sequency domain using Complex Hadamard transform (CHT) is focused in this paper. This transform provides complex domain spectra similar to the DFT [5]. In [6] claimed that CHT works similar to the DFT in transform domain on the basis that sequency of CHT works analogous to the frequency of DFT. Therefore, this paper provides a detailed study of the spectra of sinusoidal and chirp signals using basic modulation schemes. This paper constructed as below. Section-II provides the detail of chirp signal classification and modulation scheme. Section – III comprises on the simulation results. Section-IV depends on conclusion.

2. Chirp Modulation Schemes

It is classified into two categories linear and nonlinear, commonly are used in sonar, radar [7], spread spectrum in digital communications [8, 9] and optical transmission [10, 11] applications. Chirp signal can be interchanged by sweep signals with increasing and decreasing frequency with the specified time. Chirp rate is the instantaneous frequency which directly change with the rate of phase angle, $F_i(t)=\frac{1}{2\pi}\left(\frac{d\theta(t)}{dt}\right)$. For the signal $x(t)=A\cos\left(\theta(t)\right)$, Where, $x(t)$ is the oscillatory signal with magnitude A and phase angle is $\theta(t)$. Whereas, chirp rate depends on the instantaneous phase angle. Mainly chirp signals are categorized as below,

2.1. Linear Chirp in Sequency Domain

In linear chirp signals, their instantaneous frequency $F_i(t)$ is directly related to time such as;

$$F_i(t)=F_o+at \tag{1}$$

where F_o initial frequency at time zero and a is the chirp rate / rate of frequency , $a=\frac{F_1-F_o}{T_1}$.

Final frequency of chirp rate is F_1, therefore instantaneous phase of the oscillatory signal is,

$$\theta^{'}(t)=2\pi F_i(t) \tag{2}$$

where, $\theta(t)=\phi(t)+2\pi\int_0^t F_i(t)dt$, Therefore, frequency modulation signal becomes;

$$\begin{aligned} x(t)&=A\cos\left(\phi(t)+2\pi\int_0^T F_i(t)dt\right)\\ x(t)&=A\cos\left(\phi(t)+2\pi\int_0^T\left(F_o+at\right)dt\right)\\ x(t)&=A\cos\left(2\pi F_oT+\left(\frac{a}{2}T^2\right)+\phi(t)\right)\end{aligned} \tag{3}$$

where, $\phi(t)$ is the starting phase at time zero. Instantaneous frequency in eq. (1) that provides additional harmonics such as frequency modulation. This describes the Bessel functions. Its time-frequency relation can be viewed on spectrogram.

2.2. Nonlinear Chirp in Sequency Domain

Non-linear can be studied as exponential chirp signals having conatants T_1 and T_2 with time interval set T_2-T_1 , their frequency rate is $F(T_2)/F(T_1)$. Therefore, the instantaneous frequency of exponential chirp become, $F_i(t) = F_o a^T$, where, a is the rate of chirp with exponential increases frequency, as presented in eq. (3).

3. Simulation Results

The basic parameters are measured such as signal to noise ratio (SNR), the peak to average power ratio (PAPR) and energy spectral density (ESD). These factors are calculated at sampling frequency 8 k Hz. Whereas the data size varies from 256 to 1024 samples. Spectral analysis is observed in complex domain using discrete Fourier transform and variants of complex Hadamard transform. Sinusoidal and chirp signals are modulated using AM and FM modulations. Data samples of an input signal varies according to the transformation matrix. Whereas for sinusoidal signal, frequency is used $f_m = 400$ Hz, the carrier frequency is $f_c = 2K$ Hz, sampling frequency is $F_s = 8K$ Hz and the frequency deviation for FM is $f_\Delta = 57$ used. For the linear chirp inital frequency $f_0 = 0$ Hz and end frequency is $f_1 = 16K$ Hz. Here as, inital frequency $f_0 = 100$ Hz is supposed for two sided chirp signals. Measuring parameters are calculated as such;

- **Energy Spectral Density (ESD):** Energy distribution of frequency using DFT and sequency using CHT contents examined, that shows how transformed coefficients are spread all over the spectrum.

$$ESD = \sum_{k=-\infty}^{\infty} |X(k)|^2$$

where, k is the transform domain integer and $X(k)$ is a transform domain spectrum. Fig-1 shows the AM and FM modulated spectra of DFT and CHT, DFT represents that a sinusoidal signal produces two impulses those are shifted at the carrier frequency. Where CHT spectra represents that it has two impulses with some side bands having smaller magnitudes, but the main bands are similar to the DFT. Fig-2 shows the AM and FM spectra of CHT and WHT with data set 256. CHT generates two main lobes with many smaller magnitude side bands, but WHT did not provide similar spectra, as complex domain (DFT, SCHT and CS-SCHT).
Fig-4 shows the DFT and CSSCHT has the similar positions of the main frequency bands at the same time FM spectrum has similar energy distribution. In Fig-5 SCHT has comparable FM spectra with DFT, but AM has many sideband coefficients. Whereas, WHT represents its real basis functions as its energy spread in transform domain.
Fig. 6-7 shows that the higher data set spectra for linear chirp. Since the analogous spectra using complex orthogonal transforms, the two-sided chirp also analyzed that is presented in fig. 8.

- **Peak to Average Power Ratio (PAPR):** it is mathematically presented as below,

$$PAPR = \frac{max(o/p)^2}{\left(\sqrt{(1/N)\sum_{n=0}^{N-1}(o/p)^2}\right)^2}$$

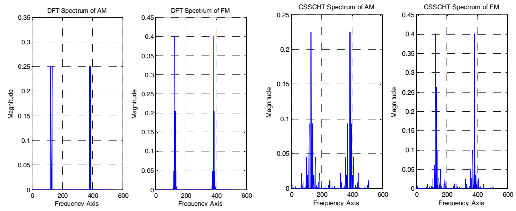

Fig.1. Spectra of Am and FM for DFT and CSSCHT with 256 data

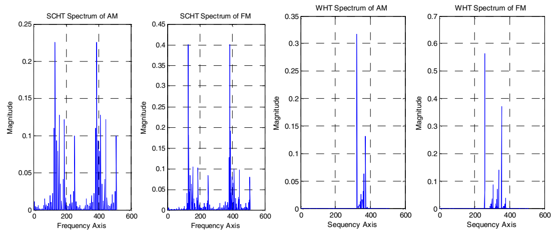

Fig.2. Spectra of AM and FM for SCHT and WHT with 256 data set

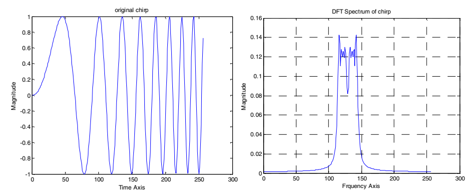

Fig. 3. Linear chirp signal and DFT spectra.

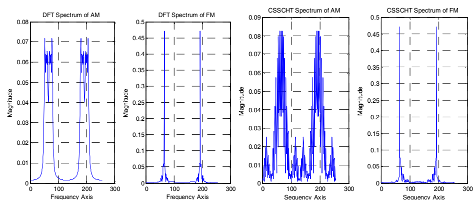

Fig.4. Linear chirp Spectra of AM and FM for DFT and CSSCHT, data set 1x256.

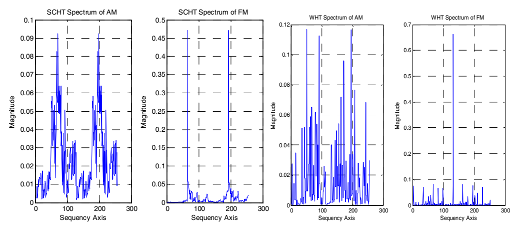

Fig.5. Spectra of AM and FM for SCHT and WHT with 256 data set

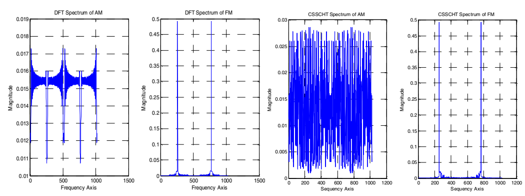

Fig. 6. Linear chirp spectra of AM and FM for DFT and CSSCHT

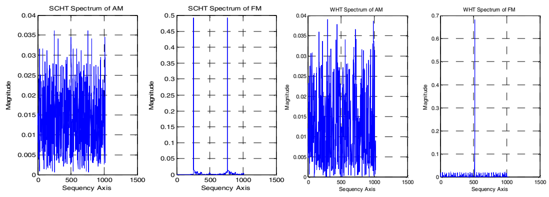

Fig.7. Linear chirp spectra of AM and FM for SCHT and WH.

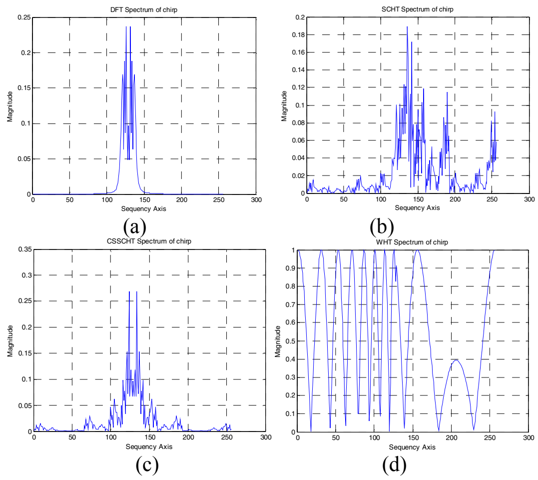

Fig. 8. (a-d)Spectra of two sided chirp in DFT, CSSCHT, SCHT and WHT

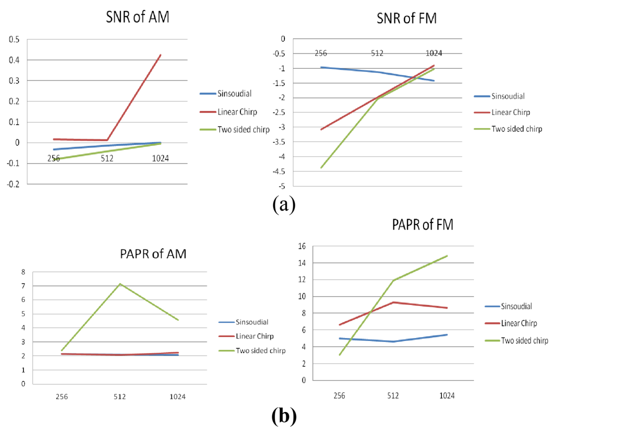

Fig. 9. (a) SNR and (b) PAPR for AM & FM at various data vectors

where, N is data size. PAPR is calculated at the demodulation side that how much average power used for transmission. In this paper PAPR measured for AM and FM for altered vectors, those containing the similar values at the receiver side.

Table 1 shows the PAPR for the modulated signals without noise and with noise. Similarly, table 2 and 3 characterize the PAPR for chirp signals.

Table 1. Modulation of sinusoidal signals SNR, PAPR and ESD with noise $F_1 = 4k$ and $F_c = 2k$

Name of Modulation	Transform	Property	Data Size		
			1x256	1x512	1x1024
AM	FWHT	SNR	-0.033	-0.014	0.0012
		PAPR	2.166	2.105	2.082
		ESD	0.252	0.521	0.250
	DFT	SNR	-0.033	-0.014	0.0012
		PAPR	2.166	2.105	2.082
		ESD	0.25	0.251	0.2501
	SCHT	SNR	-0.033	-0.014	0.0012
		PAPR	2.166	2.105	2.082
		ESD	0.252	0.521	0.2501
	CSSCHT	SNR	-0.033	-0.014	0.0012
		PAPR	2.166	2.105	2.082
		ESD	0.252	0.521	0.250
FM	FWHT	SNR	-0.965	-1.124	-1.417
		PAPR	5.036	4.617	5.4601
		ESD	0.498	0.499	0.5012
	DFT	SNR	-0.965	-1.124	-1.414
		PAPR	5.036	4.615	5.463
		ESD	0.498	0.499	0.5012
	SCHT	SNR	-0.965	-1.124	-1.4172
		PAPR	5.036	4.617	5.4601
		ESD	0.498	0.499	0.5012
	CSSCHT	SNR	-0.965	-1.124	-1.417
		PAPR	5.036	4.617	5.4601
		ESD	0.498	0.499	0.5012

Table 2. Linear Chirp signal spectra SNR, PAPR and ESD, $F_2 = 16k$ and $F_c = 2k$

Transform	Property		Data size		
			256	512	1024
FWHT	SNR	AM	0.0148	0.0113	0.424
		FM	-3.065	-1.975	-0.9051
	PAPR	AM	2.1603	2.073	2.239
		FM	6.6597	9.299	8.659
	ESD	AM	0.232	0.241	0.236
		FM	0.501	0.500	0.500
DFT	SNR	AM	0.0148	0.0113	0.4236
		FM	-3.0651	-1.975	-0.9051
	PAPR	AM	2.1603	2.073	2.239
		FM	6.6597	9.299	8.659
	ESD	AM	0.232	0.241	0.237
		FM	0.5013	0.500	0.500
SCHT	SNR	AM	0.0148	0.0113	0.4236
		FM	-3.0651	-1.975	-0.9051
	PAPR	AM	2.1603	2.073	2.239
		FM	6.6597	9.299	8.659
	ESD	AM	0.232	0.241	0.237
		FM	0.5013	0.500	0.500
CSSCHT	SNR	AM	0.0148	0.0113	0.4236
		FM	-3.0651	-1.975	-0.9051
	PAPR	AM	2.1603	2.073	2.239
		FM	6.6597	9.299	8.659
	ESD	AM	0.232	0.241	0.237
		FM	0.5013	0.500	0.500

Table 3. Linear (exponential) Chirp signal spectra SNR, PAPR and ESD, $F_1 = 100k$, $F_2 = 16k$ and $F_c = 2k$ in Hz

Transform	Property		Data size		
			256	512	1024
FWHT	SNR	AM	-0.081	-0.0416	-0.0053
		FM	-4.363	-2.036	-1.0244
	PAPR	AM	2.391	7.160	4.578
		FM	3.066	11.90	14.833
	ESD	AM	0.2055	0.2277	0.238
		FM	0.5015	0.5008	0.500
DFT	SNR	AM	-0.081	-0.0416	-0.0053
		FM	-4.363	-2.036	-1.0244
	PAPR	AM	2.391	7.160	4.578
		FM	3.066	11.90	14.833
	ESD	AM	0.2055	0.2277	0.238
		FM	0.5015	0.5008	0.500
SCHT	SNR	AM	-0.081	-0.0416	-0.0053
		FM	-4.363	-2.036	-1.0244
	PAPR	AM	2.391	7.160	4.578
		FM	3.066	11.90	14.833
	ESD	AM	0.2055	0.2277	0.238
		FM	0.5015	0.5008	0.500
CSSCHT	SNR	AM	-0.0819	-0.0416	-0.0053
		FM	-4.363	-2.036	-1.0244
	PAPR	AM	2.3915	7.160	4.578
		FM	3.066	11.90	14.833
	ESD	AM	0.205	0.2277	0.238
		FM	0.501	0.5008	0.500

- **Signal to Noise Ratio (SNR):** SNR is the signal power (SP) per noise power (NP) such as below,

$$SNR = 10 log_{10}\left(\frac{SP}{NP}\right)$$

SNR is measured for sinusoidal signals by varying data sets from 256, 512 and 1024. White Gaussian noise is added with modulated signals with 30 SNR. Table 1 shows the SNR at different data vectors for oscillator signal. It shows that similar SNR at different data sets. Where as table 2 for linear chirp and table 3 for two sided chirp has the similar SNR at different data vectors. Fig-8a show that AM is less immune to noise than FM for chirp signals, 9b represents the FM has higher peak average power than AM for two sided chirp signals.

4. Conclusions

This paper presents the spectral analysis of modulated signals in frequency and sequency domain. DFT and variants of complex Hadamard transforms are used. It is defined that variants of CHT have some similar characteristics to DFT. Therefore, this paper is focused more towards sequency domain spectra. DFT provides impulses of sinusoidal signals. So AM and narrow band FM modulation is used to show that the sequency spectral impulses are also shifted at the carrier frequency in transform domain using CHT. By varying the input signal for different data sets SNR, PAPR and ESD are measured. Noise less effected AM than FM because the FM has nonlinear characteristics. Chirps signals provide an impulse based spectra, their magnitude and power spectra are similar but phase spectra are not. PAPR is used in OFMD techniques, it is observed that two sided chirp has higher PAPR then linear chirp and sinusoidal. Chirp signals are used in spread spectrum, radar and sonar. Naturally chirp signals have lower PAPR then as impulse signal.

7. References

[1] G. M. a. F. A. D. Jabeen, "Sequency Domain Signal Processing Using Complex Hadamard Transform," *Circuits, Systems and Signal Processing,* 2015/08/04 2015.

[2] S. X. a. S. Rahardja, "Performance evaluation for quaternary DS-SSMA communications with complex signature sequences over Rayleigh-fading channels," *IEEE Transactions on Wireless Communications,* vol. 4, pp. 266–277, 2005.

[3] J. K. I. J.Cox, F. T. Leighton, and T. Shamoon, "Secure spread spectrum watermarking for multimedia," *IEEE Transactions on Image Processing,* vol. 6, pp. 1673–1687, 1997.

[4] N. Banerjee, A. Acharya, and S. K. Das, "Seamless SIP-based mobility for multimedia applications," *IEEE Network,* vol. 20, pp. 6-13, 2006.

[5] A. Aung, B. P. Ng, and S. Rahardja, "Sequency-Ordered Complex Hadamard Transform: Properties, Computational Complexity and Applications," *IEEE Transactions on Signal Processing,* vol. 56, pp. 3562-3571, 2008.

[6] A. Aung, B. P. Ng, and S. Rahardja, "Conjugate Symmetric Sequency-Ordered Complex Hadamard Transform," *IEEE Transactions on Signal Processing,* vol. 57, pp. 2582-2593, 2009.

[7] M. Kronauge and H. Rohling, "New chirp sequence radar waveform," *IEEE Transactions on Aerospace and Electronic Systems,* vol. 50, pp. 2870-2877, 2014.

[8] B. Sklar, *Digital Communications: Fundamentals and Applications (2nd Edition)* January 21, 2000.

[9] L. M. Smith, "Wideband Bessel function chirp signals and their application to the test and evaluation of audio systems," *41st South eastern Symposium system theory,* pp. 355 – 359, 2009.

[10] S. Betti, E. Bravi, and M. Giaconi, "Effects of intermodulation distortions due to the joint action of dynamic chirping and "dispersive" transmission of SCM optical signals," *IEEE Photonics Technology Letters,* vol. 11, pp. 680-682, 1999.

[11] M. Tanaka and M. Shigematsu, "Chirp compensation using four-wave mixing and its application to 10-Gb/s directly modulated signal transmission over SMF," *IEEE Photonics Technology Letters,* vol. 16, pp. 1957-1959, 2004.