Hardware Verification: Determining the Parameters of the Modified Izhikevich Neuron Model with Genetic Algorithm

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Abstract

The nonlinear function in Izhikevich neuron model (IzNM) makes difficult the digital hardware realizations of the model, so this parabolic function has been transformed to piecewise linear (PWL) functions in the literature. Some coefficients have been identified in the PWL functions by utilizing the classical step size method, but the values of these coefficients depend on the sensitivity of the step size considerably. In this study, the coefficients of the PWL functions in the modified IzNM are determined by using Genetic Algorithm (GA). After the parameter determination, the modified IzNM is simulated with the parameters, which are determined by both classical step size and GA. Also, the original and modified IzNMs exhibiting "tonic spiking" and "tonic bursting" behaviors are realized with digital programmable device, namely FPGA. Thus, it is tested the utility of the intelligent search algorithms in the neuronal structures and verified the adaptability of their results to the hardware implementations.

1. Introduction

To describe the dynamic behavior of an individual neuron or interaction between neurons, several biological neuron models have been reported in the literature [1, 2]. Some important aspects about a real neuron are represented by variable parameters in the biological neuron models. Setting the appropriate values of the parameters is an important research topic in neuronal modeling studies. These values can be determined by either the real biological signals recorded from the living organism or the simulation results of the biological neuron models [3]. However, the studies on parameter determination are restricted because of the increasing complexity of systems due to number of parameters and necessity of long simulation times. To overcome these challenges, optimization methods offer alternative solution apparatus and some application examples are reported to literature [4-7].

Izhikevich neuron model (IzNM) is a very suitable model for observing different neural dynamics according to the parameter adjustments [2]. In this model, described by Eq.1, (v) is the membrane potential, (u) is the recovery parameter and (I) is the membrane input current. (a) describes the time scale recovery variable and (b) is the sensitivity of (u). (c) and (d) are the after-spike reset parameters. [2].

$$
\begin{aligned}
&\dot{v} = 0.04v^2 + 5v + 140 - u + I \\
&\dot{u} = a(bv - u) \qquad\qquad (1) \\
&v \geq 30mV \quad \Longrightarrow \quad \begin{array}{l} v \leftarrow c \\ u \leftarrow u + d \end{array}
\end{aligned}
$$

By adjusting these parameters to suitable values in the model equations, the different dynamics of the neurons can be observed via numerical or experimental setups. The calculated values of the parameters in Eq.1 for twenty different neuron dynamics are listed in Table 1.

Table 1. The parameter values of the IzNM for observing the different neuronal dynamics [2].

NEURON DYNAMICS	a	b	c	d	I
(1) Tonic Spiking	0.02	0.2	-65	6	14
(2) Phasic Spiking	0.02	0.25	-65	6	0.5
(3) Tonic Bursting	0.02	0.2	-50	2	15
(4) Phasic Bursting	0.02	0.25	-55	0.05	0.6
(5) Mixed Mode	0.02	0.2	-55	4	10
(6) Spike Freq. Adapt.	0.01	0.2	-65	8	30
(7) Class1	0.02	-0.1	-55	6	0
(8) Class 2	0.2	0.26	-65	0	0
(9) Spike Latency	0.02	0.2	-65	6	7
(10) Subthreshold Oscill.	0.05	0.26	-60	0	0
(11) Resonator	0.1	0.26	-60	-1	0
(12) Integrator	0.02	-0.1	-55	6	0
(13) Rebound Spike	0.03	0.25	-60	4	0
(14) Rebound Burst	0.03	0.25	-52	0	0
(15) Threshold Variability	0.03	0.25	-60	4	0
(16) Bistability	1	1.5	-60	0	-65
(17) DAP	1	0.2	-60	-21	0
(18) Accommodation	0.02	1	-55	4	0
(19) Inhibition-Induced Spiking	-0.02	-1	-60	8	80
(20) Inhibition-Induced Bursting	-0.026	-1	-45	0	80

IzNM is the one of the most studied biological neuron models in the literature in order to observe the fire patterns of the neurons by simulating with the numerical tools [2, 8] and to use the biological neuron models for some applications that required real time signals by emulating with the hardware realizations [9-15]. The parabolic nonlinear function in the IzNM makes difficult the digital hardware realizations of the model especially for spiking neural network applications. For the implementation easiness, this parabolic function has been transformed to the second, third and fourth orders piecewise linear (PWL) functions in the literature [15] and some coefficients have been identified by utilizing the classical step size method. However, the values of the nonlinear function coefficients depend on the sensitivity of the step size considerably in this method. The stochastic optimization methods can also be used for determining the nonlinear function coefficients of modified IzNM by yielding closer behaviors of the original IzNM. In this study, the coefficients of the second, third and fourth order PWL functions in the modified IzNM are determined by using Genetic Algorithm for twenty different neuronal dynamical behaviors which can be observed with the original IzNM. After the values of the determined coefficients and the total errors are presented for classical step size and GA methods, the neuronal dynamics of the modified IzNM simulated with the new PWL function parameters are also given together for the comparisons. Additionally, the original and modified IzNMs exhibiting “tonic spiking” and “tonic bursting” behaviors are realized with digital programmable device, namely FPGA, by using new coefficients obtained from classical step size and GA methods. Thus, it is tested the utility of the intelligent search algorithms in the neuronal structures and verified the adaptability of their results to the hardware implementations.

In this context the linearization process in modified IzNM is examined in section 2. The results for the GA based parameter estimation of the second, third and fourth order PWL functions are presented in section 3. The numerical simulation and implementation results of original and modified IzNMs using the classical step size and GA based coefficients are given in section 4. The results are discussed in the last section.

2. The Modified Izhikevich Neuron Model

Since neural models, which include nonlinear expressions, are hard to implement with digital devices, the nonlinear expressions in these systems are converted to piecewise linear (PWL) functions. As an example, three different piecewise linear approximations are suggested for improving computational efficiency as in Fig.1.

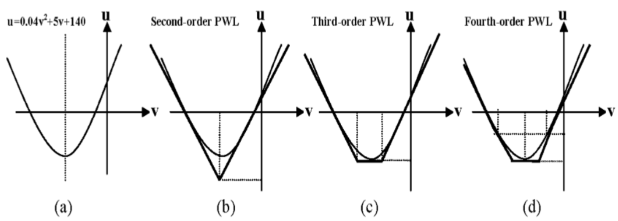


Fig. 1. (a) Original parabolic function and (b), (c) and (d), three different piecewise linear approximations of parabolic function in IzNM [15].

Second-order PWL function: The parabolic function in Eq.1 is reduced into two linear parts and it is formulated as in Eq.2. In this modified model, the results similar to that in the original Izhikevich neuron model are obtained via this approximation using two variable parameters (k_1 and k_2).

Third-order PWL function: The parabolic function is separated into three linear parts and this characteristic is defined in Eq.3. This approximation has three variable parameters (k_1, k_2 and k_3).

Fourth-order PWL function: The mathematical description for the fourth-order piecewise linear function is given in Eq.4. This approximation has also three variable parameters (k_1, k_2 and k_3).

$$\dot{v} = k_1|v + 62.5| - k_2 - u + I \quad (2)$$

$$\dot{v} = k_1(|v + 62.5 + k_2| + |v + 62.5 - k_2|) - k_3k_2k_1 - u + I \quad (3)$$

$$\dot{v} = k_2(|v+62.5+k_3| + |v+62.5-k_3|) - k_1|v+62.5| - 4k_2k_3 - u + I \quad (4)$$

$$\dot{u} = a(bv - u)$$

$$v \geq 30mV \quad \Longrightarrow \quad \begin{array}{l} v \leftarrow c \\ u \leftarrow u + d \end{array}$$

3. The Determination of the (k) Coefficients in PWL Function Based Izhikevich Neuron Models

(k) coefficients in the PWL functions of the modified IzNM must be determined so as to get the closest behaviors to the original IzNM. Thus, an error minimization algorithm in Eq.5 has been used to obtain the optimum (k) coefficients with classical step size method and the total error between the original model and the modified model is calculated by using this cost function in [15].

$$CF = \frac{1}{N}\sum_{i=1}^{N} \frac{(v_{orijinal}(i) - v_{PWL}(i))^2}{v^2_{orijinal}(i)} \quad (5)$$

where (N) represents the total number of samples recorded from membrane potential (v). (k) coefficients are determined in [15] by screening the range of two values with specific step sizes, for example, while (k_1) parameter in the second-order PWL model has been searched in the 0.1–8 range with a step size of 0.01, the range of (k_2) parameter is 10–25 with a step size of 1. This screening process is time consuming operation and its sensitivity is very low. The optimum result in the range of two values depends entirely on the sensitivity of the step size.

Here, the errors between original and modified models have been recalculated for twenty different dynamical behaviors of the neuron by using the (k) coefficients, which are identified via the classical step size method in [15]. The results of the total error and the values of (k) coefficients, which are calculated by using classical step size method, are listed in Table 2a, b and c. In these tables, K1, K2 and K3 demonstrate the (k_1), (k_2) and (k_3) coefficients in Eq.2, 3, 4, respectively. The searching ranges for (k) parameters are chosen as follows: for second-order PWL $0.1 \leq K1 \leq 8$, $15 \leq K2 \leq 25$, for third-order PWL $0.1 \leq K1 \leq 2$, $1 \leq K2 \leq 10$, $1 \leq K3 \leq 15$ and lastly for fourth–order PWL $0.1 \leq K1 \leq 1$, $0.1 \leq K2 \leq 2$, $1 \leq K3 \leq 15$.

Genetic Algorithm (GA) is a heuristic method based on natural selection and natural genetic rules. Genetic algorithm is not complicated and it is a frequently used optimization technique. The “Continuous Genetic Algorithm” method is utilized in this study [16]. In all applications of the second, third and fourth-order PWL functions, population size is selected 80, natural selection parameter is chosen 0.5, mutation rate is adjusted to 0.5 and maximum iteration value is limited to 30. This algorithm is run thirty times for each of twenty behaviors of a neuron. The minimum errors, the mean errors and the standard deviation results of these multiple running are listed in the Table 2a, b and c.

Table 2. The minimum errors of classical step size and GA and the mean error and the standard deviations results of GA for the a) second-order, b) third-order, c) fourth-order PWL function based modified IzNMs.

NEURON DYNAMICS	SECOND-ORDER PWL RESULTS							
	MINIMUM ERROR		MEAN ERROR	STANDARD DEVIATION	CLASS. STEP SIZE		GENETIC ALGORITHM	
	CLASS. STEP SIZE	GENETIC ALGORITHM			K1	K2	K1GA	K2GA
1	0,606255	0,015177	0,153493	0,115865	0.75	20	1,502017	17,0799
2	0,005980	0,001174	0,001079	0,000374	0.5	18	1,3409	18,172469
3	2,441346	2,161087	2,253511	0,043695	0.625	20	0,766598	22,117849
4	0,011044	0,001189	0,001083	0,000377	0.5	20	1,333684	17,620264
5	1,296370	0,630827	0,631619	0,213080	0.5	18	1,062211	16,556166
6	0,782609	0,248393	0,359274	0,136082	0.375	18	0,439003	15,670866
7	0,481390	0,000706	0,000961	0,000333	0.375	18	1,447675	24,957325
8	0,003935	0,000245	0,000220	0,000077	0.625	18	2,537539	16,546262
9	1,225281	0,176095	0,259673	0,078838	0.625	18	2,15815	16,355418
10	0,002609	0,000681	0,000617	0,000216	0.875	18	1,328652	16,711857
11	0,002590	0,000417	0,000375	0,000132	0.875	18	2,300197	16,726957
12	0,006840	0,000697	0,000970	0,000338	0.875	18	1,448089	24,991552
13	0,002080	0,000688	0,000627	0,000219	0.875	18	1,367674	19,47318
14	0,009914	0,000666	0,000606	0,000211	0.375	18	1,366991	19,454125
15	0,010572	0,000689	0,000697	0,000009	0.375	18	1,369713	19,47909
16	33,572347	2,697413	11,238162	5,351065	2	22	1,887221	24,466831
17	0,003298	0,000293	0,000266	0,000093	0.625	18	1,513677	24,998376
18	1218,688631	21,227813	81,848267	51,983749	0.625	18	0,678149	16,091984
19	0,184478	0,014809	0,016916	0,005604	0.625	18	1,510666	19,051889
20	7026,349614	0,524270	1,249766	1,456515	0.625	18	1,656807	20,7228

(a)

NEURON DYNAMICS	THIRD-ORDER PWL RESULTS									
	MINIMUM ERROR		MEAN ERROR	STANDARD DEVIATION	CLASS. STEP SIZE			GENETIC ALGORITHM		
	CLASS. STEP SIZE	GENETIC ALGORITHM			K1	K2	K3	K1GA	K2GA	K3GA
1	0,535426	0,025010	0,198152	0,100141	0.625	5,8	6,4	0,638734	2,388345	12,602596
2	0,001980	0,000692	0,000910	0,000149	0.625	5,8	6,4	0,752855	5,956922	5,642808
3	2,815762	1,005573	1,220962	0,074936	0.625	5,8	6,4	0,723495	8,404468	4,524986
4	0,007682	0,000632	0,000811	0,000131	0,5	7	6,5	0,789779	5,192309	5,979160
5	0,802490	0,148857	0,257607	0,033538	0,5	7	6,5	1,769850	8,011323	1,099061
6	0,638911	0,042585	0,267850	0,162220	0,5	7	6,5	1,769755	5,998277	3,527963
7	0,008604	0,000067	0,000075	0,000006	0,5	7	6,5	1,725960	9,901799	4,516681
8	0,002919	0,000081	0,000150	0,000036	0,5	7	6,5	1,372375	5,716821	4,070843
9	1,634890	0,149904	0,188798	0,045911	0,5	7	6,5	0,544802	9,631955	5,036701
10	0,003202	0,000276	0,000375	0,000071	0,5	7	6,5	1,310639	6,638004	3,867157
11	0,003058	0,000134	0,000215	0,000049	0,5	7	6,5	1,340558	7,158508	3,695079
12	0,008604	0,000068	0,000078	0,000006	0,5	7	6,5	1,761989	9,654933	4,638365
13	0,002807	0,000452	0,000563	0,000075	0,5	7	6,5	0,716540	5,387002	6,178109
14	0,003011	0,000438	0,000516	0,000077	0,5	7	6,5	1,162380	6,369355	4,184676
15	0,002807	0,000467	0,000585	0,000094	0,5	7	6,5	0,761407	5,063705	6,185978
16	36,621608	3,945762	4,521987	0,166177	1,25	12	3	0,690068	5,091898	5,757308
17	0,002321	0,000012	0,000043	0,000022	0,5	7	6,5	1,431785	6,879198	3,609803
18	1556,806146	43,393221	69,408774	15,334013	0,5	7	6,5	0,188749	8,854111	8,900077
19	2,113775	0,015048	0,033069	0,017465	0,5	7	6,5	0,780814	2,996242	8,692797
20	351,570790	0,073095	0,419004	0,474384	0,5	7	6,5	1,440480	5,631057	7,222465

(b)

NEURON DYNAMICS	FOURTH-ORDER PWL RESULTS									
	MINIMUM ERROR		MEAN ERROR	STANDARD DEVIATION	CLASS. STEP SIZE			GENETIC ALGORITHM		
	CLASS. STEP SIZE	GENETIC ALGORITHM			K1	K2	K3	K1GA	K2GA	K3GA
1	0,720661	0,003315	0,074279	0,101141	0,375	0,75	11	0,494766	0,745709	11,263335
2	0,000233	0,000231	0,000244	0,000017	0,375	0,75	11	0,334941	0,764337	10,761411
3	3,134090	3,382634	3,523856	0,052589	0,375	0,75	11	0,206153	0,705690	14,861282
4	0,000243	0,000217	0,000230	0,000023	0,375	0,75	11	0,290132	0,846335	9,648156
5	1,174979	0,072030	0,415641	0,164276	0,375	0,75	11	0,211509	0,615164	7,869807
6	0,878395	0,006736	0,156806	0,133672	0,375	0,75	11	0,554991	0,815752	10,432550
7	0,001238	0,000050	0,000051	0,000001	0,375	0,75	11	0,837928	1,356390	14,682758
8	0,000203	0,000060	0,000079	0,000016	0,375	0,75	11	0,297647	1,282029	6,367713
9	0,668595	0,074222	0,181934	0,037039	0,375	0,75	11	0,901725	0,555279	14,731146
10	0,000228	0,000154	0,000166	0,000018	0,375	0,75	11	0,261176	1,006265	8,086007
11	0,000212	0,000093	0,000110	0,000018	0,375	0,75	11	0,344221	1,077887	7,542965
12	0,001238	0,000049	0,000052	0,000001	0,375	0,75	11	0,895997	1,334911	14,988213
13	0,000257	0,000177	0,000183	0,000010	0,375	0,75	11	0,461635	0,781303	10,912178
14	0,000248	0,000156	0,000158	0,000002	0,375	0,75	11	0,457748	0,838668	10,157537
15	0,000257	0,000177	0,000183	0,000008	0,375	0,75	11	0,460675	0,780238	10,928544
16	36,520340	0,292237	7,041474	2,787030	0,375	0,75	11	0,573539	0,432787	14,024722
17	0,000223	0,000012	0,000016	0,000004	0,375	0,75	11	0,524375	1,200292	7,473017
18	2026,417595	44,163314	82,065404	26,896005	0,375	0,75	11	0,804305	0,258246	4,143061
19	0,157636	0,010183	0,016330	0,006759	0,375	0,75	11	0,175049	0,960562	9,032416
20	2,357791	0,073790	0,289192	0,327149	0,375	0,75	11	0,555458	1,171006	12,609934

(c)

Also, the (k) coefficients providing the lowest error value within these thirty runs are reported to the Table 2a, b and c for second-, third-, and fourth-order PWL functions, respectively. In these tables, K_{1GA}, K_{2GA} and K_{3GA} represent the (k_1), (k_2) and (k_3) coefficients in Eq.2, 3, 4, respectively for genetic algorithm based searching. In the GA method, (k) coefficients have been searched in the same ranges with the classical step size method for a smooth comparison. In addition to these selections, Eq.5 is also used as a cost function to determine the minimum error in all estimation methods.

The lowest error values for both classical step size and GA are reported in the first parts of the tables. The mean errors and standard deviation results of thirty runs for GA are given in the tables to make a proper performance comparison. These results are not available for classical step size method in the tables, because multiple running processes aren't applied to this method. In the last part of the tables, the values of the "k" coefficients determined by the two searching methods are presented separately. Taking into consideration all results mentioned above, stochastic method exhibits more successful performance then the classical step size method in the parameter determination process for the modified IzNM.

4. The Hardware Verification of the Modified Izhikevich Neuron model

To overcome digital implementation difficulty, parabolic expression in the IzNM is converted to three approximate PWL nonlinear functions and the coefficients in these modified models are estimated separately by using classical step size method and GA as mentioned in previous section. After the parameter estimation process, two characteristic neuronal behaviors (tonic spiking and tonic bursting) of a neuron, which is modeled by original and modified IzNMs, are realized with FPGA (field programmable gate array) device in this section. FPGA is preferred for its rapid prototyping and reconfigurability features. The programmable digital device-based implementations of second-, third- and fourth- order PWL function based modified models are made by utilizing the new coefficients and the modified IzNMs are implemented with FPGA device. Thus, it is tested the utility of the intelligent search algorithms in the neuronal structures and verified the adaptability of their results to the hardware implementations.

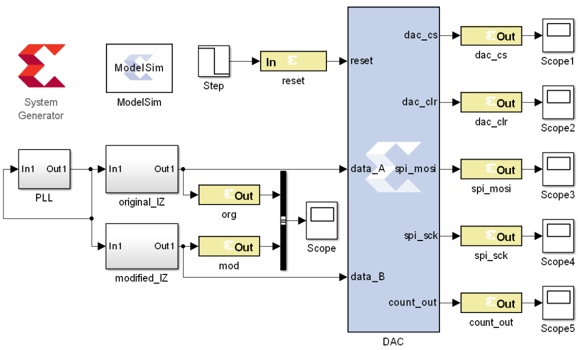
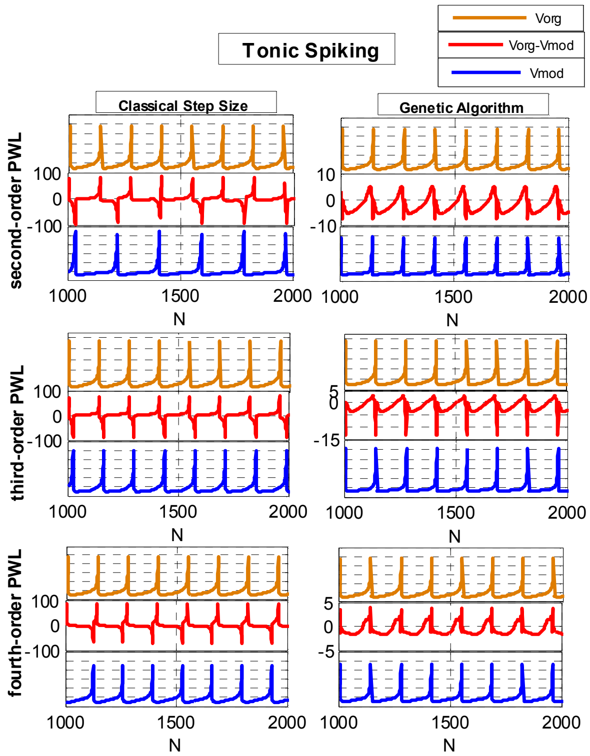
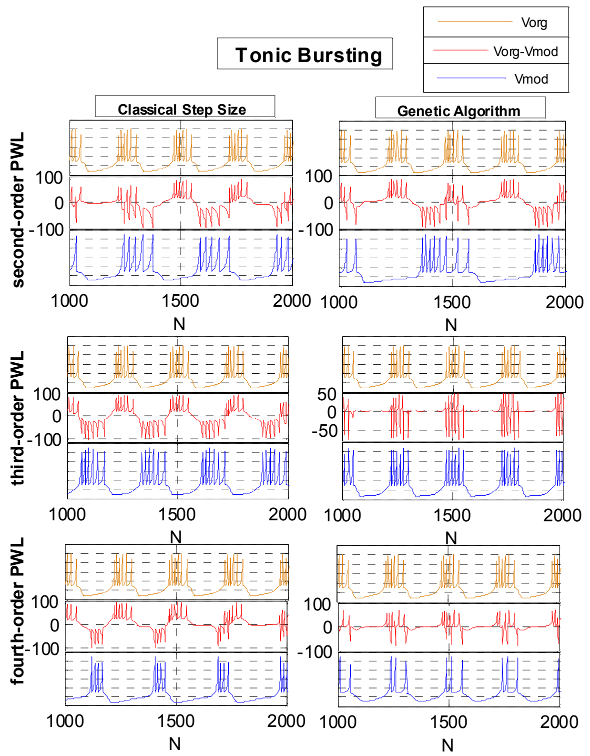


Fig. 2. A design scheme on 'System Generator for DSP' software of the original and the modified IzNMs.

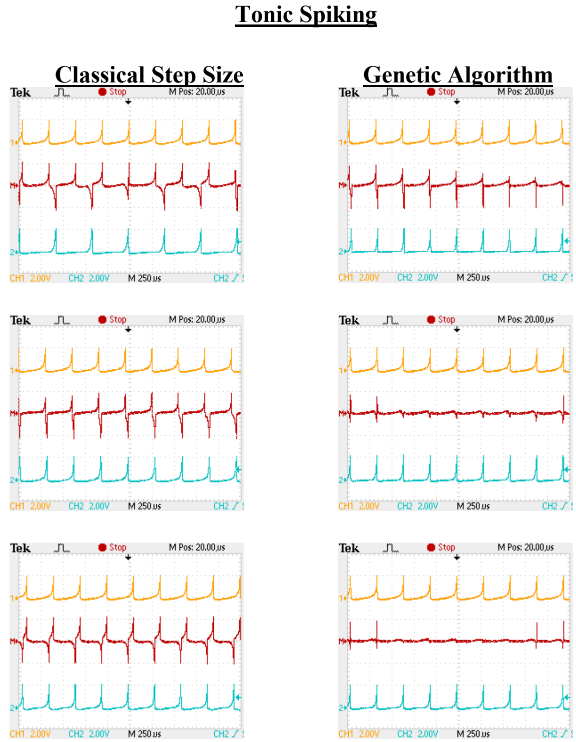


(a)

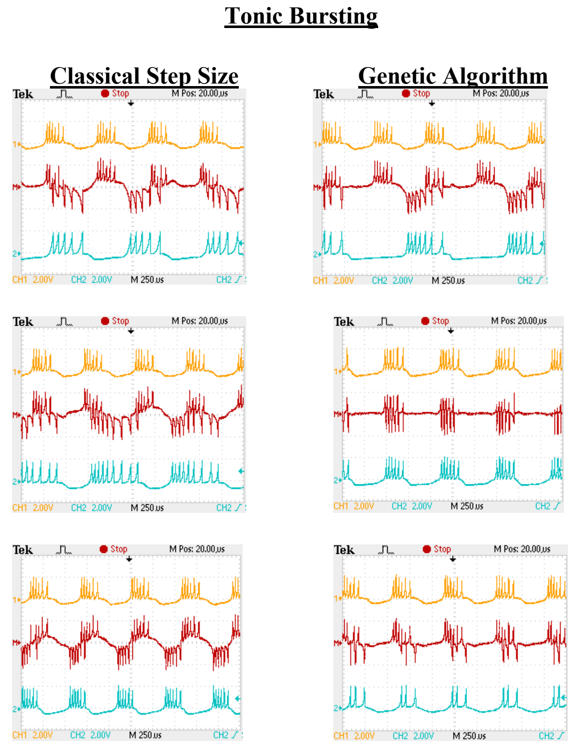


(b)

Fig. 3. The comparative simulation results of the a) second-,b) third- and c) fourth-order PWL function based modified IzNM using the (k) coefficients, which are determined by I) classical step size, II) Genetic algorithm based searching for a) "tonic spiking", b) "tonic bursting" behaviors.



(a)



(b)

Fig. 4. The comparative FPGA implementation results of the a) second-,b) third- and c) fourth-order PWL function based modified IzNM using the (k) coefficients, which are determined by I) classical step size, II) Genetic algorithm based searching**,** a) "tonic spiking", b) "tonic bursting" behaviors.

The numerical simulation tools generally provide an automated transition to the hardware description languages (HDLs) for FPGA platforms. Some program modules such as "System Generator for DSP-XILINX™ (SG_{DSP})" convert the models constructed by the MATLAB-SIMULINK™ program into HDL codes that can be used for FPGA realizations and it allows skipping the "HDL modeling".

In this study, the used FPGA platform is Spartan-3AN, which is a product of the XILINX and the original and modified IzNMs are implemented by using SG_{DSP} program as seen in Fig.2. The simulation results for 'tonic spiking' and 'tonic bursting' dynamical behaviors of the IzNM are illustrated in Fig. 3a and 3b, respectively.

In the implementation process, Euler method is used for the discretization and the discretization constant is chosen as h=0.2. Also, 32-bit fixed point arithmetic (Q 32.18) is used in arithmetic operations. The experiments of the 'tonic spiking' and 'tonic bursting' behaviors are realized by FPGA device for the hardware verifications. The results are seen in Fig. 4.

6. Conclusions

In this study, (k) coefficients of the PWL function based modified Izhikevich neuron models have been determined by utilizing GA for three PWL function approximations. All of the neuron dynamics of original Izhikevich neuron model have been compared with the modified ones. The comparative results are presented in the tables and the study is supported by the numerical simulation analyses and programmable digital hardware verifications.

When the total errors between original and modified Izhikevich neuron model values in the tables are examined, it can be concluded that GA algorithm has the lowest error value. With this study, it is demonstrated both the effectiveness of the GA in these type applications and convergence towards the result of fewer errors.

Additionally, two neuronal behaviors are implemented with FPGA device by using both original and modified IzNM. Thus, the performances of the stochastic and classical parameter determination methods are compared with both various simulation results and experimental verifications. FPGA based experimental implementation results in Fig.4a and 4b are exactly matched with numerical simulation results in the Fig. 3a and 3b.

By inspired of this study, it is expected to be useful applying these methods to the neuronal processes such as the estimation of parameters in the other biological neuron models or the identification the synaptic weights of neuronal networks. Thus, the experimental studies will be implemented more accurately with optimum hardware thanks to the GA based modified models.

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